

Status Update for Accurate Remote Estimation: Centralized and Decentralized Schemes*

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SUMMARY In this work, we consider a remote estimation system where a remote controller estimates the status of heterogeneous sensing devices with the information delivered over wireless channels. Status of heterogeneous devices changes at different speeds. With limited wireless resources, estimating as accurately as possible requires careful design of status update schemes. Status update schemes can be divided into two classes: centralized and decentralized. In centralized schemes, a central scheduler coordinates devices to avoid potential collisions. However, in decentralized schemes where each device updates on its own, update decisions can be made by using the current status which is unavailable in centralized schemes. The relation between these two schemes under the heterogeneous devices case is unclear, and thus we study these two schemes in terms of the mean square error (MSE) of the estimation. For centralized schemes, since the scheduler does not have the current status of each device, we study policies where the scheduling decisions are based on age of information (AoI), which measures the staleness of the status information held in the controller. The optimal scheduling policy is provided, along with the corresponding MSE. For decentralized schemes, we consider deviation-based policies with which only devices with estimation deviations larger than prescribed thresholds may update, and the others stay idle. We derive an approximation of the minimum MSE under the deviation-based policies and show that it is $e/3$ of the minimum MSE under the AoI-based policies. Simulation results further show that the actual minimum MSEs of these two policies are even closer than that shown by the approximation, which indicates that the cost of collision in the deviation-based policy cancels out the gain from exploiting status deviations.

key words: remote estimation, age of information, decentralized access, mean-field model

1. Introduction

Accurate information is of paramount importance for many real-time control and cyber-physical systems [1]. For example, in autonomous driving, acquiring accurate information of the status of the surrounding environment and vehicles is the crux of safe control [2]. The acquiring process can be modeled as a remote estimation problem, in which de-

vices update the status information to a remote controller. Nonetheless, as communication resources are usually limited, making the controller informed of the current status all the time would be unrealistic. Subject to communication resources constraints, how to design status update schemes is a key problem of guaranteeing accurate remote estimation.

To this end, there are two different schemes: centralized and decentralized schemes. In centralized schemes, a central scheduler decides when and which devices should update their status. With decentralized schemes, each device makes the update decision on its own. In the case of heterogeneous sensing devices, we study the performance of these two schemes in terms of the MSE of the estimation.

For centralized schemes, the scheduler does not know the current status of each device, otherwise, there is no need to update. We consider a special type that uses age of information (AoI) [3], [4] to make scheduling decisions. AoI of a device is defined as the time elapsed since the generation time of the latest status update from this device received by the controller. As long as the time in the system is synchronized, a scheduler can easily obtain the timing information of each device and compute the corresponding AoI.

For decentralized schemes, each device exploits its current status information to make the update decision. In particular, we investigate deviation-based policies. As long as a device knows the rules the controller uses to estimate, it can obtain the deviation of its current status from the estimation. With a deviation-based policy, a device will not update unless its deviation is larger than a prescribed threshold. While the deviation-based policy enjoys the benefit of having access to the current status, however, the decentralization feature may incur collisions in wireless network.

In terms of AoI-based policies, most papers focus on optimizing AoI-related measures, e.g., minimizing the average AoI. Different scheduling policies are studied in [5]. Near-optimal policies are obtained when new status packets are always available. It is also proved in a recent work [6] that an AoI threshold-type policy is optimal. When the status packets are not always available, Ref. [7] provides a near-optimal policy based on Whittle's index. A Round-Robin scheduling is proved to be asymptotically optimal as the number of devices approaches infinity [8]. Reference [9] studies the problem where the time needed to transmit is random. Some recent papers also study the estimation error of AoI-based methods. References [10] and [11] consider the problem of how to sample a stochastic process to minimize the estimation error. They investigate Wiener process

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and Ornstein-Uhlenbeck process respectively under the single device case. It is found that a deviation-based policy outperforms AoI-based policies.

On the other hand, deviation-based policies have been widely studied in the control literature. It is observed in [12] that a deviation-based sampling strategy is better than periodic sampling strategies in terms of the estimation error. References [13]–[16] also report similar results under different system settings and communication models. However, all these papers study a single device case. Extending to the multiple-device case, the authors in [17] consider two devices, proving that an optimal policy in terms of MSE always has a threshold structure regardless of the status sensed by devices.

In this work, we study the MSE of the estimation of the AoI-based and the deviation-based policies respectively. The closest work to ours is [18]. However, they only study a homogeneous case where the status of each device changes at the same speed. We investigate a heterogeneous case where the speeds can be different. We first study the MSE under the AoI-based policies. It is proved that the MSE optimization problem is equivalent to minimizing the weighted average AoI, where the weights are based on the statistical information of status. We obtain a tight lower bound for the corresponding MSE and provide an optimal scheduling policy. Next, we analyze the performance of the deviation-based policies. In this case, devices whose estimation deviations are larger than their corresponding thresholds are called *active*. With the help of the mean-field analysis, we express the MSE as a function of the thresholds. Furthermore, we derive approximations of the optimal MSE and the corresponding thresholds. The approximation of the optimal thresholds is proportional to the square of the ratio between the number of devices and the number of channels. And the approximation of the minimum MSE of the deviation-based policy is $\frac{2}{3}$ of the minimum MSE of the AoI-based policy. Simulation results further show that the actual minimum MSEs of these two policies are even closer than what the approximation result shows.

The paper is organized as follows. In Sect. 2, we introduce the system model. The performance of AoI-based policies is analyzed in Sect. 3. Section 4 is devoted to deviation-based policies. Simulation results are presented in Sect. 5. In the end, we conclude with the future work discussed in Sect. 6.

2. System Model

We consider a network of N devices and a controller as in Fig. 1. The status of each devices is changing. According to the statistical properties, devices can be grouped into a set of classes $\mathcal{C} = \{1, 2, \dots, C\}$. The number of devices in class c is N_c , and $\sum_{c \in \mathcal{C}} N_c = N$. Let $\alpha_c = \frac{N_c}{N}$.

Let time be slotted and indexed by t . The status of the i -th device in class c evolves as an one dimension random walk as follows

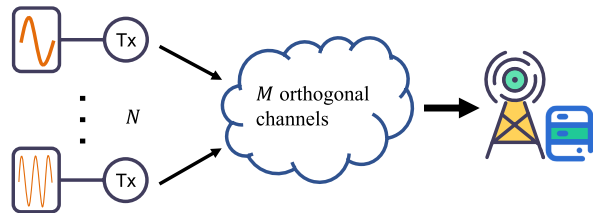


Fig. 1 System model.

$$S_{c,i}(t+1) = S_{c,i}(t) + w_{c,i}(t), \quad (1)$$

where the change $w_{c,i}(t)$ takes values $\{-1, 0, +1\}$ with probability $\{p_c, 1 - 2p_c, p_c\}$ respectively, and it is independent across time and devices.

There are M orthogonal channels. At the beginning of each time slot, each device will *decide* whether to sample and transmit a packet containing the current status to the controller, or *might be scheduled* by the controller to do so. The transmission time is set to be one time slot. We assume that $M < N$. The ratio $\frac{N}{M}$ is denoted by γ .

Since the communication resources are limited, these devices cannot all transmit simultaneously. Let $U_{c,i}(t)$ be the latest time slot in which this device has updated[†]. The AoI of this device at time slot t is denoted by $h_{c,i}(t)$, and is defined as

$$h_{c,i}(t) \triangleq t - U_{c,i}(t), \quad (2)$$

which measures the staleness of the latest information from this device received by the controller.

Let $\hat{S}_{c,i}(t)$ be the estimation of the status in the controller side. Minimum MSE estimator is adopted as,

$$\begin{aligned} \hat{S}_{c,i}(t) &= \mathbb{E} [S_{c,i}(t) | \mathcal{U}_{c,i}(t)] \\ &= S_{c,i}(U_{c,i}(t)), \end{aligned} \quad (3)$$

where $\mathcal{U}_{c,i}(t)$ is the collection of all updates from this device until time slot t .

Our target is to design status update policies to minimize the following expected sum of MSE:

$$\Delta = \limsup_{T \rightarrow \infty} \mathbb{E} \left[\frac{1}{NT} \sum_{t=1}^T \sum_{i,c} (S_{c,i}(t) - \hat{S}_{c,i}(t))^2 \right]. \quad (4)$$

Without loss of generality, we assume that the limit exists and limsup can be replaced by limit.

We first consider centralized schemes. The central scheduler makes the decisions and schedules M devices to transmit. We investigate the AoI-based policies where the scheduler makes scheduling decisions based on the AoI of the received status information.

Then we study decentralized schemes, under which each device makes the transmission decision on its own. The deviation-based policy is studied: each device computes its own status estimation deviation and decide whether it should access the channel or not. If it decides to access,

[†]In this work, *update* means a successful transmission.

then it randomly selects one out of the M channels. We assume that more than one transmission on a channel will fail due to collision. With a good deviation-based policy, devices with larger status estimation deviation should be assigned with higher priority to access the channel. However, since each device does not know the status of other devices, it is difficult to judge locally whether the deviation is large enough to deserve a transmission.

3. AoI-Based Policies

In this section, we investigate the MSE under the AoI-based policies. We first express the MSE Δ as a function of AoI.

The expectation of the squared deviation of a certain device at time slot t is

$$\begin{aligned} & \mathbb{E} \left[\left(S_{c,i}(t) - \hat{S}_{c,i}(t) \right)^2 \right] \\ &= \mathbb{E} \left[\left(S_{c,i}(t) - S_{c,i}(U_{c,i}(t)) \right)^2 \right] \\ &= \mathbb{E} \left[\left(\sum_{j=0}^{t-U_{c,i}(t)-1} w_{c,i}(j + U_{c,i}(t)) \right)^2 \right] \\ &\stackrel{(a)}{=} 2p_c h_{c,i}(t), \end{aligned} \quad (5)$$

where (a) is because the decisions of AoI-based policies are independent of the process itself. Therefore, we have

$$\Delta = \limsup_{T \rightarrow \infty} \mathbb{E} \left[\frac{1}{NT} \sum_{t=1}^T \sum_{i,c} 2p_c h_{c,i}(t) \right]. \quad (6)$$

This result shows that minimizing the MSE in AoI-based policies is in fact minimizing the weighted average AoI, where the weights are the probabilities that the status changes.

Accordingly, the optimal policy to minimize the MSE under AoI-based policies is: 1) sort these devices by the value $p_c h_{c,i}(t) \forall c \in C$ and $i \leq N_c$ in descending order. 2) schedule the top M devices.

In [5], a lower bound for the weighted AoI minimization problem is provided. It is further proved in [6] that this lower bound is tight. Thus, the minimum MSE under AoI-based policies is

$$\Delta_{\text{AoI}}^* = \gamma \left(\sum_{c \in C} \alpha_c \sqrt{p_c} \right)^2. \quad (7)$$

4. Deviation-Based Status Update

AoI-based policies only utilizes the statistic information of devices' status. In deviation-based policies, the transmission decision depends on the exact current status of each device. Another difference from AoI-based policies is that transmissions in deviation-based policies may fail due to collision.

Now, we formally describe deviation-based policies. Each class of devices is assigned with a predefined threshold

H_c . Once the deviation between the current status and the latest updated status of this device exceeds H_c , this certain device becomes *active* and it will randomly pick one channel to transmit with probability τ_c ($\tau_c \leq 1$). We seek to find the optimal parameters H_c and τ_c for all $c \in C$ to minimize the MSE Δ . To simplify the analysis, we assume that γ is large enough that the optimal H_c for any class is larger than 2.

Let $d_{c,i}(t)$ be the deviation of the current status from the estimation of the i -th device of class c in time slot t , which equals

$$\begin{aligned} d_{c,i}(t) &= |S_{c,i}(t) - \hat{S}_{c,i}(t)| \\ &= |S_{c,i}(t) - S_{c,i}(U_{c,i}(t))|. \end{aligned} \quad (8)$$

The system can be fully characterized by $\mathbf{d}(t) \triangleq \{d_{c,i}(t) | \forall i \leq N_c \text{ and } c \in C\}$, which is a Markov chain.

Here, we make the following assumptions to facilitate analysis

Assumption 1. *In the stationary region, the transmission success probability for any device is independent of the behavior of other devices, and it can be replaced by a constant probability p_s , which is a function of M , N and $\{H_c, \tau_c, \forall c \in C\}$.*

Then, the transition probability of $d_{c,i}$ is as follows:

- If $i < H_c$,

$$\begin{aligned} P(d_{c,i}(t+1) = d_{c,i}(t) + 1) &= p_c, \\ P(d_{c,i}(t+1) = |d_{c,i}(t) - 1|) &= p_c, \\ P(d_{c,i}(t+1) = d_{c,i}(t)) &= 1 - 2p_c. \end{aligned}$$

- If $i \geq H_c$,

$$\begin{aligned} P(d_{c,i}(t+1) = d_{c,i}(t) - 1) &= p_c(1 - \tau_c p_s), \\ P(d_{c,i}(t+1) = d_{c,i}(t) + 1) &= p_c(1 - \tau_c p_s), \\ P(d_{c,i}(t+1) = 0) &= \tau_c p_s(1 - 2p_c), \\ P(d_{c,i}(t+1) = 1) &= 2\tau_c p_s p_c, \\ P(d_{c,i}(t+1) = d_{c,i}(t)) &= 1 - 2p_c(1 - \tau_c p_s) + \tau_c p_s. \end{aligned}$$

Theoretically, we can compute the steady status distribution of $\mathbf{d}(t)$ given H_c and τ_c , for all c , and thus obtain the corresponding Δ . Nonetheless, the transition probability of $\mathbf{d}(t)$ is too complex to compute, let alone its steady status distribution.

Instead, we resort to study the population process of the system. Due to the exchangeability property, we can describe the system in terms of the number of the devices that belong to the same class and have the same deviation $d_{c,i}(t)$. In particular, the population process $\{x_{c,k}^{(N)}(t)\}$ is defined as

$$x_{c,k}^{(N)}(t) \triangleq \frac{1}{N} \sum_{i=1}^{N_c} \mathbb{I}(d_{c,i}(t) = k),$$

where $\mathbb{I}(\cdot)$ is the indicator function and the superscript (N) indicates that the number of devices is N . Let $\mathbf{x}^{(N)}(t) \triangleq$

$$\frac{dx_{c,0}(t)}{dt} = -2p_c x_{c,0}(t) + p_c x_{c,1}(t) + \tau_c p_s (1 - 2p_c) \sum_{k \geq H_c} x_{c,k}(t), \quad (9)$$

$$\frac{dx_{c,1}(t)}{dt} = -2p_c x_{c,1}(t) + 2p_c x_{c,0}(t) + p_c x_{c,2}(t) + 2\tau_c p_s p_c \sum_{k \geq H_c} x_{c,k}(t), \quad (10)$$

$$\frac{dx_{c,k}(t)}{dt} = -2p_c x_{c,k}(t) + p_c x_{c,k+1}(t) + p_c x_{c,k-1}(t), \quad \text{if } 1 < k < H_c - 1, \quad (11)$$

$$\frac{dx_{c,H_c-1}(t)}{dt} = -2p_c x_{c,H_c-1}(t) + p_c (1 - \tau_c p_s) x_{c,H_c}(t) + p_c x_{c,H_c-2}(t), \quad (12)$$

$$\frac{dx_{c,H_c}(t)}{dt} = -2p_c (1 - \tau_c p_s) x_{c,H_c}(t) + p_c (1 - \tau_c p_s) x_{c,H_c+1}(t) + p_c x_{c,H_c-1}(t) - \tau_c p_s x_{c,H_c}(t), \quad (13)$$

$$\frac{dx_{c,k}(t)}{dt} = -2p_c (1 - \tau_c p_s) x_{c,k}(t) + p_c (1 - \tau_c p_s) x_{c,k+1}(t) + p_c (1 - \tau_c p_s) x_{c,k-1}(t) - \tau_c p_s x_{c,k}(t), \quad \text{if } k > H_c. \quad (14)$$

$\{x_{c,k}^{(N)}(t)\}$. The second assumption is

Assumption 2. We embed the discrete time Markov chain (DTMC) $\mathbf{d}(t)$ into a continuous time Markov chain (CTMC) with transition rate 1. And we approximate the original population process by the CTMC counterpart. With slight abuse of notation, we still use $\mathbf{x}^{(N)}(t)$ to represent the CTMC population process.

In the CTMC, for example, the transition rate from status H_c to $H_c + 1$ is $p_c(1 - \tau_c p_s)$. With this approximation, the probability of more than one transition happening in an infinitesimal time interval is zero, due to the continuity of the exponential distribution. In particular, the possible transitions are listed as follows:

- A class c device with estimation error 0 has its error changed to 1. This happens with rate $2p_c N x_{c,0}^{(N)}(t)$. Once this event happens, the population process will change as

$$x_{c,0}^{(N)} \rightarrow x_{c,0}^{(N)} - \frac{1}{N}, \quad x_{c,1}^{(N)} \rightarrow x_{c,1}^{(N)} + \frac{1}{N}.$$

- A class c device with estimation error k ($0 < k < H_c$) has its error increased by 1. This event happens with rate $p_c N x_{c,k}^{(N)}(t)$.
- A class c device with estimation error k ($0 < k < H_c$) has its error decreased by 1. This event happens with rate $p_c N x_{c,k}^{(N)}(t)$.
- Let $k \geq H_c$. The event that a class c device with error n transmits successfully and the current doesn't change during the transmission happens with rate $\tau_c p_s (1 - 2p_c) N x_{c,k}^{(N)}(t)$.
- The event that a class c device with error $k \geq H_c$ transmits successfully and the current changes during the transmission happens with rate $2\tau_c p_s p_c N x_{c,k}^{(N)}(t)$.
- The event that a class c device with error $k \geq H_c$ doesn't transmit or fails to transmit, and the current status changes by +1 happens with rate $p_c (1 - \tau_c p_s) N x_{c,k}^{(N)}(t)$. The same rate for the event of changing by -1.

Computing the stationary distribution of the $\mathbf{x}^{(N)}(t)$ is difficult because the status space is very large. So we consider the corresponding mean-field model $\mathbf{x}(t) \triangleq \{x_{c,k}(t)\}$ instead. This model is characterized by a set of ordinary differential equations as (9) to (14).

Note that given the initial status $\mathbf{x}(0)$, the process $\mathbf{x}(t)$ is deterministic. The following theorem shows that the estimation deviation distribution $\mathbf{x}^{(N)}(t)$ converges in distribution to $\mathbf{x}(t)$ as $N \rightarrow \infty$, if the ratio γ is fixed, which means that the number of channels M scales up with N .

Theorem 1. Assume that $\mathbf{x}^{(N)}(0)$ converges to $\mathbf{x}(0)$ as $N \rightarrow \infty$, and the fraction α_c for each class is fixed. Then $\mathbf{x}^{(N)}(t)$ converges in distribution to $\mathbf{x}(t)$ as $n \rightarrow \infty$, which is the solution to the ODEs (9)–(14) with initial value $\mathbf{x}(0)$. Furthermore, $\mathbf{x}(t) \xrightarrow{t \rightarrow \infty} \mathbf{x}^*$.

This theorem can be proved by first showing that the $\mathbf{x}^{(N)}(t)$ converges to $\mathbf{x}(t)$ in any finite time interval. And then showing that $\lim_{t \rightarrow \inf}$ and $\lim_{N \rightarrow \inf}$ can be interchanged. The proof is standard and thus is omitted here. We refer the readers to [19] and [20] for the complete proof.

4.1 Mean-Field Analysis

The equilibrium point of the ODEs (9)–(14) can be used to study the MSE Δ in (4). Since it satisfies

$$\frac{d\mathbf{x}^*}{dt} = \mathbf{0}, \quad (15)$$

we have the following results

Theorem 2. the equilibrium point \mathbf{x}^* satisfies these equations

$$x_{c,k}^* = 2x_{c,0}^* - ka_c b_c + 2p_c a_c b_c, \quad \text{if } k < H_c, \quad (16)$$

$$x_{c,k}^* = q^{k-H_c} x_{c,H_c}^*, \quad \text{if } k \geq H_c, \quad (17)$$

$$x_{c,H_c-1}^* = (1 - a_c p_c) x_{c,H_c}^* + a_c b_c, \quad (18)$$

$$x_{c,H_c-1}^* = \frac{1 - a_c p_c}{q} x_{c,H_c}^*, \quad (19)$$

$$a_c b_c = \frac{2(1-q)x_{c,0}^*}{(1-q)(H_c - 2p_c) + q}, \quad (20)$$

where

$$a_c \triangleq \frac{\tau_c p_s}{p_c}, \quad b_c \triangleq \sum_{k \geq H_c} x_{c,k}^*, \quad g \triangleq \frac{a_c}{1 - a_c p_c}.$$

q is the root of the following equation and $q < 1$,

$$(g + 2 - q)q = 1. \quad (21)$$

Proof. See Appendix A. ■

Since $\sum_{k=0}^{\infty} x_{c,k}^* = \alpha_c$, the following relation holds,

$$\begin{aligned} \left(2p_c(H_c - 1) - \frac{H_c(H_c - 1)}{2} - \frac{q(H_c - 1) - 2p_c q}{(1-q)(1 - a_c p_c)} \right) a_c b_c \\ + \left(2H_c - 1 + \frac{2q}{(1-q)(1 - a_c p_c)} \right) x_{c,0}^* = \alpha_c. \end{aligned} \quad (22)$$

Assume that τ_c is given, and thus q is fixed. We can obtain $x_{c,0}^*$ by solving (22), and thus \mathbf{x}^* based on Theorem 2.

Actually, the transmission success probability p_s can be computed approximately if τ_c and H_c are given. p_s also represents the probability that a certain channel is being idle before a transmission occurs, thus

$$\begin{aligned} p_s &= \prod_{c \in C} \left(1 - \frac{\tau_c}{M} \right)^{b_c N} \\ &= \prod_{c \in C} \left(1 - \frac{\tau_c}{M} \right)^{b_c \gamma M} \\ &\stackrel{(a)}{=} e^{-\gamma \sum_{c \in C} b_c \tau_c}, \end{aligned} \quad (23)$$

where the equation (a) is obtained by letting $M \rightarrow \infty$, during which b_c is constant.

4.2 Performance Evaluation

Now, we want to investigate the influence of τ_c and H_c on the MSE Δ . With equilibrium point \mathbf{x}^* , the MSE for class c device is

$$\begin{aligned} \Delta_c &= \sum_{k=1}^{H_c-1} k^2 (2x_{c,0}^* - k a_c b_c + 2p_c a_c b_c) + \sum_{k=H_c}^{\infty} k^2 x_{c,k}^* q^{k-H_c} \\ &= b_c \frac{q(1+q)}{(1-q)^2} + 2b_c H_c \frac{q}{(1-q)} + b_c H_c^2 \\ &\quad + \frac{H_c(H_c-1)(2H_c-1)}{3} x_{c,0}^* - \frac{(H_c-1)^2 H_c^2}{4} a_c b_c \\ &\quad + \frac{H_c(H_c-1)(2H_c-1)}{3} p_c a_c b_c. \end{aligned} \quad (24)$$

Replacing the term $a_c b_c$ by (20), we have

$$\begin{aligned} \Delta_c &= b_c \frac{q(1+q)}{(1-q)^2} + 2b_c H_c \frac{q}{(1-q)} + b_c H_c^2 \\ &\quad + \frac{(1-q)H_c^2 + (1+3q)H_c - 2q}{6((1-q)(H_c - 2p_c) + q)} H_c(H_c-1)x_{c,0}^*, \end{aligned} \quad (25)$$

and $\Delta = \sum_{c \in C} \Delta_c$.

However, optimizing Δ as a function of τ_c and H_c directly is complicated. In this part, we only consider the regime where the following conditions are satisfied

1. Condition 1

$$\frac{q}{1-q} \ll H_c.$$

2. Condition 2

$$2p_c \ll H_c.$$

When γ is large, which means that the number of devices per channel is large, the first condition is satisfied when τ_c and H_c are close to the optimal one. See Appendix B for further justification.

With this approximation, (20) gives

$$a_c b_c \approx \frac{2x_{c,0}^*}{H_c - 2p_c}. \quad (26)$$

Substituting $\frac{2x_{c,0}^*}{H_c - 2p_c}$ for $a_c b_c$ in (22), we have

$$x_{c,0}^* \approx \frac{\alpha_c}{H_c + 2p_c}. \quad (27)$$

From (23), (26) and (27), the following relation holds,

$$b_c \tau_c e^{-\gamma \sum_{c \in C} b_c \tau_c} \approx \frac{2p_c \alpha_c}{H_c^2}. \quad (28)$$

As for (25), we only keep the term of the highest order of H_c and assume that $b_c \ll 1$. Then

$$\Delta \approx \sum_{c \in C} \frac{\alpha_c H_c^2}{6}. \quad (29)$$

Therefore,

$$\begin{aligned} \left(\sum_{c \in C} b_c \tau_c e^{-\gamma \sum_{c \in C} b_c \tau_c} \right) \times \Delta &\approx \left(\sum_{c \in C} \frac{\alpha_c H_c^2}{6} \right) \left(\frac{2p_c \alpha_c}{H_c^2} \right) \\ &\stackrel{(a)}{\geq} \left(\sum_{c \in C} \alpha_c \sqrt{\frac{p_c}{3}} \right)^2, \end{aligned} \quad (30)$$

where the inequality (a) is due to the Cauchy-Schwarz inequality. And the equality holds when

$$H_c = \left(\frac{2 \sum_{c \in C} \alpha_c \sqrt{p_c}}{G e^{-\gamma G}} \right)^{\frac{1}{2}} p_c^{\frac{1}{4}}, \quad (32)$$

where $G \triangleq \sum_{c \in \mathcal{C}} b_c \tau_c$, and G represents the expected fraction of the devices accessing the channels, i.e., system load. Also, when H_c takes the value as in (32), the MSE is

$$\Delta \approx \frac{(\sum_{c \in \mathcal{C}} \alpha_c \sqrt{p_c})^2}{3Ge^{-\gamma G}}. \quad (33)$$

Remark 1. This expression implies that the MSE is determined by the system load G . Therefore, devices of different classes can adopt the same access probability τ without harming the MSE performance, as long as $Ge^{-\gamma G}$ is minimized. Thus, the optimal $\tau = \frac{1}{\gamma \sum_{c \in \mathcal{C}} b_c}$, which is the reciprocal of the number of active devices per channel.

When $G = \frac{1}{\gamma}$, the minimum MSE under deviation-based policies can be approximated as

$$\Delta_{\text{Approx}}^* \approx \frac{\gamma e (\sum_{c \in \mathcal{C}} \alpha_c \sqrt{p_c})^2}{3}. \quad (34)$$

The corresponding thresholds are

$$H_c = \left(2\gamma e \sum_{c \in \mathcal{C}} \alpha_c \sqrt{p_c} \right)^{\frac{1}{2}} p_c^{\frac{1}{4}}. \quad (35)$$

Therefore, the ratio between the Δ_{Approx}^* and Δ_{AoI}^* is

$$\frac{\Delta_{\text{Approx}}^*}{\Delta_{\text{AoI}}^*} = \frac{e}{3}, \quad (36)$$

5. Simulation

In the simulation part, we first investigate whether assumptions 1 and 2 hold and the accuracy of the mean-field model. We then present the performance of the AoI-based and the deviation-based policies. For decentralized schemes, we consider a slotted ALOHA type policy for comparison where each device may transmit regardless of its deviation, which means all devices are active. The access probability is $\frac{M}{N}$, as Remark 1 suggests.

In Fig. 2–3, we check the correctness of assumptions 1 and 2. The simulation time horizon is 10^6 time slots. There are 5 channels and 300 devices. Devices are divided into two classes, the status transition probability for class 1 is 0.45, and it is 0.3 for class 2. The number of devices in each class is 150. Thresholds are set to be 10 and access probability is 0.2. Theoretical point distribution of the status estimation is based on the results in Theorem 2. Note that in computing the theoretical results, p_s is obtained from the simulation data. The fact that the theoretical distribution matches perfectly with the empirical distribution justifies the correctness of assumption 1 and 2, and also the accuracy of the mean-field model.

To study the performance of the deviation-based policies, we need to set $\tau = \frac{1}{\gamma \sum_{c \in \mathcal{C}} b_c}$. However, this result only holds in the stationary regime. Due to the randomness inherent in the system, we need to set the access probability

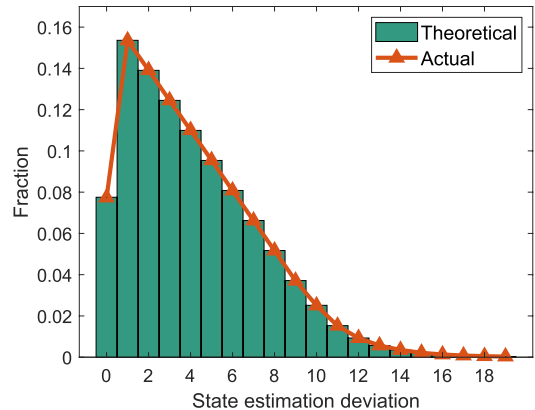


Fig. 2 The theoretical and empirical point distribution of the status estimation error for class 1 devices.

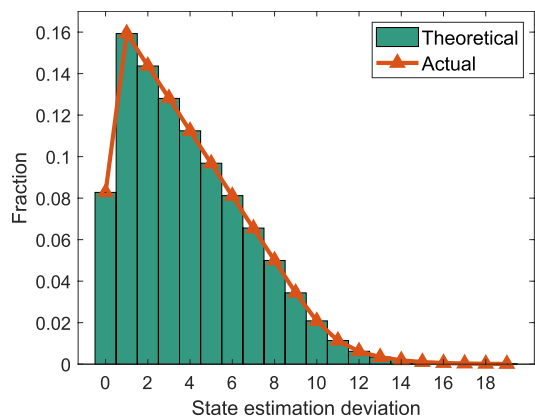


Fig. 3 The theoretical and empirical point distribution of the status estimation error for class 2 devices.

τ to be $\max\{1, \frac{M}{N_{\text{active}}}\}$, where N_{active} is the number of active devices in the system. In this work, we do not bother to design a sophisticated mechanism to estimate the number of active devices. Instead, it is assumed that there is a genie informing each device in the system about the number of active devices.

In Fig. 4 and 5, we present the performance of these two status update approaches, along with the approximated result in (34) and the baselines. Simulation time horizon is 10^6 time slots. M , the number of channels, is 8. And recall that $\gamma = \frac{N}{M}$. We only consider the case with 1 class whose transition probability p_c is e^{-1} . The threshold is

$$H = \left\lfloor \sqrt{2\gamma e p_c} \right\rfloor. \quad (37)$$

As shown in Fig. 4, the performance of deviation-based policy is close to that of the AoI-based policy, and both increase proportionally to γ . Also, both approaches are far better than the slotted ALOHA approach where each device access the channels with probability $\frac{1}{\gamma}$ regardless of its deviation. In Fig. 5, we present the details of Fig. 4. We observe that the MSE of the deviation-based policy is larger than the approximation result (34), and is slightly smaller than that of the AoI-based policy. The curve of the deviation-based policy

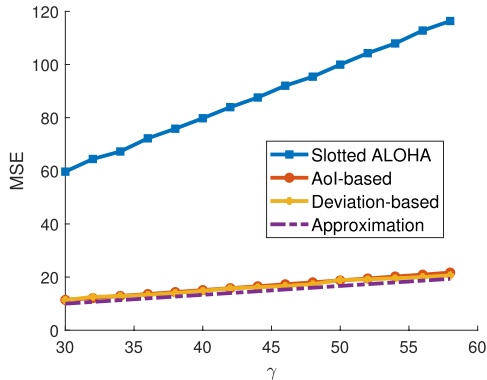


Fig. 4 The MSEs under slotted ALOHA, AoI-based and deviation-based policies with respect to varying number of devices.

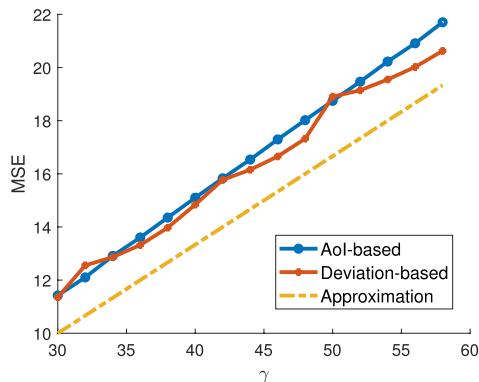


Fig. 5 Details of the MSEs of deviation-based and AoI-based policies.

Table 1 Minimum MSE and thresholds.

γ	Δ_{Opt}	Δ_{D}^*	H	q
32	12.39	12.56	7	0.45
36	13.33	13.32	8	0.33
40	14.93	14.84	8	0.44
44	16.23	16.15	9	0.31
48	17.30	17.32	9	0.40
52	19.17	19.14	10	0.29
56	20.00	20.02	10	0.36

fluctuates because the threshold H must be an integer.

To further justify our result, we also search for the optimal thresholds and the minimum MSE under deviation-based policies by brute-force search. The result is summarized in Table 1. In this table, Δ_{Opt} is obtained by searching the range of all possible thresholds. The column H is the corresponding optimal threshold. The column q presents the value of q when H is taken as the optimal value. And Δ_{D}^* is obtained by using the thresholds in (37). Actually, the thresholds in (37) are the same as the optimal thresholds. Also, we notice that the value of $\frac{q}{1-q}$ is small compared with H .

6. Conclusion

In this paper, we considered a remote estimation problem where heterogeneous sensing devices update status to a re-

mote controller. The relation between centralized and decentralized status update schemes was studied in terms of the MSE of the estimation. For centralized schemes, we investigated AoI-based policies, where a controller schedules devices to update based on their AoI and statistical information. We showed that the problem of minimizing the MSE is equivalent to minimizing the weighted average AoI. The minimum MSE was obtained, along with the corresponding scheduling policy. For decentralized schemes, we proposed a deviation-based policy, where devices try to update only when the status estimation deviations exceed the corresponding thresholds. We derived approximations of the minimum MSE and the optimal thresholds. The theoretical analysis suggested that the approximation of the minimum MSE under the deviation-based policy is $\frac{\epsilon}{3}$ of that under the AoI-based policy. Simulation results further showed that the actual minimum MSEs of these two schemes were also close.

In this work, we modeled the status of each device as a symmetric random walk, which is a martingale. Future work includes extending this model to more general cases. For example, the step of change could be different, and the process itself may no longer be stationary. Furthermore, we will explore other decentralized access protocols like CSMA. Another interesting topic is to quantify the value of the timing information in a status update system, which helps to study centralized schemes.

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Appendix A: Proof for Theorem 2

For simplicity,

$$a_c \triangleq \frac{\tau_c p_s}{p_c}, \quad b_c \triangleq \sum_{k \geq H_c} x_{c,k}^*$$

The equations (9) to (11) can be written as

$$-2x_{c,0}^* + x_{c,1}^* + (1 - 2p_c)a_c b_c = 0, \quad (\text{A} \cdot 1)$$

$$-2x_{c,1}^* + 2x_{c,0}^* + x_{c,2}^* + 2p_c a_c b_c = 0, \quad (\text{A} \cdot 2)$$

$$-2x_{c,k}^* + x_{c,k+1}^* + x_{c,k-1}^* = 0, \quad \text{if } 1 < k < H_c - 1. \quad (\text{A} \cdot 3)$$

Solving these equations gives

$$x_{c,k}^* = 2x_{c,0}^* - k a_c b_c + 2p_c a_c b_c, \quad \text{if } k < H_c. \quad (\text{A} \cdot 4)$$

When $k \geq H_c + 1$, we have

$$x_{c,k+1}^* = (g + 2)x_{c,k}^* - x_{c,k-1}^*, \quad (\text{A} \cdot 5)$$

where g is denoted as

$$g = \frac{a_c}{1 - a_c p_c}. \quad (\text{A} \cdot 6)$$

Let q be the root of the equation and $q < 1$

$$(g + 2 - q)q = 1, \quad (\text{A} \cdot 7)$$

then,

$$x_{c,k+1}^* - q x_{c,k}^* = (g + 2 - q)(x_{c,k}^* - q x_{c,k-1}^*), \quad k \geq H_c + 1. \quad (\text{A} \cdot 8)$$

Therefore

$$\begin{aligned} x_{c,k}^* - q x_{c,k-1}^* &= (g + 2 - q)^{k-H_c-1} (x_{c,H_c+1}^* - q x_{c,H_c}^*) \\ &= r s^{k-H_c-1}, \end{aligned} \quad (\text{A} \cdot 9)$$

where,

$$r = x_{c,H_c+1}^* - q x_{c,H_c}^*, \quad s = g + 2 - q. \quad (\text{A} \cdot 10)$$

Thus

$$x_{c,k}^* - \frac{r s}{s - q} s^{k-H_c-1} = q(x_{c,k-1}^* - \frac{r s}{s - q} s^{k-1-H_c-1}), \quad (\text{A} \cdot 11)$$

which can be written as,

$$x_{c,k}^* = \frac{r}{s - q} s^{k-H_c} + (x_{c,H_c}^* - \frac{r}{s - q}) q^{k-H_c}, \quad k \geq H_c + 1. \quad (\text{A} \cdot 12)$$

Since $q < 1$, we have $s > 1$. Because $x_{c,k}^* \rightarrow 0$ as $k \rightarrow \infty$, therefore, we must have $r = 0$ which gives

$$x_{c,H_c+1}^* = q x_{c,H_c}^*. \quad (\text{A} \cdot 13)$$

Finally, we have

$$x_{c,k}^* = q^{k-H_c} x_{c,H_c}^*. \quad (\text{A} \cdot 14)$$

From (12), we have

$$-2x_{c,H_c-1}^* + (1 - a_c p_c)x_{c,H_c}^* + x_{c,H_c-2}^* = 0. \quad (\text{A} \cdot 15)$$

Since $x_{c,H_c-1}^* = x_{c,H_c-2}^* - a_c b_c$, (A·15) gives

$$x_{c,H_c-1}^* = (1 - a_c p_c)x_{c,H_c}^* + a_c b_c. \quad (\text{A} \cdot 16)$$

From (13), we have

$$-2(1 - a_c p_c)x_{c,H_c}^* + (1 - a_c p_c)x_{c,H_c+1}^* + x_{c,H_c-1}^* - a_c x_{c,H_c}^* = 0. \quad (\text{A} \cdot 17)$$

Substituting $q x_{c,H_c}^*$ for x_{c,H_c+1}^* leads to

$$\begin{aligned} x_{c,H_c-1}^* &= (2 - 2a_c p_c + a_c - q + a_c p_c q)x_{c,H_c}^* \\ &= \frac{1 - a_c p_c}{q} x_{c,H_c}^*. \end{aligned} \quad (\text{A} \cdot 18)$$

Combining (A·4), (A·16) and (A·18), we obtain

$$a_c b_c = \frac{2(1 - q)x_{c,0}^*}{(1 - q)(H_c - 2p_c) + q}. \quad (\text{A} \cdot 19)$$

$$H + \frac{(2p+t)(H-1)}{H-2p+t} + \frac{2t(t+1)}{(1-ap)(H-2p+t)} = \frac{1}{x_0} \quad (\text{A} \cdot 20)$$

Appendix B: Justification for Condition 1

When the thresholds H_c for all c are relatively small, it is always better to increase the access probability to be $\frac{M}{N \sum_{c \in C} b_c}$, with which the system can achieve maximum throughput. And $\frac{M}{N \sum_{c \in C} b_c}$ is in fact the reciprocal of the number of active devices per channel. On the other hand, as long as the maximum throughput is achieved, it is always better to increase the thresholds to minimize the MSE. These two observations imply that is the region around the best choice of the parameters, $\frac{M}{N \sum_{c \in C} b_c}$ should be near to 1, and we denote it by k . Then $\tau_c \approx \frac{1}{k}$.

Based on the definitions, we have

$$\frac{q}{1-q} = \frac{1}{2} \sqrt{1 + \frac{4}{g}} - \frac{1}{2}, \quad (\text{A} \cdot 21)$$

and

$$g \approx \frac{1}{p_c(ke-1)} \leq \frac{2}{ke-1}. \quad (\text{A} \cdot 22)$$

Hence

$$\frac{q}{1-q} \leq \frac{1}{2} \sqrt{2ke-1} - \frac{1}{2}. \quad (\text{A} \cdot 23)$$

When $k = 2$, this is 0.5532, which is far less than H_c . Table 1 in the simulation part also supports this claim.



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