PAPER

Matrix Completion ESPRIT for DOA Estimation Using Nonuniform Linear Array

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SUMMARY A novel matrix completion ESPRIT (MC-ESPRIT) algorithm is proposed to estimate the direction of arrival (DOA) with nonuniform linear arrays (NLA). By exploiting the matrix completion theory and the characters of Hankel matrix, the received data matrix of an NLA is tranformed into a two-fold Hankel matrix, which is a treatable for matrix completion. Then the decision variable can be reconstructed by the inexact augmented Lagrange multiplier method. This approach yields a completed data matrix, which is the same as the data matrix of uniform linear array (ULA). Thus the ESPRIT-type algorithm can be used to estimate the DOA. The MC-ESPRIT could resolve more signals than the MUSIC-type algorithms with NLA. Furthermore, the proposed algorithm does not need to divide the field of view of the array compared to the existing virtual interpolated array ESPRIT (VIA-ESPRIT). Simulation results confirm the effectiveness of MC-ESPRIT.
key words: *direction of arrival (DOA), nonuniform linear array, matrix completion*

1. Introduction

Array signal processing is being widely used to estimate the parameters of signals in numerous areas such as sonar, radar and wireless communications [1]. And the direction of arrival (DOA) estimation problem, which has drawn considerable attention, is an important aspect of array signal processing. In this paper, we consider the case of Nonuniform Linear Arrays (NLA). In practice, some of the sensors in a uniform array may stop functioning, which yields an NLA. In this case, the array should be treated as nonuniform in order to optimize the DOA sestimator. Another application of NLA is the design of high performance and low cost arrays with reduced number of sensors. Reducing the number of sensors decreases the production cost as well as the computational time. This is due to the fact that nonregular geometry provides almost the same Root Mean Square Error (RMSE) performance as the equivalent Uniform Linear Array (ULA) with the same number of array elements.

Subspace decomposition-based methods such as multiple signal classification (MUSIC) based algorithms [2] have been proposed. However, MUSIC based algorithms are extremely in terms of computational complexity due to spectral peaks search to obtain the DOAs. Since the ESPRIT [3] can avoid the spectral searching procedure of the MUSIC based algorithms and thus reduce much of the computational load, various ESPRIT-type algorithms [4], [5] have been presented to estimate the DOA. But the ESPRIT-type algorithms require special sensor array configuration, and it can be applied only to uniform linear arrays (ULA). However, it was found that linear arrays with nonuniform spacing could achieve higher performance for DOA estimation [6], [7]. In order to extend the ESPRIT-type methods to nonuniform linear arrays (NLA), a series of methods with interpolated array [8], [9] are proposed. Their basic idea is using the interpolation technique to estimate the outputs of a virtual array from the received data of real array, so the outputs of a virtual array can be used to estimate DOA through ESPRIT, so they are called as virtual interpolated array ESPRIT (VIA-ESPRIT). However, the interpolated ESPRIT procedure must divide the field of view of the array into several sectors, if the DOA of target is beyond a certain sector, there should be a sharp drop in angle estimated performance. [10] dealt with the problem of DOA estimation for multiple uncorrelated signals incident on partially augmentable antenna arrays, and the maximum-entropy (ME) positive-definite (p. d.) completion algorithm for partially specified Toeplitz covariance matrices was proposed for NLA.

Matrix completion (MC) [11] is a new technique which can be applied to recover a low-rank matrix from subset of the matrix entries by minimizing the nuclear norm of the matrix. Many algorithms have been proposed to solve the MC problem. According to the nature of optimization problem, two main categories are formed. One in which the nuclear norm minimization is explored, such as singular value thresholding (SVT) [12], accelerated proximal gradient (APG) [13] and augmented Lagrange multiplier (ALM) [14], et al. The other in which an approximation error objective function on a Grassmann manifold is minimized, such as subspace evolution and transfer (SET) [15] and Grassmanian rank-one update subspace estimation (GROUSE) [16], et al. Compared with ULA, the received data of NLA can be considered as a low-rank matrix. Therefore, through solving the MC problem, the received data of NLA is able to recover a completed data matrix, which is the same as the data matrix of ULA.

In this paper, we combine the theory of matrix completion with the ESPRIT, and a novel algorithm called matrix completion ESPRIT (MC-ESPRIT) is proposed to deal with the problem of the interpolated ESPRIT method. By reshap-

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ing the received data of NLA into a low-rank Hankel matrix, a virtual completed data matrix can be obtained by solving a MC problem. Because the virtual completed data matrix is the same as the received data of ULA with the same array aperture, ESPRIT can be exploited to estimate DOA finally.
Notation: $(\cdot)^T$ denotes the transpose operator; rank$[\cdot]$ denotes the rank of a matrix; $\langle\cdot\rangle$ denotes the inner product of matrixs; C denotes the plural sets.

2. Problem Formulation

Consider an NLA consisting of N omnidirectional sensors whose positions $d = [d_1, d_2, \cdots, d_N]$ are the integer times to $\lambda/2$ (λ is the wavelength of the signal), and take the first sensor as the reference that means $d_1 = 0$. Assume that there are P noncoherent far-field targets within the same range locating at $\theta_p, p = 1, 2, \cdots, P$, where θ_p is DOA of the pth target with respect to the array normal. Therefore, the outputs of the entire sensors can be expressed as

$$\begin{aligned} \boldsymbol{x}(t_l) &= [x_1(t_l), x_2(t_l), \cdots, x_N(t_l)]^T \\ &= \boldsymbol{A}\boldsymbol{s}(t_l) + \boldsymbol{n}(t_l), \qquad l = 1, 2, \cdots, L \end{aligned} \tag{1}$$

where $\boldsymbol{x}(t_l) \in C^{N\times 1}$, $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_P)]$ is an $N \times P$ matrix composed of P steering vectors, $\boldsymbol{a}\left(\theta_p\right) = \left[1, e^{j\frac{2\pi d_2}{\lambda}\sin\theta_p}, \cdots, e^{j\frac{2\pi d_N}{\lambda}\sin\theta_p}\right]^T$, $\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_P(t)]^T$ are the envelopes of the reflected signals. The noise vector $\boldsymbol{n}(t)$ is modeled as a zero-mean, spatially complex white Gaussian distribution with covariance matrix $\sigma_n^2\boldsymbol{I}_N$. L is the number of snapshots.

In order to use MC theory to recover a completed data matrix of virtual ULA, we can set the received data as zero for the position without sensors. Assume that a virtual ULA has the same array aperture with the NLA, and d_N denotes the array aperture of the NLA. So the array aperture of the NLA is $d_N = (M-1)\times\lambda/2$ $(M > N)$, where M denotes the number of sensors for the virtual ULA. Denote the received data of the virtual ULA as $\boldsymbol{y}(t_l)$, so the dimension of $\boldsymbol{y}(t_l)$ is $M\times 1$, For example, if the four sensors of NLA are situated at positions $d = [0, 1, 3, 6]\times\lambda/2$, its geometry and the geometry of virtual ULA are illustrated in Fig. 1.

Then the received data of the virtual ULA can be written as

$$\boldsymbol{y}(t_l) = [x_1(t_l), x_2(t_l), 0, x_3(t_l), 0, 0, x_4(t_l)]^T \tag{2}$$

Therefore, the received data matrix of virtual ULA with

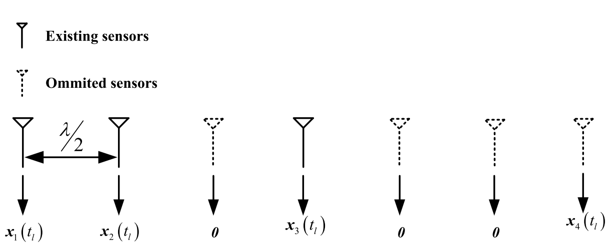


Fig. 1 The geometry of NLA with $d = [0, 1, 3, 6] \times \lambda/2$.

multiple snapshots can be expressed as

$$\boldsymbol{Y} = [\boldsymbol{y}(t_1), \boldsymbol{y}(t_2), \cdots, \boldsymbol{y}(t_L)] \tag{3}$$

3. MC-ESPRIT

First of all, we recall the basic theory of MC. The strong incoherent property (SIP) is one of the basic conditions for MC, the requirement of the recovery matrix is that the singular value vector of the matrix is independent to the orthonormal basis of its space. As long as there are no all zero rows or columns, the SIP is generally satisfied. If the data matrix is a low-rank matrix and meets the condition of SIP, one could recover the data matrix by solving the following optimization problem

$$\begin{array}{ll} \text{minimize} & \text{rank}(\boldsymbol{M}) \\ \text{subject to} & \text{P}_\Omega(M) = \text{P}_\Omega(\boldsymbol{Y}) \end{array} \tag{4}$$

where $\boldsymbol{M}$ is the decision variable and rank$(\boldsymbol{M})$ is equal to the rank of the matrix $\boldsymbol{M}$. Ω is the set of effective element positions for $\boldsymbol{Y}$. For arbitrary $(i, j) \in \Omega$, there always exists $\boldsymbol{Y}_{ij}$ belong to $\boldsymbol{Y}$, and $\boldsymbol{Y}_{ij}$ is non-zero element, so we call is an effective element position. $\text{P}_\Omega(\boldsymbol{M})$ is the orthogonal projection of $\boldsymbol{M}$ onto the subspace of matrices that vanish outside Ω. Similar to the l_0-norm problem of compressive sensing (CS), the above problem is NP-hard, and the NP refers to a nondeterministic polynomial. The so-called uncertainty is that a certain number of operations can be used to solve problems that can be solved in polynomial time. In general, the NP problems are problems where the correctness of their solution can be "easily checked", which means there exists a polynomial checking algorithm. If all the problems in NP are reduced by turing to one problem, the problem is called NP-hard. Therefore, Eq. (4) can be transferred as

$$\begin{array}{ll} \text{minimize} & \|\boldsymbol{M}\|_* \\ \text{subject to} & \text{P}_\Omega(\boldsymbol{M}) = \text{P}_\Omega(\boldsymbol{Y}) \end{array} \tag{5}$$

where $\|\boldsymbol{M}\|_*$ denotes the nuclear norm (or sum of all singular values) of a matrix $\boldsymbol{M}$.

One can observe that the received data matrix $\boldsymbol{Y}$ have rows whose elements are all zero, which does not satisfy the condition of SIP. Thus, we cannot use the above optimization procedure to recover the completed data matrix of virtual ULA. According to reference [17], $\boldsymbol{Y}$ should be transposed, we have $\tilde{\boldsymbol{Y}} = \boldsymbol{Y}^T$. And then a two-fold Hankel matrix can be defined as

$$\boldsymbol{Y}_e = \begin{bmatrix} \tilde{\boldsymbol{Y}}_1 & \tilde{\boldsymbol{Y}}_2 & \cdots & \tilde{\boldsymbol{Y}}_{L-k_1+1} \\ \tilde{\boldsymbol{Y}}_2 & \tilde{\boldsymbol{Y}}_3 & \cdots & \tilde{\boldsymbol{Y}}_{L-k_1+2} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\boldsymbol{Y}}_{k_1} & \tilde{\boldsymbol{Y}}_{k_1+1} & \cdots & \tilde{\boldsymbol{Y}}_L \end{bmatrix} \tag{6}$$

Where k_1 $(1 \le k_1 \le L)$ is called pencil parameter, and each block matrix of $\boldsymbol{Y}_e$ is a $k_2 \times (M - k_2 + 1)$ Hankel matrix, which is defined as

$$\tilde{\boldsymbol{Y}}_m = \begin{bmatrix} \tilde{\boldsymbol{Y}}_{m,1} & \tilde{\boldsymbol{Y}}_{m,2} & \cdots & \tilde{\boldsymbol{Y}}_{m,M-k_2+1} \\ \tilde{\boldsymbol{Y}}_{m,2} & \tilde{\boldsymbol{Y}}_{m,3} & \cdots & \tilde{\boldsymbol{Y}}_{m,M-k_2+2} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\boldsymbol{Y}}_{m,k_2} & \tilde{\boldsymbol{Y}}_{m,k_2+1} & \cdots & \tilde{\boldsymbol{Y}}_{m,M} \end{bmatrix} \tag{7}$$

where m satisfies $1 \le m \le L$, and $1 \le k_2 \le M$ is another pencil parameter. $\tilde{\boldsymbol{Y}}_{m,k}$ denotes the $(m,k)th$ element of the matrix $\tilde{\boldsymbol{Y}}$.

According to the expression of $\tilde{\boldsymbol{Y}}$, the block $\tilde{\boldsymbol{Y}}_m$ can be expressed as

$$\tilde{\boldsymbol{Y}}_m = \boldsymbol{Z}_L \boldsymbol{D}_m \boldsymbol{Z}_R \tag{8}$$

$$\boldsymbol{Z}_L = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_P \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{k_2-1} & z_2^{k_2-1} & \cdots & z_P^{k_2-1} \end{bmatrix} \tag{9}$$

$$\boldsymbol{Z}_R = \begin{bmatrix} 1 & z_1 & \cdots & z_1^{M-k_2} \\ 1 & z_2 & \cdots & z_2^{M-k_2} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_P & \cdots & z_P^{M-k_2} \end{bmatrix} \tag{10}$$

$$\boldsymbol{D}_m = \begin{bmatrix} s_1(t_m) & 0 & \cdots & 0 \\ 0 & s_2(t_m) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & s_P(t_m) \end{bmatrix} \tag{11}$$

where $z_p = e^{j\pi \sin\theta_p}, p = 1, 2, \cdots, P$.

Substituting Eq. (8) into Eq. (6), one can obtain the following

$$\boldsymbol{Y}_e = \begin{bmatrix} \boldsymbol{Z}_L \\ \boldsymbol{Z}_L \boldsymbol{D}_1 \\ \vdots \\ \boldsymbol{Z}_L \boldsymbol{D}_{k_1-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{Z}_R & \boldsymbol{D}_1 \boldsymbol{Z}_R & \cdots & \boldsymbol{D}_{k_1-1} \boldsymbol{Z}_R \end{bmatrix} \tag{12}$$

Equation (12) indicates that $\boldsymbol{Y}_e$ is low-rank (rank $(\boldsymbol{Y}_e) \le P$) when L is supposed to be much greater than P, so $\boldsymbol{Y}_e$ satisfies the condition of SIP. Therefore, we can recover the decision variable $\boldsymbol{M}$ through the low-rank MC algorithm that is presented in Eq. (5) by using $\boldsymbol{Y}_e$ instead of $\boldsymbol{Y}$. The optimization problem of Eq. (5) can be solved by inexact augmented Lagrange multiplier (IALM) method [18] or the classic singular value thresholding (SVT) method. Because the IALM has better stability and smaller computational load than the SVT method, we choose the IALM to recover the decision variable $\boldsymbol{M}$. The IALM function and algorithm for MC-ESPRIT can be expressed as below.

◆ Formulation

$$\begin{array}{ll} \text{minimize} & \|\boldsymbol{M}\|_* \\ \text{subject to} & \boldsymbol{M} + \boldsymbol{E} = \boldsymbol{Y}, \mathrm{P}_\Omega(\boldsymbol{E}) = 0, \mathrm{P}_\Omega(\boldsymbol{M} = \mathrm{P}_\Omega(\boldsymbol{Y}) \end{array} \tag{13}$$

where E is a temporary matrix for Lagrange operation whose entries are also 0 if $(i, j) \in \Omega$.

◆ Function

$$L(\boldsymbol{M}, \boldsymbol{E}, \boldsymbol{D}, \mu) = \|\boldsymbol{M}\|_* + \langle \boldsymbol{D}, \boldsymbol{Y} - \boldsymbol{M} - \boldsymbol{E} \rangle + \frac{\mu}{2} \|\boldsymbol{Y} - \boldsymbol{M} - \boldsymbol{E}\|_F^2 \tag{14}$$

where $\boldsymbol{D}$ is a temporary sparse matrix.

◆ Algorithm

Observation samples $\boldsymbol{Y}$

$$k = 0,$$
$$\boldsymbol{M}_0 = \boldsymbol{0}, \boldsymbol{E}_0 = \boldsymbol{0}, \mu_0 = 1/\|\boldsymbol{Y}\|_2 > 0 \tag{15}$$

While not converged do

$$k > 0,$$
$$(\boldsymbol{U}, \boldsymbol{S}, \boldsymbol{V}) = svd\left[\boldsymbol{Y} - \boldsymbol{E}_0 + \mu_0^{-1} \boldsymbol{D}_k\right] \tag{16}$$
$$\boldsymbol{M}_{k+1} = \boldsymbol{U}_k diag\left[\max\left(0, \sigma_i - \mu_k^{-1}\right)\right] \boldsymbol{V}_k^T \tag{17}$$
$$\boldsymbol{E}_{k+1} = \mathrm{P}_\Omega\left(\boldsymbol{Y} - \boldsymbol{M}_{k+1} - \boldsymbol{D}_k/\mu_k\right) \tag{18}$$
$$\boldsymbol{D}_{k+1} = \boldsymbol{D}_k + \mu_k\left(\boldsymbol{Y} - \boldsymbol{M}_{k+1} - \boldsymbol{E}_{k+1}\right) \tag{19}$$
$$\mu_{k+1} = 1.6\mu_k \tag{20}$$

where σ_i denotes the ith element of $\boldsymbol{S}$.

End while

$$\frac{\|\mathrm{P}_\Omega(\boldsymbol{Y} - \boldsymbol{M}_k)\|_2}{\|\mathrm{P}_\Omega(\boldsymbol{Y})\|_2} < \varepsilon \tag{21}$$

where ε is the stopping criteria.

◆ Output $\boldsymbol{M}$

Because $\boldsymbol{M}$ has the same structure of $\boldsymbol{Y}_e$, the completed data matrix of virtual ULA can be obtained by inverse transform of two-fold Hankel by the recovered data matrix $\boldsymbol{M}$. Therefore, the DOAs can be obtained through the completed data of virtual ULA by exploiting the ESPRIT-type methods.

4. Discussions

4.1 Cramer-Rao Bound (CRB)

The CRB is derived for the angle estimation with the completed data of virtual ULA, the ith diagonal element of the Fisher information matrix (FIM) with respect to the ith DOA can be written as

$$\boldsymbol{F}_{ii} = L\mathrm{tr}\left\{\boldsymbol{R}^{-1} \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{\Psi}_i} \boldsymbol{R}^{-1} \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{\Psi}_i}\right\} \tag{22}$$

where $\boldsymbol{R}$ is the covariance matrix of the completed data of virtual ULA, $\boldsymbol{\Psi}_i$ denotes the ith DOA of the estimated DOAs of signals.

According to [19], the CRB of linear array can be expressed as

$$CRB(\theta_i) = \frac{1}{2L(2\pi \sin\theta_i/\lambda)^2} \left(\frac{1}{\mathrm{SNR}} + \frac{1}{N \times \mathrm{SNR}^2}\right) \frac{1}{N \times \mathrm{AV}} \tag{23}$$

where $G = N \times \mathrm{AV}$ is defined as topological gain of linear array, $\mathrm{AV} = \frac{1}{N}\sum_{n=1}^{N}(x_n)^2$, x_n is the position of the nth element of linear array.

From the Eq. (23), we can see that the CRB of NLA is lower than that of the ULA with the same number of elements for higher topological gain. Whereas its CRB is higher than that of the ULA with the same apeture for lower topological gain. For the same reason, the parameter estimation performance of NLA is the same. This can be verified by the later simulation in Fig. 6 and Fig. 7.

4.2 Computational Load

The comparisons of computational load for the proposed MC-ESPRIT, VIA-ESPRIT and the algorithms in [7], [10] are shown in this section. Because the MC-ESPRIT and VIA-ESPRIT both use the ESPRIT algorithm, whose computational load of ESPRIT algorithm is $o\left(M^2L + M^3 + (M-1)P^2 + 6P^3\right)$, to estimate the DOAs, the difference of their computational loads focuses on how to obtain the completed data of virtual ULA. For MC-ESPRIT, it use the IALM method to recover the data matrix $\boldsymbol{M}$, and then one can obtain the completed data of virtual ULA. The computational load of IALM method is $K \times o\left(4(M-k_2)^2(k_1+1) + 10(k_1+1)^3 + 4P\sqrt{P} + 8\left(\sqrt{P}\right)^3\right)$, where K is the number of iteration. For VIA-ESPRIT, it exploits interpolation technique to estimate the completed data of virtual ULA, and the computational load of interpolation is $o\left(2N^2Q + N^3 + MNQ\right)$, where Q is the number of interpolated step. In general, L is supposed to be much greater than N, and $1 \le k_1 \le L$, so $10(k_1+1)^3$is bigger than N^3 in most case. Through comparing expression of the computational loads of MC-ESPRIT and VIA-ESPRIT, one can find that the MC-ESPRIT has higher computational load than the VIA-ESPRIT.

For the algorithm in [7], [10], they both exploit Root-MUSIC algorithm to estimate the DOAs of signals, and the computaional load of Root-MUSIC algorithm is $o\left\{M^2L + M^3 + 36(M-1)\left[M(M-P) + M + (M-1)\log_2(M-1) + (M-2)^2\right]\right\}$. As we known that the algorithm in [10] and the MC-ESPIRT both use matrix completion technology to obtain the completed data of virtual ULA, so the computational load of the MC-ESPRIT is lower than the algorithm in [10]. Besides, the algorithm in [7] exploits the Canonical Polyadic Decomposition (CPD) of higher-order tensors and the Root-MUSIC to estimate the DOAs, so its computational load must be higher than the MC-ESPRIT.

4.3 Implementation and Remarks

Based on the above theoretical analysis, the procedure of the proposed MC-ESPRIT can be summarized as follow.

Step 1 Extend the receive vector data $\boldsymbol{x}(t_l) \in C^{N\times 1}$ to a virtual ULA received data $\boldsymbol{y}(t_l) \in C^{M\times 1}$ by setting the received data as zero for the position without sensors, then the received data matrix $\boldsymbol{Y}$ of virtual ULA with multiple snapshots can be obtained according to Eq. (3).

Step 2 Transform received data matrix $\boldsymbol{Y}$ to a two-fold Hankel matrix $\boldsymbol{Y}_e$ according to Eq. (6).

Step 3 Exploit the IALM method to recovery $\boldsymbol{Y}_e$ exactly.

Step 4 Inverse-transform of the two-fold Hankel $\boldsymbol{Y}_e$ to the completed data matrix of virtual ULA.

Step 5 Use ESPRIT-type method to estimate the DOAs based on the completed data matrix of virtual ULA.

Remark 1: In order to obtain the interpolation matrix, the VIA-ESPRIT algorithm should divide the field of view of the array into several sectors, if the DOA of target is outside the sector, VIA-ESPRIT could not estimate the DOA correctly. Whereas our MC-ESPRIT does not limit by the sectors, it can estimate the DOA in the field of view of the array.

Remark 2: The proposed MC-ESPRIT algorithm can resolve M-1 narrow-band signals if the number of sensors in virtual ULA is M ($M > N$), while the classic MUSIC algorithm based on the received data of N-element NLA can estimate $N-1$ narrow-band signals. Therefore, our MC-ESPRIT could resolve more signals than the classic MUSIC algorithm based on the received data of NLA.

5. Simulation Results

In the first simulation, assume that there exist P=4 uncorrelated stationary signals, which are located at angles $-10°, 0°, 5°, 20°$ for a 5-element NLA, whose positions are $d = [0, 1, 4, 9, 11] \times \lambda/2$. The number of snapshots $L = 256$ is given during the simulations. The estimated results of DOAs by the MC-ESPRIT, VIA-ESPRIT and the method in [10] are shown in Fig. 2, Fig. 3 and Fig. 4 with $SNR = 20\,dB$ and 50 Monte-Carlo trials for the four targets.

It can be seen from Fig. 2, Fig. 4 and Fig. 5 that the proposed algorithm and the method proposed in [7] and [10] can estimate the DOAs correctly without dividing the field of view of the array into several sectors. Figure 3 indicates that the VIA-ESPRIT can effectively estimate DOAs when the angles of signals are lay in the sector $[-10°, 20°]$. On the contrary, the VIA-ESPRIT cannot estimate the DOAs correctly beyond the interpolated sector.

In the second simulation, the probabilities of successful detection of the MC-ESPRIT, VIA-ESPRIT, the method proposed in [7], [10] and the MUSIC (ULA with 5, 12) are evaluated. The ULA is the same as the first simulation. Assume that there are two closely-spaced targets located at 0° and 5°, which are said to be successfully resolved if and only if the absolute errors of DOA for the two targets are within 0.1°. The interpolated sector of the VIA-ESPRIT is $[0°, 10°]$, and the grid of the VIA-ESPRIT is 0.2°. Figure 6 displays the probability of successful detection as functions of SNR with $L = 256$. For each SNR, 500 Monte Carlo experiments are run.

It is observed that MUSIC with 12-element ULA has the lowest SNR threshold among the five algorithms, whose rea-

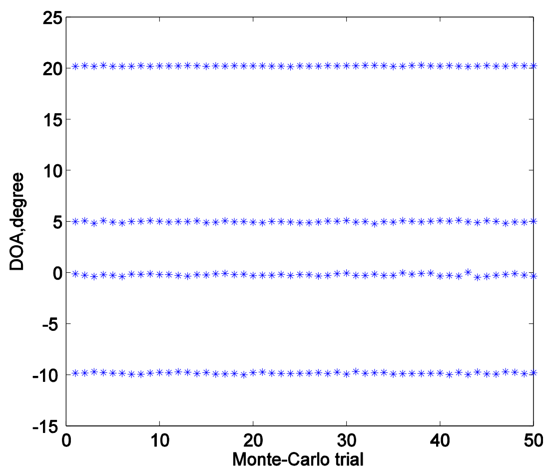
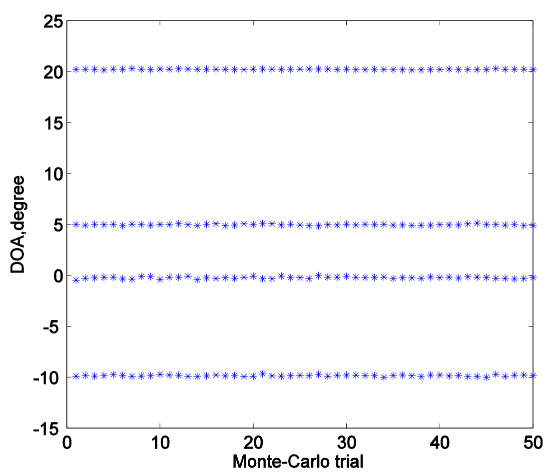
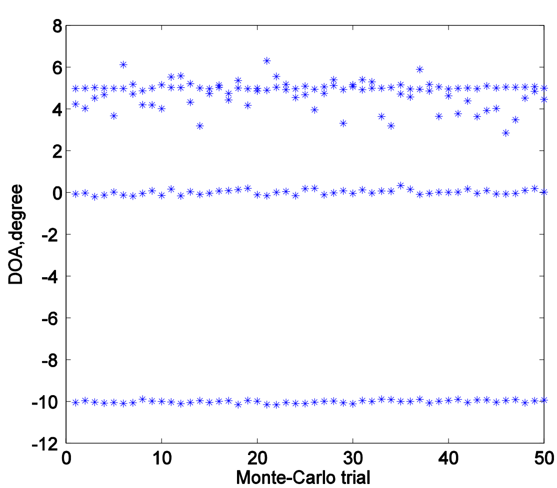


Fig. 2 The estimated results of the MC-ESPRIT.



(a) VIA-ESPRIT with sector [-10°, 20°]



(b) VIA-ESPRIT with sector [-10°, 10°]

Fig. 3 The estimated results of the VIA-ESPRIT.

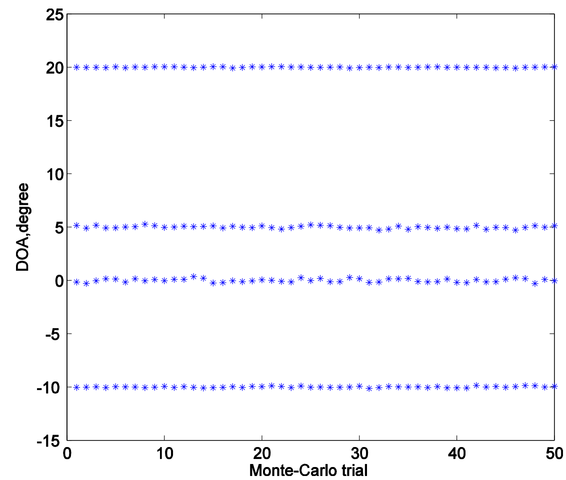


Fig. 4 The estimated results of the method proposed in [7].

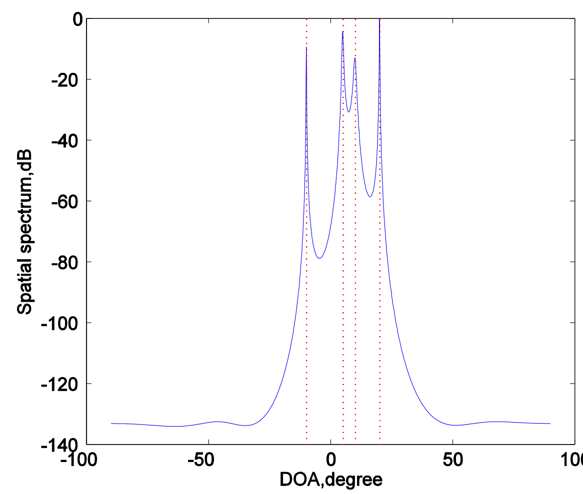


Fig. 5 The estimated results of the method proposed in [10] (Dot-dashed vertical lines indicate the four exact DOA's).

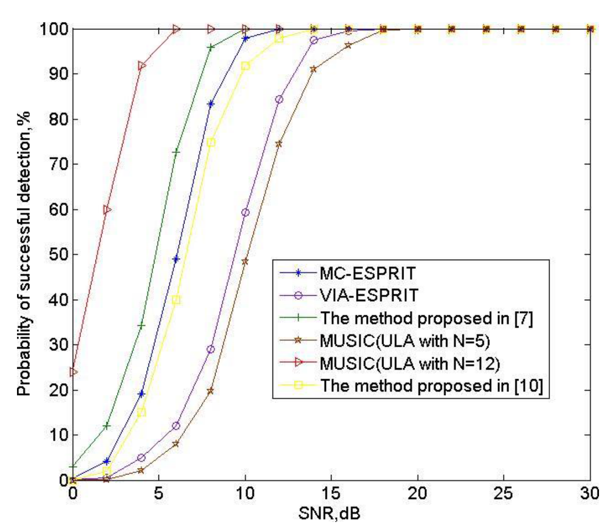


Fig. 6 Probability of successful detection versus SNR.

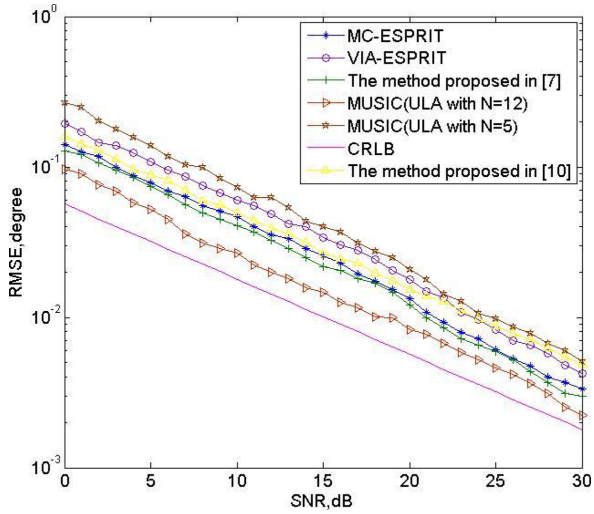


Fig. 7 RMSEs of angle estimation versus SNR.

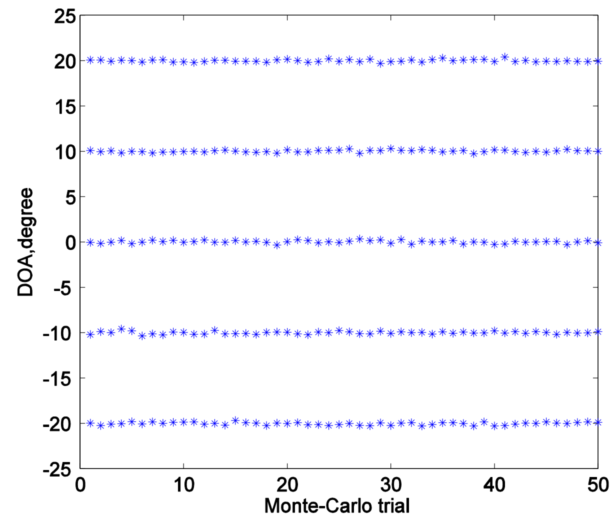


Fig. 8 Estimated results of five targets with MC-ESPRIT.

son is that the MC-ESPRIT, VIA-ESPRIT and the method in [7], [10] exist transformed errors when they transform the NLA to ULA with the same aperture. On the other hand, for low SNRs, the MC-ESPRIT shows great improved resolution for two closely-spaced targets as compared to the VIA-ESPRIT algorithm. The reason is that the VIA-ESPRIT obtains the completed data of virtual ULA through discretizing the interpolated sector, so its transformed errors must be affected by the size of grid. The bigger grid, the bigger transformed error. Wheras the MC-ESPRIT uses the matrix completion theory to get the completed data of virtual ULA, it does not refer to discretize the interpolated sector.

The simulated conditions of the third simulation are the same as the second one. Define that the performance of angle estimation is evaluated by RMSE defined as RMSE $= \sqrt{\frac{1}{P}\sum_{p=1}^{P} E\left[\left(\hat{\theta}_p - \theta_p\right)^2\right]}$, where $\hat{\theta}_p$ and θ_p are the estimated/true DOA, respectively. The RMSE of the MC-ESPRIT, VIA-ESPRIT, the method in [7], [10] and MUSIC (ULA with $N = 5$ and $N = 12$) are evaluated. Figure 7 displays the RMSEs of the six algorithms as functions of SNR. For each SNR, 200 Monte Carlo experiments are run.

It is observed that the proposed MC-ESPRIT outperforms the VIA-ESPRIT, the MUSIC with 5-element ULA and the method proposed in [10]. In addition, the performance of the proposed algorithm is almost the same as the method in [7].

In the fourth simulation, there are five targets located at $-20°$, $-10°$, $0°$, $10°$ and $20°$. The estimated results of five targets are presented in Fig. 8 for an NLA with $d = [0, 1, 3, 6] \times \lambda/2$ using the proposed MC-ESPRIT. The SNR is 15 dB and the number of pulses is $L = 256$, where 50 Monte-Carlo simulations are used. It can be seen from Fig. 8 that the proposed algorithm can estimate more signals than the number of sensors in NLA, so it can resolve more signals than the classic MUSIC algorithm based on the received data of NLA.

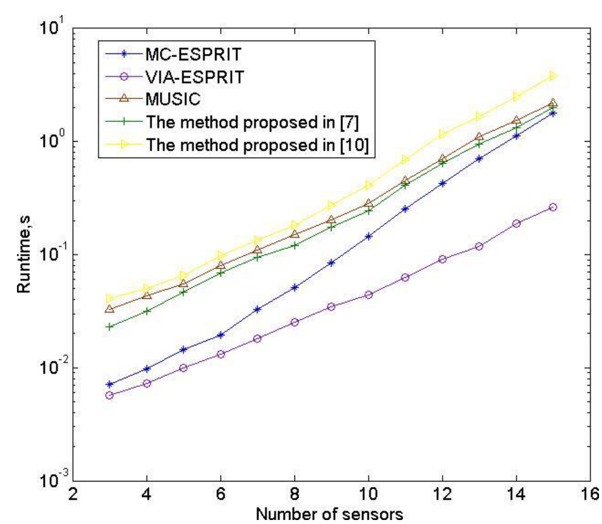


Fig. 9 Runtime of MC-ESPRIT, VIA-ESPRIT, MUSIC and the method proposed in [7], [10] versus the number of sensors.

For the fifth simulation, we compare the computational complexity of the MC-ESPRIT with the VIA-ESPRIT, MUSIC and the method in [7], [10]. Figure 8 presents an evaluation of the computational load using TIC and TOC instruction in MATLAB for the proposed algorithm, VIA-ESPRIT and MUSIC algorithm. The TIC and TOC instruction can be used to calculate the runtime of an algorithm. Simulations are conducted in MATLAB 2015b on a Core i5, 2.5 GHz, 8 GB RAM PC, and all results are given via 50 Monte-Carlo trials. The runtimes are plotted versus the number of sensors in Fig. 9.

We can observe from Fig. 9 that the VIA-ESPRIT algorithm has the lowest computation load, while the method in [7] has largest computation load, which is consistent with the results of previous theoretical analysis. When the number of sensors is small, the computation load of MC-ESPRIT is smaller than the method proposed in [10], and almost the same as the VIA-ESPRIT. With the increase of the sensors number, the computation load of MC-ESPRIT is gradually similar to the method proposed in [10].

6. Conclusion

In this paper, the theory of matrix completion is exploited to ESPRIT-type method, and then a novel DOA estimation algorithm called MC-ESPRIT has been presented for NLA. Based on matrix completion theory and the characters of Hankel matrix, a completed data matrix of virtual ULA, which has more elements than the real NLA, is recovered. Therefore, the proposed algorithm could estimate more targets than the MUSIC-type algorithms with NLA. At the same time, MC-ESPRIT does not limit by the sectors, which ensures it to estimate the DOA in the field of view of the array. Simulation results demonstrate that MC-ESPRIT provides better performance than VIA-ESPRIT when the SNR is low.

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