PAPER

# A Novel Low Complexity Lattice Reduction-Aided Iterative Receiver for Overloaded MIMO

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**SUMMARY** This paper proposes a novel low complexity lattice reduction-aided iterative receiver for overloaded MIMO. Novel noise cancellation is proposed that increases an equivalent channel gain with a scalar gain introduced in this paper, which results in the improvement of the signal to noise power ratio (SNR). We theoretically analyze the performance of the proposed receiver that the lattice reduction raises the SNR of the detector output signals as the scalar gain increases, when the Lenstra–Lenstra–Lova's (LLL) algorithm is applied to implement the lattice reduction. Because the SNR improvement causes the scalar gain to increase, the performance is improved by iterating the reception process. Computer simulations confirm the performance. The proposed receiver attains a gain of about 5 dB at the BER of $10^{-4}$ in a $6 \times 2$ overloaded MIMO channel. Computational complexity of the proposed receiver is about $1/50$ as much as that of the maximum likelihood detection (MLD).
***key words:*** *noise cancelling, hard-input soft-output, iterative detection, serial interference cancellation, low complexity*

## 1. Introduction

A lot of iterative receivers have been proposed and intensively investigated since the emergence of the Turbo codes [1], [2]. Most of them apply soft-input soft-output (SISO) iterative decoding, because SISO plays a crucial role in improving transmission performance. In those decoders, soft input or soft output signals are exchanged between the component decoders where the maximum *a posteriori* probability (MAP) decoding can be performed by using the soft input signals as prior information. The exchange of the soft signals is iterated many times to improve the performance [3]. The Turbo principle has been applied to a detector concatenated with a channel decoder, e.g., Turbo equalizers, where soft input signals and soft output signals are also exchanged between a decoder and a detector [4]–[6].

On the other hand, transmission speed in wireless communication systems has reached several Gbps. The fifth generation cellular system has been standardized to provide users with more than 1 Gbps even in mobile communication environments. Many techniques have been investigated for the provision of such a high speed wireless communication, for instance, multiple input and multiple output (MIMO), multi-user MIMO (MU-MIMO), resource allocation, non-orthogonal multiple access, and so on. Especially, massive MIMO has drawn lots of attention, because massive MIMO is regarded as one of the most powerful techniques for increasing the network throughput [7]. Many independent signal streams are transmitted simultaneously for terminals in systems with massive MIMO. As the number of signal streams for a terminal increases, the receiver on the terminal becomes more complex. Therefore, linear detectors have been considered for the receiver because of their low computational complexity. When linear detectors are applied to the receiver, the number of the signal streams has to be at most equal to that of receive antennas on a terminal, which limits the user throughput. To enhance the user throughput, the number of the signal streams has been considered to exceed that of the receive antennas. Such a system is called “Overloaded MIMO” [8], [9]. Because the performance of linear detectors are so deteriorated due to lack of the freedom in overloaded MIMO channels, non linear detectors have been considered to achieve superior performance [10], [11]. Non-linear detectors concatenated with a channel decoder has been shown to achieve superb transmission performance [9], [11]. SISO iterative reception makes a nonlinear detector concatenated with a channel decoder achieve more superior performance [12], [13]. However, non-linear detectors such as the maximum likelihood detector (MLD) and the MAP estimator execute a brute force search. In principle, the complexity of the soft input MLD is necessary to obtain the soft input signal for one bit [14]. The computational complexity of SISO iterative receivers with nonlinear detectors grows exponentially to a prohibitive revel for terminals as the number of streams increases. The high computational complexity prohibits nonlinear detectors to be applied to terminals. Though some computational complexity reduction techniques have been proposed [15], the complexity is still high due to non-linear signal processing included in the receivers. Because the SISO reception requires a functionality similar to the MLD to obtain the soft input signals provided to the channel decoder, the SISO reception is also difficult to use in terminals.

This paper proposes a novel lattice-reduction (LR)-aided iterative receiver for overloaded MIMO. Though the proposed receiver applies an LR-aided serial interference canceller (SIC) as a linear detector for low computational complexity [16]–[18], the proposed receiver achieves superior performance. Though we reveal that the LR-aided SIC can overcome the difficulty due to the lack of the freedom in

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overloaded MIMO systems†, the bit error rate (BER) performance is much worse than that of the MLD††. To improve the performance, we propose noise cancellation and a scalar gain for the receiver to raise the signal to noise power ratio (SNR) of the linear detector input signals. The noise canceller feeds channel decoder output signals back to increase an equivalent channel gain with a scalar gain introduced in the paper. This feedback loop characterizes the proposed receiver. While conventional iterative receivers also employ similar feedback loops to provide soft information for cancelling interference signals because of the MLD or the MAP estimation in use, the proposed iterative receiver uses the feedback loop for cancelling not interference but the noise in the detector based on the MMSE. While the hard detection signals are provided to the channel decoder, the soft output signals from the decoder are fed back for the noise cancellation. Besides, the scalar gain is optimized for the performance improvement whenever the detection and decoding is iterated. In the proposed receiver, the noise cancellation and the optimized scalar gain enables the receiver to achieve better transmission performnace as the number of the iteration increases. Because the linear detector with the hard input decoding is applied in the proposed receiver, hence, the complexity of the proposed iterative receiver is expected to be much less than that of the conventional iterative receivers.

The next section describes a system model of overloaded MIMO. Section 3 explains the proposed low complexity iterative receiver, and Sect. 4 analyzes the performance of the proposed receiver, theoretically. In Sect. 5, the performance of the proposed receiver is confirmed by computer simulation. Finally, Sect. 6 concludes this paper.

Throughout this paper, $(\mathbf{A})^{-1}$, $\{\mathbf{A}\}_m$ and superscript T denote an inverse matrix, an $m$th column vector of a matrix $\mathbf{A}$, and transpose of a matrix or a vector, respectively. $\mathrm{E}[\beta]$, $\Re[\alpha]$, and $\Im[\alpha]$ represent the ensemble average of a variable $\beta$, a real part and an imaginary part of a complex number $\alpha$.

## 2. System Model

We assume that $N_\mathrm{T}$ signal streams are emitted from $N_\mathrm{T}$ antennas on a transmitter without any precoding, and are received at $N_\mathrm{R}$ antennas on a receiver, where the number of the streams, $N_\mathrm{T}$, is more than that of the receive antennas, $N_\mathrm{R}$. In the system, information bit sequence is provided to a channel encoder, and the output is fed to an interleaver. The interleaver output signals are modulated to modulation signals, e.g., the quaternary phase shift keying (QPSK) or the quadrature amplitude modulation (QAM) signals†††. Let $x_{\mathrm{c},i} \in \mathbb{C}$ denote a signal transmitted from the $i$th antenna, a transmission signal vector $\mathbf{X}_\mathrm{c} \in \mathbb{C}^{N_\mathrm{T}\times 1}$ is defined as $\mathbf{X}_\mathrm{c} = (x_{\mathrm{c},1} \cdots x_{\mathrm{c},N_\mathrm{T}})^\mathrm{T}$. While QPSK modulation signals are expressed as a complex number in a lowpass equivalent system, the signals can be written as a two-dimensional real vector. The transmission signal vector $\mathbf{X}_\mathrm{c}$ is also written as a real vector $\mathbf{X} \in \mathbb{R}^{2N_\mathrm{T}\times 1}$, which is defined as $\mathbf{X} = (x_1\ x_2 \cdots x_{2N_\mathrm{T}})^\mathrm{T}$ where $x_k$ is defined as $x_{2i} = \Im[x_{\mathrm{c},i}]$ and $x_{2i-1} = \Re[x_{\mathrm{c},i}]$, $i = 1, .., N_\mathrm{T}$. When the transmission signal vector is expressed in a real number, received signal vector $\mathbf{Y} \in \mathbb{R}^{2N_\mathrm{R}\times 1}$ in the MIMO channel is expressed as follows††††.

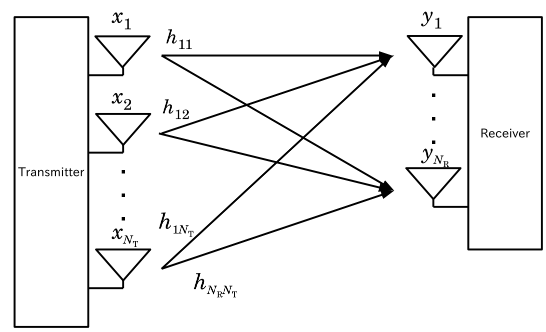


**Fig. 1** System model.

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N} \tag{1}$$

In (1), $\mathbf{N} \in \mathbb{R}^{2N_\mathrm{R}\times 1}$ represents the additive white Gaussian noise (AWGN) vector, which is defined as $\mathbf{N} = (n_1 \cdots n_{2N_\mathrm{R}})^\mathrm{T}$ where $n_{2i-1}$ and $n_{2i}$ denote the AWGNs seen in the I-channel and Q-channel connected to the $i$th receive antenna. In addition, $\mathbf{H} \in \mathbb{R}^{2N_\mathrm{R}\times 2N_\mathrm{T}}$ represents a channel matrix defined in the following equation.

$$\mathbf{H} = \begin{pmatrix} \mathbf{h}_{1,1} & \mathbf{h}_{1,2} & \cdots & \mathbf{h}_{1,N_\mathrm{T}} \\ \mathbf{h}_{2,1} & \mathbf{h}_{2,2} & & \mathbf{h}_{2,N_\mathrm{T}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{h}_{N_\mathrm{R},1} & \mathbf{h}_{N_\mathrm{R},2} & \cdots & \mathbf{h}_{N_\mathrm{R},N_\mathrm{T}} \end{pmatrix}. \tag{2}$$

$\mathbf{h}_{k,i} \in \mathbb{R}^{2\times 2}$ in (2) denotes a submatrix defined as,

$$\mathbf{h}_{k,i} = \begin{pmatrix} \Re[h(k,i)] & -\Im[h(k,i)] \\ \Im[h(k,i)] & \Re[h(k,i)] \end{pmatrix}. \tag{3}$$

In (3), $h(k,i)$ represents a channel impulse response in a lowpass equivalent system between the $k$th receive antenna and the $i$th transmit antenna.

Conventional minimum mean square error (MMSE) filters or SICs are so deteriorated in overloaded MIMO systems due to lack of the freedom. Though non-linear detectors such as the MLD achieve superior performance, a brute force search performed in the MLD is prohibitive in a terminal.

†To our best knowledge, the performance has not been shown before. We think that this is a part of originality in this paper.

††This performance is shown in the Sect. 5. This performance gap is the main problem to overcome in this paper.

†††As is shown in Sect. 3.3, the proposed receiver never takes advantage of characteristics of modulation signals. Hence, the proposed receiver can cope with any modulation signals.

††††System models are sometimes expressed in a real number, especially when the LLL algorithm is applied. When the LLL algorithm is expressed in a real number [19], the other part of the system is also described in a real number for convenience [10]–[13], [15], [17].

In the next section, we propose a low complexity receiver that achieves superior performance even in overloaded MIMO systems, which can be implemented with much lower complexity than that with the MLD.

## 3. Proposed Low Complexity Iterative Receiver

### 3.1 Detection With Noise Cancellation

As is described above, lack of the freedom causes MMSE filters or SICs to degrade in overloaded MIMO systems. Inspired by [16], we take an approach to extend a channel matrix as,

$$\underline{\mathbf{H}}_n = \begin{pmatrix} \mathbf{H} \\ \gamma_n \mathbf{I}_{2N_\mathrm{T}} \end{pmatrix}, \tag{4}$$

where $\gamma_n \in \mathbb{R}$, $\mathbf{I}_{2N_\mathrm{T}} \in \mathbb{R}^{2N_\mathrm{T}\times 2N_\mathrm{T}}$, and $\underline{\mathbf{H}}_n \in \mathbb{R}^{2(N_\mathrm{T}+N_\mathrm{R})\times 2N_\mathrm{T}}$ represent a scalar gain, the identity matrix, and an extended channel matrix at $n$th iteration stage. While the size of the channel matrix $\mathbf{H}$ is $2N_\mathrm{R} \times 2N_\mathrm{T}$, that of the extended channel matrix $\underline{\mathbf{H}}$ is $2(N_\mathrm{R}+N_\mathrm{T}) \times 2N_\mathrm{T}$. In a word, whereas the channel matrix $\mathbf{H}$ is fat in overloaded MIMO systems, the extended channel matrix $\underline{\mathbf{H}}$ stays thin in spite of the combination of $N_\mathrm{T}$ and $N_\mathrm{R}$. This suggests that linear detectors could achieve better performance in channels with the extended channel matrix.

Such a channel is given by extending the received signal vector with zero padding.

$$\underline{\mathbf{Y}}_0 = \begin{pmatrix} \mathbf{Y} \\ \mathbf{0}_{2N_\mathrm{T}} \end{pmatrix} = \underline{\mathbf{H}}_n \mathbf{X} + \begin{pmatrix} \mathbf{N} \\ -\gamma_n \mathbf{X} \end{pmatrix}, \tag{5}$$

where $\underline{\mathbf{Y}}_0 \in \mathbb{R}^{2(N_\mathrm{T}+N_\mathrm{R})\times 1}$ and $\mathbf{0}_{2N_\mathrm{T}} \in \mathbb{R}^{2N_\mathrm{T}\times 1}$ denote an extended received signal vector and the $2N_\mathrm{T}$-dimensional null vector. Whereas the first term in the right hand of (5) is obviously seen as a signal vector, the second term can be regarded as a noise vector. However, the second term consists of not only the AWGN but also the transmission signal vector.

This paper proposes noise cancellation to remove the signal component in the noise vector in (5) with the estimate of the transmission signal vector obtained from the channel decoder. Let $\bar{\mathbf{X}}_{\mathrm{a},n} \in \mathbb{R}^{2N_\mathrm{T}\times 1}$ denote the estimate of the transmission signal vector that is provided by the channel decoder at the $n$th iteration stage, the proposed canceller outputs an extended received signal vector $\underline{\mathbf{Y}}_n \in \mathbb{R}^{2(N_\mathrm{T}+N_\mathrm{R})\times 1}$ at the $n$th iteration stage, which is is expressed as follows.

$$\begin{aligned} \underline{\mathbf{Y}}_n &= \underline{\mathbf{Y}}_0 + \begin{pmatrix} \mathbf{0}_{2N_\mathrm{R}} \\ \gamma_n \bar{\mathbf{X}}_{\mathrm{a},n} \end{pmatrix} \\ &= \underline{\mathbf{H}}_n \mathbf{X}_n + \begin{pmatrix} \mathbf{N} \\ \gamma_n \left(\bar{\mathbf{X}}_{\mathrm{a},n} - \mathbf{X}\right) \end{pmatrix}. \end{aligned} \tag{6}$$

This paper applies the LR to the extended channel matrix for performance improvement.

$$\begin{aligned} \mathbf{\Phi}_n &= \underline{\mathbf{H}}_n \mathbf{T}_n = \mathbf{Q}_n \mathbf{R}_n \\ &= \begin{pmatrix} \mathbf{Q}_n^{(1)} \mathbf{Q}_n^{(2)} \end{pmatrix} \begin{pmatrix} \mathbf{R}_n^{(1)} \\ \mathbf{R}_n^{(2)} \end{pmatrix} \end{aligned} \tag{7}$$

In (7), $\mathbf{\Phi}_n \in \mathbb{R}^{2(N_\mathrm{T}+N_\mathrm{R})\times 2N_\mathrm{T}}$, $\mathbf{T}_n \in \mathbb{R}^{2N_\mathrm{T}\times 2N_\mathrm{T}}$, $\mathbf{Q}_n \in \mathbb{R}^{2(N_\mathrm{T}+N_\mathrm{R})\times 2(N_\mathrm{T}+N_\mathrm{R})}$, and $\mathbf{R}_n \in \mathbb{R}^{2(N_\mathrm{T}+N_\mathrm{R})\times 2N_\mathrm{T}}$ represent an extended channel matrix with lattice reduction, a unimodular matrix[†], a unitary matrix, i.e., $\mathbf{Q}_n^\mathrm{T}\mathbf{Q}_n = \mathbf{I}_{2N_\mathrm{U}+2N_\mathrm{T}}$, and a right upper triangular matrix. In addition, $\mathbf{Q}_n^{(1)} \in \mathbb{R}^{2(N_\mathrm{T}+N_\mathrm{R})\times 2N_\mathrm{T}}$, $\mathbf{Q}_n^{(2)} \in \mathbb{R}^{2(N_\mathrm{T}+N_\mathrm{R})\times 2N_\mathrm{R}}$, $\mathbf{R}_n^{(1)} \in \mathbb{R}^{2N_\mathrm{T}\times 2N_\mathrm{T}}$, and $\mathbf{R}_n^{(2)} \in \mathbb{R}^{2N_\mathrm{R}\times 2N_\mathrm{T}}$ denote a right submatrix and a left submatrix of the unitary matrix $\mathbf{Q}_n$, an upper submatrix and a lower submatrix of the upper triangular matrix $\mathbf{R}_n$, respectively. Because the rank of the channel matrix $\mathbf{\Phi}_n$ is at most $2N_\mathrm{T}$, the submatrices $\mathbf{R}_n^{(1)}$ and $\mathbf{R}_n^{(2)}$ are a upper right triangular matrix and the null matrix, respectively.

This paper applies an SIC, one of linear receivers, for computational complexity reduction, which has the almost same complexity as MMSE filters. As is done in conventional SICs, the received signal is fed to a spatial filter with the submatrix $\mathbf{Q}_n^{(1)}$ as,

$$\begin{aligned} \mathbf{V}_n^{(2N_\mathrm{T})} &= \left(\mathbf{Q}_n^{(1)}\right)^\mathrm{T} \underline{\mathbf{Y}}_n \\ &= \mathbf{R}_n^{(1)} \mathbf{Z}_n + \left(\mathbf{Q}_n^{(1)}\right)^\mathrm{T} \underline{\mathbf{N}}_n. \end{aligned} \tag{8}$$

In (8), $\mathbf{V}_n^{(2N_\mathrm{T})} \in \mathbb{R}^{2N_\mathrm{T}\times 1}$, $\underline{\mathbf{N}}_n \in \mathbb{R}^{2(N_\mathrm{T}+N_\mathrm{R})\times 1}$ and $\mathbf{Z}_n \in \mathbb{R}^{2N_\mathrm{T}\times 1}$ represent a spatial filter output signal vector, the extended noise vector described as the second term in the lower right hand side of (6), and a signal vector in z-domain. They are defined as $\mathbf{V}_n^{(2N_\mathrm{T})} = \left(v_n^{(2N_\mathrm{T})}(1)\cdots v_n^{(2N_\mathrm{T})}(2N_\mathrm{T})\right)^\mathrm{T}$, $\underline{\mathbf{N}}_n = \left(\mathbf{N}^\mathrm{T} \quad \gamma_n \left(\bar{\mathbf{X}}_{\mathrm{a},n} - \mathbf{X}\right)^\mathrm{T}\right)^\mathrm{T}$, and $\mathbf{Z}_n = \mathbf{T}_n^{-1}\mathbf{X}$, respectively. The vector $\mathbf{V}_n^{(2N_\mathrm{T})}$ is fed to the following successive canceller. When the LLL algorithm [19] is used for the LR, the diagonal elements of the upper triangular submatrix $\mathbf{R}_n^{(1)}$ are expected to be aligned in ascending order. Therefore, the successive canceller achieves superior transmission performance without ordering as follows.

$$\begin{aligned} \bar{z}_n(m) &= \lfloor v_n^{(m)}(m)/r_n(m,m) \rceil \\ \mathbf{V}_n^{(m-1)} &= \mathbf{V}_n^{(m)} - \mathbf{R}_n^{(1)}(m)\bar{z}_n(m) \\ & \qquad m = 2N_\mathrm{T}, ..., 2. \end{aligned} \tag{9}$$

In (9), $\lfloor a \rceil \in \mathbb{R}$, and $r_n(m,m) \in \mathbb{R}$ represent a nearest integer of a scalar $a$ and an $(m,m)$th entry of the submatrix $\mathbf{R}_n^{(1)}$. Let $\bar{\mathbf{Z}}_n \in \mathbb{R}^{2N_\mathrm{T}\times 1}$ represent an estimated signal vector in z-domain that is defined as $\bar{\mathbf{Z}}_n = (\bar{z}_n(1)\cdots\bar{z}_n(2N_\mathrm{T}))$, an estimated transmission signal vector $\bar{\mathbf{X}}_{\mathrm{d},n} \in \mathbb{R}^{2N_\mathrm{T}\times 1}$ is obtained as,

$$\bar{\mathbf{X}}_{\mathrm{d},n} = \mathbf{T}_n \bar{\mathbf{Z}}_n, \tag{10}$$

The estimated transmission signal vector $\bar{\mathbf{X}}_{\mathrm{d},n}$ is fed to the channel decoder via the deinterleaver. Because $\bar{\mathbf{Z}}_n$ is a hard

[†]Unimodular matrices have integers as entries, and determinants of which are ±1. When the LLL algorithm is used for the LR, a unimodular matrix transforms a channel matrix to satisfy the requirement of the LLL.

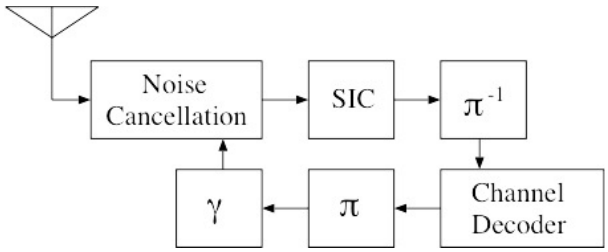


**Fig. 2** Configuration of proposed iterative receiver.

decision signal vector as is expressed in (9), the estimated transmission signal vector $\bar{\mathbf{X}}_{\mathrm{d},n}$ is a hard decision vector. Hence, hard input decoding is carried out in the channel decoder.

The channel decoder can outputs the estimate of the transmission signal sequence, which is transformed to the transmission signal vectors via the interleaver. The transmission signal vectors are provided to the noise canceller where the vector is used as $\bar{\mathbf{X}}_{\mathrm{a},n+1}$ at the $n+1$ iteration stage. The detector at the $n+1$ iteration stage provides the estimated signal vector $\bar{\mathbf{X}}_{\mathrm{d},n+1}$ to the channel decoder. In a word, the detection and the decoding processes are iterated in the proposed iterative receiver. While most conventional iterative receivers apply similar feedback loops, those receivers cancel signals except for a signal of interest with the use of the feedback loop, because those receivers apply the MAP detection or the MLD for signal detection. In a word, they do not touch the signal of interest. In contrast with this, the proposed receiver uses the feedback loop for the noise cancellation, which results in the SNR improvement as is shown in Sect. 3.3. Our detection approach is completely different from that taken in the conventional receivers.

The configuration of the proposed receiver is drawn in Fig. 2. In the figure, $\pi$ and $\pi^{-1}$ denote the interleaver and the deinterleaver, respectively. In the figure, the block named as “Noise cancellation” performs the noise cancellation defined in (6), and the block named as “$\gamma$” optimizes the gain factor based on the soft information fed by the channel decoder, which is defined in (15). As is described above, non-linear signal processing such as a brute force search is not used in the proposed receiver except in the channel decoding, and hard input decoding is applied in the proposed receiver.

### 3.2 Initial Reception

While the proposed receiver feeds the estimate of the transmission signal sequence coming out from the channel decoder back to the canceller, the receiver has nothing to feed back at fist. The receiver puts the null vector $\mathbf{0}_{2N_{\mathrm{R}}} \in \mathbb{R}^{2N_{\mathrm{T}}}$ [†] into the canceller at the first stage, i.e.,

$$\bar{\mathbf{X}}_{\mathrm{a},0} = \mathbf{0}_{2N_{\mathrm{T}}} \tag{11}$$

[†] Although the null vector can be also defined in a complex number, the vector is dared to be defined in a real number, because it is simpler to define all variables in a real number.

The detector at the first stage in the proposed receiver is almost reduced to a conventional LR-aided SIC receiver.

### 3.3 Optimization of Scalar Gain

As is shown in Sect. 3.1, the scalar gain $\gamma_n$ plays an important role in the proposed iterative receiver. The gain $\gamma_n$ should be set to a value that minimizes the BER. If we try to reduce bit errors, we have to assume a situation where the estimate of the transmission signal sequence is not always correct. In a word, some estimated signals are not equal to the transmit signals,

$$\Delta\mathbf{X}_{\mathrm{a},n} = \bar{\mathbf{X}}_{\mathrm{a},n} - \mathbf{X} \neq \mathbf{0}_{2N_{\mathrm{T}}}. \tag{12}$$

$\Delta\mathbf{X}_{\mathrm{a},n} \in \mathbb{R}^{2N_{\mathrm{T}}\times 1}$ denotes an error vector. Although the canceller tries to remove the signal component in the extended noise vector, the noise is still left in the output of the canceller, because of the decoding errors defined in (12).

Since the output vector from the canceller is provided to the spatial filtering in the proposed receiver, on the other hand, the noise should pass through a whitening filter before the spatial filtering. Because the transmission signal vector and the AWGN is not correlated each other, the correlation matrix $\mathbf{C}_n \in \mathbb{R}^{2(N_{\mathrm{T}}+N_{\mathrm{R}})\times 2(N_{\mathrm{T}}+N_{\mathrm{R}})}$ of the extended noise vector can be calculated as,

$$\begin{aligned}\mathbf{C}_n &= \mathrm{E}\left[\underline{\mathbf{N}}_n\left(\underline{\mathbf{N}}_n\right)^{\mathrm{T}}\right] \\ &= \begin{pmatrix} \sigma^2\mathbf{I}_{2N_{\mathrm{R}}} & \mathbf{O} \\ \mathbf{O}^{\mathrm{T}} & \gamma_n^2\mathrm{E}\left[\Delta\mathbf{X}_{\mathrm{a},n}\left(\Delta\mathbf{X}_{\mathrm{a},n}\right)^{\mathrm{T}}\right]\end{pmatrix} \quad (n \geq 1). \end{aligned} \tag{13}$$

In (13), $\sigma^2 \in \mathbb{R}$ and $\mathbf{O} \in \mathbb{R}^{2N_{\mathrm{R}}\times 2N_{\mathrm{T}}}$ denote variance of the AWGN and the null matrix. Because signal errors happen with the uniform distribution and are independent, the submatrix in the lower left position of the correlation matrix $\mathbf{C}_n$ can be rewritten as,

$$\gamma_n^2\mathrm{E}\left[\Delta\mathbf{X}_{\mathrm{a},n}\left(\Delta\mathbf{X}_{\mathrm{a},n}\right)^{\mathrm{T}}\right] = \gamma_n^2\mathrm{E}\left[||\Delta\bar{\mathbf{x}}_{\mathrm{a},n}||^2\right]\mathbf{I}_{2N_{\mathrm{T}}}, \tag{14}$$

$\mathrm{E}\left[||\Delta\bar{\mathbf{x}}_{\mathrm{a},n}||^2\right] \in \mathbb{R}$ in (14) represents a variance of the elements of the vector $\Delta\mathbf{X}_{\mathrm{a},n}$ ; $\mathrm{E}\left[||\Delta\bar{\mathbf{x}}_{\mathrm{a},n}||^2\right] = \mathrm{E}\left[||\Delta\mathbf{x}_{\mathrm{a},n}(1)||^2\right] = \cdots = \mathrm{E}\left[||\Delta\mathbf{x}_{\mathrm{a},n}(2N_{\mathrm{T}})||^2\right]$ where $\Delta\mathbf{x}_{\mathrm{a},n}(m) \in \mathbb{R}$ represents the $m$th element of the vector $\Delta\mathbf{X}_{\mathrm{a},n}$. Therefore, the following setting of $\gamma_n$ can make the noise vector white.

$$\gamma_n = \sqrt{\frac{\sigma^2}{\mathrm{E}\left[||\Delta\bar{\mathbf{x}}_{\mathrm{a},n}||^2\right]}} \tag{15}$$

However, the probability density function (PDF) of $\Delta\mathbf{x}_{\mathrm{a},n}$ is not known. Since we can see that the PDF of $\Delta\mathbf{X}_{\mathrm{a},n}$ is related to the BER, we propose that the variance $\mathrm{E}\left[||\Delta\bar{\mathbf{x}}_{\mathrm{a},n}||^2\right]$ is approximately obtained as follows[††].

$$\mathrm{E}\left[||\Delta\bar{\mathbf{x}}_{\mathrm{a},n}||^2\right] = 4\sigma_{\mathrm{x}}^2\,\mathrm{Pr}\left(\left|\mathrm{LLR}\left(\bar{\mathbf{x}}_{\mathrm{a},n}\right)\right| < \Upsilon\right), \tag{16}$$

[††] Let $p_{\mathrm{a},n}$ denote an error probability of a signal $\bar{\mathbf{x}}_{\mathrm{a},n}$, $\mathrm{E}\left[||\Delta\bar{\mathbf{x}}_{\mathrm{a},n}||^2\right]$ can be rewritten as $\mathrm{E}\left[||\Delta\bar{\mathbf{x}}_{\mathrm{a},n}||^2\right] = p_{\mathrm{a},n}||\bar{\mathbf{x}}_{\mathrm{a},n} - \mathbf{x}_{\mathrm{a},n}||^2$, because $\Delta\bar{\mathbf{x}}_{\mathrm{a},n}$ can be defined as $\Delta\bar{\mathbf{x}}_{\mathrm{a},n} = \bar{\mathbf{x}}_{\mathrm{a},n} - \mathbf{x}_{\mathrm{a},n}$. $\mathbf{x}_{\mathrm{a},n}$ takes $\pm s$

where $\mathrm{LLR}(x) \in \mathbb{R}$, $\sigma_{\mathrm{x}}^2 \in \mathbb{R}$, $\Upsilon \in \mathbb{R}$, and $\Pr(a) \in \mathbb{R}$ represent a log-likelihood ratio of a bit $x$, power of the transmission signal $x_i$, a threshold, and a probability that an event $a$ happens. The threshold is optimized through computer simulation shown in Sect. 5.

If the extended noise vector $\underline{\mathbf{N}}_n$ is made white by the optimized scalar gain, the extended noise vector can be dealt as the AWGN vector. In other words, even though the error vector $\Delta\mathbf{X}_{\mathrm{a},n}$ is included in the extended noise vector, the optimization of the scalar gain makes the extended noise vector behave as if the extended noise vector consisted of only the AWGN. The optimization enables the receiver to achieve the optimum transmission performance.

## 4. Performance Analysis

This paper applies the Lenstra–Lenstra–Lova's (LLL) algorithm to implement the LR [19]. The LLL algorithm makes the matrix $\mathbf{\Phi}_n$ satisfy the following characteristics associated with the orthogonality deficiency $\mathrm{od}(\mathbf{\Phi}_n)$ (see Appendix A).

$$\begin{aligned}\sqrt{1-\mathrm{od}(\mathbf{\Phi}_n)} &= \sqrt{\frac{\det\left[\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{\Phi}_n\right]}{\prod_{m=1}^{2N_{\mathrm{T}}}\|\{\mathbf{\Phi}_n\}_m\|^2}} \\ &\geq \frac{2^{2N_{\mathrm{T}}}}{\sqrt{\prod_{m=1}^{2N_{\mathrm{T}}}\epsilon_m(\delta)}} = \mathrm{c}(\delta), \qquad (17) \\ \epsilon_m(\delta) &= 3 + \frac{\xi(\delta)^m - 1}{\xi(\delta) - 1}\end{aligned}$$

where $\det[\mathbf{B}]$, $\delta \in \mathbb{R}$, $\xi(\delta) \in \mathbb{R}$, and $\mathrm{c}(\delta) \in \mathbb{R}$ represent a determinant of a matrix $\mathbf{B}$, a parameter used in the LLL algorithm, a function with $\delta$ defined as $(\xi(\delta))^{-1} = \delta - \frac{1}{4}$, and a lower bound of the orthogonality deficiency. Even though $\mathrm{c}(\delta)$ looks dependent on the input matrix $\mathbf{\Phi}_n$ in (17), the lower bound $\mathrm{c}(\delta)$ depends only on $\delta$, which is shown in Appendix A. Because $\delta$ is set as $1/4 < \delta < 1$ [17], by increasing $\delta$ near by 1, the right hand side of (17) can come close to 1, which means that the channel is made orthogonal by the LLL algorithm.

As is seen in (9), the performance of the SIC depends on the diagonal entries of the upper triangular matrix $\mathbf{R}_n^{(1)}$. The $m$th diagonal entry $r_n(m,m)$ at the $m$th iteration stage satisfies the following inequality (See Appendix B).

$$\mathrm{M}_{2N_{\mathrm{T}}}\left[\|r_n(m,m)\|^2\right] \geq \sqrt[2N_{\mathrm{T}}]{\mathrm{c}^2(\delta)\prod_{m=1}^{2N_{\mathrm{T}}}\left(\|\mathbf{H}\{\mathbf{T}\}_m\|^2 + \gamma_n^2\|\{\mathbf{T}\}_m\|^2\right)} \qquad (18)$$

In (18), $\mathrm{M}_N[a_m]$ denotes a moving average of $N$ samples, which is defined as $\mathrm{M}_N[a_m] = \frac{1}{N}\sum_{i=1}^{N} a_i$. Therefore, the SNR $\Gamma_n$ of the SIC output signals is guaranteed by the following inequality (see Appendix C) †.

$$\begin{aligned}\Gamma_n &\approx \frac{\mathrm{E}\left[\|r_m(m,m)\|^2\right]}{\mathrm{E}\left[\|n_m\|^2\right]} \\ &\geq \frac{\sqrt[N_{\mathrm{T}}]{\mathrm{c}(\delta)}\gamma_n^2}{\sigma^2} \qquad (19)\end{aligned}$$

Because $\mathrm{c}(\delta)$ depends on the parameter $\delta$ as shown in Appendix A, the lower bound $\mathrm{c}(\delta)$ can be regarded as a constant. Therefore, the SNR is simply improved by the increase of the scalar gain $\gamma_n$. If the scalar $\gamma_n$ is increased because of the performance improvement of the channel decoder, the SIC output performance is improved, which causes the channel decoding performance improvement. Hence, the performance is expected to improve by iterating the SIC and the channel decoding.

wherer $s$ represents amplitude of the signal $\mathbf{x}_{\mathrm{a},n}$. Since we assume that $\bar{\mathbf{x}}_{\mathrm{a},n}$ is not equal to $\mathbf{x}_{\mathrm{a},n}$, $\mathrm{E}\left[\|\Delta\bar{\mathbf{x}}_{\mathrm{a},n}\|^2\right]$ is reduced as $\mathrm{E}\left[\|\Delta\bar{\mathbf{x}}_{\mathrm{a},n}\|^2\right] = 4\sigma_{\mathrm{x}}^2 p_{\mathrm{a},n}$. Because the probability $p_{\mathrm{a},n}$ is not known, the probability is approximately estimated with $\Pr\left(\left|\mathrm{LLR}(\bar{\mathbf{x}}_{\mathrm{a},n})\right| < \Upsilon\right)$ in this paper.

## 5. Computer Simulation

**Table 1** Parameters in computer simulation.

| | |
|---|---|
| $(N_{\mathrm{T}}, N_{\mathrm{R}})$ | (6, 2) |
| Modulation | QPSK/Single Carrier |
| Packet length | 1,536 bits |
| Channel | Rayleigh fading |
| Channel estimation | Ideal |
| $\delta$ in LLL | 0.9 |
| Error correction coding | Convolutional code ($R = 1/2$, $K = 3$) |
| Interleaver | Block interleaver ($24 \times 64$) |
| Decoding | Hard input Viterbi algorithm |
| Number of iterations | 4 |

The performance of the proposed receiver is evaluated by computer simulation in an $6\times 2$ overloaded MIMO channel. Overloading ratio $\frac{N_{\mathrm{T}}}{N_{\mathrm{R}}} = 3$. Rayleigh fading based on Jakes' model is applied as a channel model. As is described above, the LLL algorithm and QPSK are used for the LR and the modulation, respectively. The rate half convolutional code with constraint length of 3 [20] is applied for the channel coding. The soft output Viterbi algorithm [20] is used to generate the estimate of the transmission signal sequence, which is fed back to the SIC as the estimate of the transmission signal vectors via the interleaver. The number of iteration is set to 4, because the performance seems to converge at the 4th iterations, which is shown in the following. Simulation parameters are summarized in Table 1.

### 5.1 SNR of SIC Output Signals

The SNR distribution of the output signals from the SIC is shown in the Fig. 3 where the horizontal axis is the SNR and the vertical axis is the cumulative distribution function (CDF). In the figure, "$i$th" denotes the performance when the detection is $i$ times iterated. $E_{\mathrm{b}}/N_0$ is 20 dB. In addition, "no-iteration" means the performance at the first stage. The

†Performance degradation caused by the error propagation in SICs is not taken into account in this performance analysis.

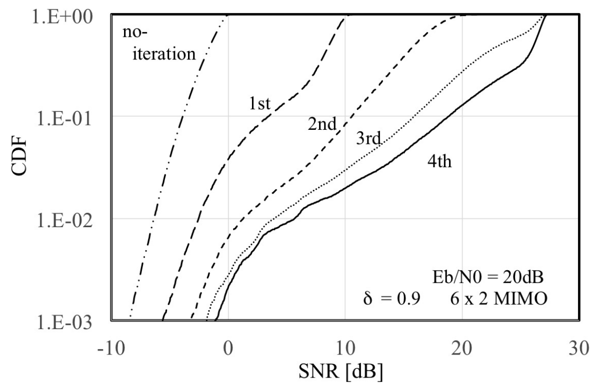


**Fig. 3** SNR distribution.

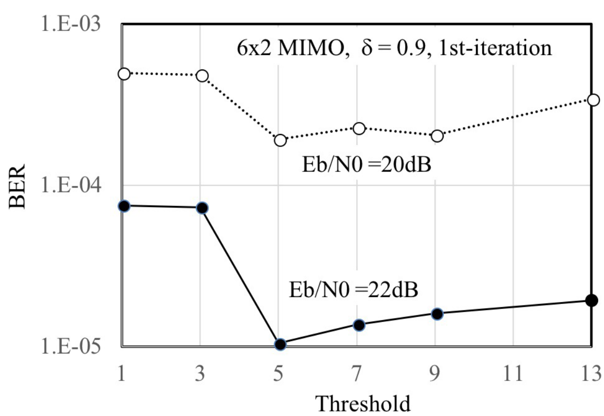


**Fig. 4** BER vs. threshold.

figure shows that the SNR is improved as the number of the iterations increases. While the average of the SNR is improved about 20 dB, the gain is reduced to about 8 dB at the 0.1% outage, which limits the transmission performance improvement.

## 5.2 Optimization of Threshold

The BER performance of the proposed receiver is evaluated with respect to the threshold $\Upsilon$ defined in (16), which is shown in Fig. 4. This performance is obtained at the 1st iteration stage. As is shown in the figure, the BER is minimized at the threshold of 5 in spite of the $E_b/N_0$†. This value was used in the previous section and is used in the following performance evaluation.

## 5.3 BER Performance

Figure 5 shows the BER performance of the SIC output signals with respect to the number of the iterations. The horizontal axis and the vertical axis are $E_b/N_0$ and the BER in the figure, respectively. In the figure, the performance of the MLD is added as a reference. "no-iteration" means

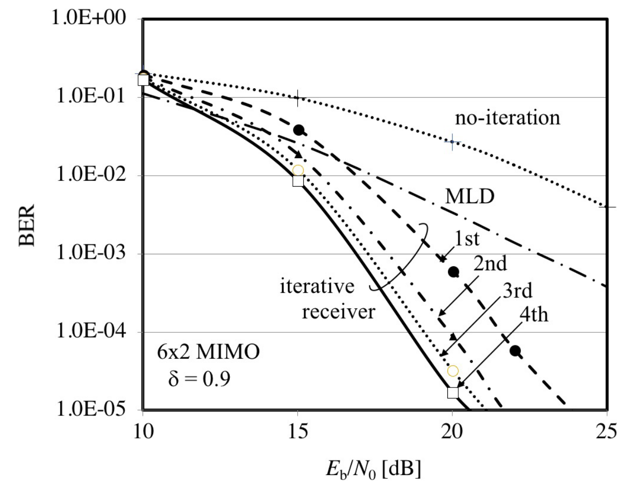


**Fig. 5** BER of demodulator output.

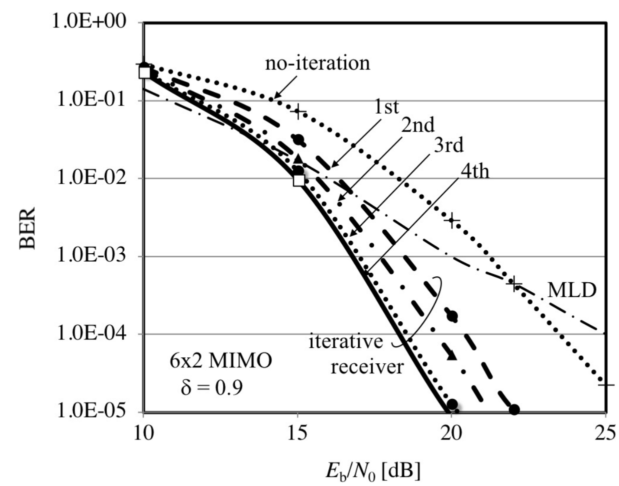


**Fig. 6** BER of decoder output.

the performance of the proposed receiver at the first stage. Although the MLD outperforms the proposed receiver with "no-iteration", the proposed receiver achieves better BER performance than the MLD as the number of iterations increases, as far as $E_b/N_0$ is greater than about 18 dB.

Figure 6 shows the BER performance of the channel decoder output signals with respect to the number of the iterations. Similar to the performance of the SIC output signals shown in Fig. 5††, the performance is improved as the number of the iterations increases. Actually, it takes 3 or 4 itaration for the performnace to converge. In fact, the receiver achieves a gain of about 5 dB at the BER of $10^{-4}$ by the 4 times iterations.

## 5.4 Performance Comparison

Whereas the soft output signals from the channel decoder is

†We can see a coincidence that the optimum threshold value is the same to the free distance of the code (rate=$\frac{1}{2}$, constraint length $K = 3$). It is one of our future works to analyze the coincidence.

††The size of the block interleaver is optimized for the proposed receiver. In addition, $\delta = 0.9$ is used in the LLL algorithm. Those could be the reason why the performance of the receiver with no iteration is better than that of the MLD.

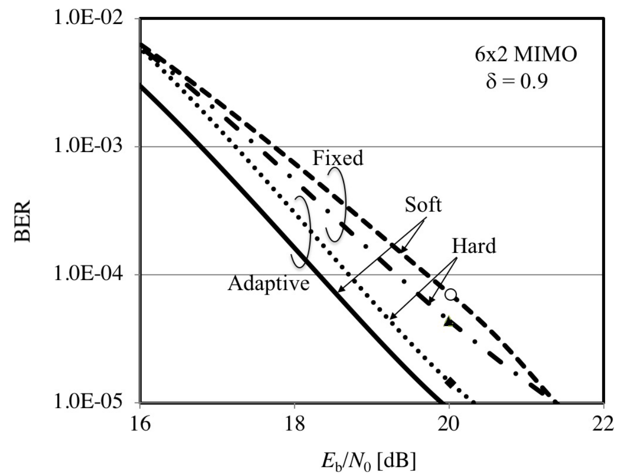


**Fig. 7** Comparison with other variants.

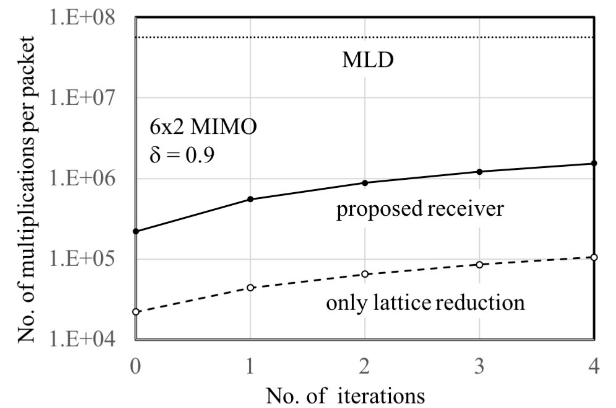


**Fig. 8** Complexity of receivers.

fed beck to the canceller via the interleaver, the hard output signals can be also used for the canceller. Therefore, the performance of them is compared in the Fig. 7. In addition, the performance of the proposed receiver with a fixed scalar gain is compared with that with the adaptive gain defined in (15). In the receiver with a fixed gain, the gain $\gamma_n$ is set as $\gamma_n = \sigma$. Obviously, the proposed receiver with the adaptive gain defined in (15) outperforms the other variants. Especially, the adaptive gain makes proposed receiver achieve a gain of about 1.5 dB when the soft output signals are used for the cancellation.

### 5.5 Complexity

The complexity of the proposed receiver is compared with the MLD in terms of the number of multiplications per packet in Fig. 8. In the figure, the horizontal axis means the number of the iterations and the vertical axis is the number of multiplications executed in a packet. The number of multiplications that the LLL algorithm executes in the proposed receiver is drawn to show the computational complexity of the filtering. Though the packet length is not so long. the complexity needed for the LLL is only 10% of the proposed receiver. Although the complexity of the proposed receiver grows as the number of the iterations increases, the complexity of the receiver with 4 iterations is about 1/50 as much as that of the MLD.

The computational complexity of the proposed receiver is almost independent of modulation schemes, since the proposed receiver is classified into linear receivers. In contrast, the computational complexity depends on the number of the streams. Since the complexity of the SIC is in proportion to $N_T^2$, the complexity of the proposed receiver is proportional to $N_T^2$ multiplied with the number of the iterations. In the complexity evaluation, the complexity of the LLL algorithm is not taken into account, because the LLL algorithm is executed only once in a packet. On the other hand, the complexity of the MLD is proportional to $M^{N_T}$ to demodulate $N_T$ symbols when the modulation scheme with $M$ bit/Hz is applied. Obviously, the proposed receiver becomes much less complex than the MLD as the number of the streams or the cardinality of the modulation signals increases. This performance is proven in Fig. 8. However, the LLL algorithm has a polynomial complexity. Even if the LLL algorithm is executed once in a packet, the complexity for the LLL might dominate that of the receiver as the number of the streams grows high. Such complexity analysis is one of our future works.

## 6. Conclusion

This paper has proposed a novel low complexity lattice reduction-aided iterative receiver for overloaded MIMO. The proposed receiver feeds back output signals from a channel decoder for noise cancellation. A scalar gain is introduced in the noise cancellation for noise whitening. The scalar gain is increased by the improvement of the BER performance of the channel decoder output signals. Because the BER performance of the decoder output signals is usually better than that of the input signals, the scalar gain gets bigger than that at the previous iteration stage in the proposed receiver. It is theoretically analyzed that the SNR of the SIC used in the proposed receiver is improved by the increase of the scalar gain. In other words, the increase of the scalar gain enhances the detection performance, which improves the performance of the decoder output signals. Therefore, the BER performance is more improved as the number of the interactions increases. Since the proposed receiver applies the LLL-aided SIC and hard input channel decoding even in overloaded MIMO channels, the complexity of the proposed receiver is much less than the that of MLD.

The BER performance is verified in a $6 \times 2$ overloaded MIMO channel. The proposed receiver attains a gain of about 5 dB at the BER of $10^{-4}$ by the 4 times iterations. The complexity of the proposed receiver with the 4 times iterations is about 1/50 as much as that of the MLD.

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## Appendix A: Orthogonality Deficiency

Even if the rectangular matrix $\mathbf{R}_n$ is provided to the LLL algorithm, the LLL algorithm deals with only the right upper triangular matrix $\mathbf{R}_n^{(1)}$. When the $(l, m)$ entry of the right upper triangular matrix $\mathbf{R}_n^{(1)}$ is denoted by $r_n(l, k)$, the lattice reduction makes the right upper triangular matrix $\mathbf{R}_n^{(1)}$ satisfy the following property.

$$|r_n(l,k)| \le \frac{1}{2}|r_n(l,l)| \qquad (1 \le l < k < 2N_{\mathrm{T}}) \qquad \text{(A· 1)}$$

$$\delta \|r_n(l-1,l-1)\|^2 \le \|r_n(l,l)\|^2 + \|r_n(l-1,l)\|^2. \qquad \text{(A· 2)}$$

If (A· 1) is taken into account, when $\delta$ is set to be bigger than 0.25, (A· 2) can be rewritten as follows.

$$\begin{aligned}\|r_n(l,l)\|^2 &\le \left(\delta - \frac{1}{4}\right)^{-1} \|r_n(l+1,l+1)\|^2 \\ &\le \left(\delta - \frac{1}{4}\right)^{l-h} \|r_n(h,h)\|^2 \ \ (h > l)\end{aligned} \qquad \text{(A· 3)}$$

By using the inequality written in (A· 3), we can show that the norm of the $l$th column in the upper triangular matrix $\mathbf{R}_n^{(1)}$ satisfies the following inequality.

$$\begin{aligned}\|\{\mathbf{R}_n^{(1)}\}_l\|^2 &= \|r_n(l,l)\|^2 + \sum_{k=1}^{l-1}\|r_n(k,l)\|^2 \\ &\le \|r_n(l,l)\|^2 + \frac{1}{4}\sum_{k=1}^{l-1}\|r_n(k,k)\|^2 \\ &\le \|r_n(l,l)\|^2 + \frac{1}{4}\sum_{k=1}^{l-1}\left(\delta - \frac{1}{4}\right)^{k-l}\|r_n(l,l)\|^2 \\ &\le \frac{1}{4}\epsilon_l(\delta)\|r_n(l,l)\|^2,\end{aligned} \qquad \text{(A· 4)}$$

where $\epsilon_l(\delta)$ is defined as,

$$\epsilon_l(\delta) = \left(3 + \frac{\xi^l - 1}{\xi - 1}\right). \qquad \text{(A· 5)}$$

Furthermore, the denominator in the middle of (17) is rewritten by using the inequality in (A· 4).

$$\begin{aligned}\prod_{m=1}^{2N_{\mathrm{T}}}\|\{\mathbf{\Phi}_n\}_m\|^2 &= \prod_{m=1}^{2N_{\mathrm{T}}}\|\{\mathbf{R}_n^{(1)}\}_m\|^2 \\ &\le \prod_{m=1}^{2N_{\mathrm{T}}}\epsilon_m(\delta)\|r_n(m,m)\|^2\end{aligned} \qquad \text{(A· 6)}$$

(17) can be derived by using (A· 6).

$$\sqrt{1-\mathrm{od}(\mathbf{\Phi}_n)} = \sqrt{\frac{\det\left[\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{\Phi}_n\right]}{\prod_{m=1}^{2N_{\mathrm{T}}} \|\{\mathbf{R}_n^{(1)}\}_m\|^2}} = \sqrt{\frac{\prod_{m=1}^{2N_{\mathrm{T}}} \|r_n(m,m)\|^2}{\prod_{m=1}^{2N_{\mathrm{T}}} \|\{\mathbf{R}_n^{(1)}\}_m\|^2}} \geq \frac{2^{2N_{\mathrm{T}}}}{\sqrt{\prod_{m=1}^{2N_{\mathrm{T}}} \epsilon_m(\delta)}} = \mathrm{c}(\delta) \quad (\mathrm{A}\cdot 7)$$

Because $\epsilon_m(d)$ is a function with $\delta$ as is defined in (A· 5), $\mathrm{c}(\delta)$ in (A· 7) is not dependent on the input matrix, but only on the parameter $\delta$.

## Appendix B: Upper Bound of the Equivalent Channel Gain

When the extended channel matrix with lattice reduction, $\mathbf{\Phi}_n$, is given, (17) can be calculated as,

$$\sqrt{\frac{\prod_{m=1}^{2N_{\mathrm{T}}} \|r_n(m,m)\|^2}{\prod_{m=1}^{2N_{\mathrm{T}}} \|\{\mathbf{R}_n^{(1)}\}_m\|^2}} \geq c(\delta) \quad (\mathrm{A}\cdot 8)$$

Because arithmetic mean is always bigger than its geometric mean, the following relationship can be derived.

$$\prod_{m=1}^{2N_{\mathrm{T}}} \|r_n(m,m)\|^2 \leq \left(\frac{1}{2N_{\mathrm{T}}}\sum_{m=1}^{2N_{\mathrm{T}}} r_n(m,m)\right)^{2N_{\mathrm{T}}} \quad (\mathrm{A}\cdot 9)$$

From the definition of the right upper triangular matrix $\mathbf{R}_n^{(1)}$, the following can be obtained.

$$\mathrm{c}(\delta)^2 \prod_{m=1}^{2N_{\mathrm{T}}} \|\{\mathbf{R}_n^{(1)}\}_m\|^2 = \mathrm{c}(\delta)^2 \prod_{m=1}^{2N_{\mathrm{T}}} \left(\|\mathbf{H}\{\mathbf{T}\}_m\|^2 + \gamma_n^2\|\{\mathbf{T}\}_m\|^2\right) \quad (\mathrm{A}\cdot 10)$$

(18) can be easily derived from (A· 9) and (A· 10).

## Appendix C: Channel Gain Guaranteed by the LLL Algorithm

$\|\mathbf{H}\{\mathbf{T}_n\}_m\|^2$, $\gamma_n^2$ and, $\|\{\mathbf{T}_n\}_m\|$ are positive. Hence, the following term included in the right hand side of (18) can be rewritten

$$\prod_{m=1}^{2N_{\mathrm{T}}} \left(\|\mathbf{H}\{\mathbf{T}_n\}_m\|^2 + \gamma_n^2\|\{\mathbf{T}_n\}_m\|^2\right) \geq \prod_{m=1}^{2N_{\mathrm{T}}} \|\mathbf{H}\{\mathbf{T}_n\}_m\|^2 + \left(\gamma_n^2\right)^{2N_{\mathrm{T}}} \prod_{m=1}^{2N_{\mathrm{T}}} \|\{\mathbf{T}_n\}_m\|^2 \quad (\mathrm{A}\cdot 11)$$

On the other hand, the determinant of $(\mathbf{H}\mathbf{T}_n)^{\mathrm{T}}\mathbf{H}\mathbf{T}_n$ is calculated as $\det\left[(\mathbf{H}\mathbf{T}_n)^{\mathrm{T}}\mathbf{H}\mathbf{T}_n\right] = \det\left[\mathbf{T}_n^{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\mathbf{H}\mathbf{T}_n\right] = \|\det[\mathbf{T}]\|^2\det\left[\mathbf{H}^{\mathrm{T}}\mathbf{H}\right]$. Because the determinant of unimodular matrix is defined as $\det[\mathbf{T}_n] = \pm 1$, unimodular matrices do not change the determinant of $\mathbf{H}^{\mathrm{T}}\mathbf{H}$, i.e., $\det\left[(\mathbf{H}\mathbf{T}_n)^{\mathrm{T}}\mathbf{H}\mathbf{T}_n\right] = \det\left[\mathbf{H}^{\mathrm{T}}\mathbf{H}\right]$. On the other hand, if an input matrix $\mathbf{A}$ is orthogonal, $\sqrt{1-\mathrm{od}(\mathbf{A})}$ becomes 1, which is the upper bound of $\sqrt{1-\mathrm{od}(\mathbf{A})}$. If (17) is taken into account, the following can be derived.

$$\prod_{m=1}^{N} \|\mathbf{H}\{\mathbf{T}_n\}_m\|^2 \geq \det\left[(\mathbf{H}\mathbf{T}_n)^{\mathrm{T}}\mathbf{H}\mathbf{T}_n\right] = \det\left[\mathbf{H}^{\mathrm{T}}\mathbf{H}\right] \quad (\mathrm{A}\cdot 12)$$

By taking account of (A· 12), (A· 11) is rewritten as,

$$\prod_{m=1}^{2N_{\mathrm{T}}} \|\mathbf{H}\{\mathbf{T}_n\}_m\|^2 + \left(\gamma_n^2\right)^{2N_{\mathrm{T}}} \prod_{m=1}^{2N_{\mathrm{T}}} \|\{\mathbf{T}_n\}_m\|^2 \geq \det\left[\mathbf{H}^{\mathrm{T}}\mathbf{H}\right] + \left(\gamma_n^2\right)^{2N_{\mathrm{T}}} \quad (\mathrm{A}\cdot 13)$$

When the system is overloaded, i.e., $N_{\mathrm{T}} > N_{\mathrm{R}}$, the matrix $\mathbf{H}^{\mathrm{T}}\mathbf{H}$ has some zero eigenvalues. Therefore, the determinant of $\mathbf{H}^{\mathrm{T}}\mathbf{H}$ is reduced to zero. On the other hand, as is defined in (8), multiplying the matrix $\mathbf{Q}_n^{(1)}$ with the received signal vector $\underline{\mathbf{Y}}_n$ produces the vector $\mathbf{V}_n^{(2N_{\mathrm{T}})}$. While the multiplication transforms the extended channel matrix into the right upper triangular matrix, the multiplication keeps the power of the vectors, because the matrix is a the right submatrix of the unitary matrix $\mathbf{Q}_n$. As is described in the Sect. 3.3, on the other hand, the scalar gain $\gamma_n$ makes the extended noise vector $\underline{\mathbf{N}}_n$ white. Hence, the power of the noise included in the SIC is the same to that in the AWGN. In a word. $\mathrm{E}\left[\|n_m\|^2\right] = \sigma^2$.

If the inequality in (A· 13), the following discussion and the power of the extended noise vector described above are taken into account, the inequality in (19) is easily derived from (18).

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