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The Gaussian MIMO Broadcast Channel under Receive Power Protection Constraints*

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SUMMARY A Gaussian MIMO broadcast channel (GMBC) models the MIMO transmission of Gaussian signals from a transmitter to one or more receivers. Its capacity region and different precoding schemes for it have been well investigated, especially for the case wherein there are only transmit power constraints. In this paper, a special case of GMBC is investigated, wherein receive power constraints are also included. By imposing receive power constraints, the model, called protected GMBC (PGMBC), can be applied to certain scenarios in spatial spectrum sharing, secretive communications, mesh networks and base station cooperation. The sum capacity, capacity region, and application examples for the PGMBC are discussed in this paper. Sub-optimum precoding algorithms are also proposed for the PGMBC, where standard user precoding techniques are performed over a BC with a modified channel, which we refer to as the "protection-implied BC." In the protection-implied BC, the receiver protection constraints have been implied in the channel, which means that by satisfying the transmit power constraints on the protection implied channel, receiver protection constraints are guaranteed to be met. Any standard single-user or multi-user MIMO precoding scheme may then be performed on the protection-implied channel. When SINR-matching duality-based precoding is applied on the protection-implied channel, sum-capacity under full protection constraints (zero receive power), and near-sum-capacity under partial protection constraints (limited non-zero receive power) are achieved, and were verified by simulations.
key words: *MIMO systems, broadcast channels, capacity region, precoding, Lagrangian duality, dirty paper coding, wireless distributed network, spectrum sharing, cognitive radio*

1. Introduction

The Gaussian MIMO broadcast channel (GMBC) models a single transmitter with multiple antennas sending private, independent, Gaussian-distributed information streams to multiple receivers with multiple antennas. The capacity region, which is the union of all achievable information rate vectors under all multiuser transmission strategies, have been well investigated for the GMBC under transmit power constraints. Analysis for the GMBC began for the case wherein a transmit sum-power (SP) constraint is imposed. In [1], an outer bound for the capacity region was derived for the Gaussian BC case. This region was first obtained by Caire and Shamai [2] for the single antenna case. Later, this was extended to the MIMO case by Vishwanath, Jindal, and Goldsmith using the idea of "dirty paper coding" (DPC) which was proposed by Costa [3]. Vishwanath and Tse [4] showed that the Sato bound was tight for the GMBC.

Yu [5] further described the sum-capacity for the multi-antenna case. Under the SP constraint, capacity-region achieving precoding was proposed in [6], by establishing duality of the GMBC with a Gaussian multiple-access channel (GMAC) with a total sum-power constraint. This has been called "SINR-matching" duality, because the BC and MAC transmit covariance matrices yield the same receiver SINRs in the downlink and uplink. These transmit covariances along with "dirty paper coding" [3], were used to obtain the capacity-region achieving precoding.

In [7], Yu showed that the sum capacity bounds of the GMBC and the GMAC have a Lagrangian minimax duality and the duality was extended to GMBCs with arbitrary linear constraints. He applied the minimax duality in [8] for the case of multiple transmit antenna constraints. Finally, in [9], Weingarten derived the capacity region for GMBC for arbitrary linear constraints.

A class of linear constraints is the imposition of receive power limits on one or more receivers. We may refer to a GMBC with both transmit power and receive power constraints as the *protected Gaussian MIMO broadcast channel* (PGMBC). In the PGMBC framework, when a receiver is fully protected, its receive power is constrained to be zero. When it is partially protected, its receive power is limited below a certain power level. When it is not protected, no constraints are imposed on its receive power.

In this paper, the capacity regions of three categories of the PGMBC are discussed. The first category is the partial protection PGMBC, wherein all of the protected receivers are partially protected (non-zero receive power limit). Next is the full protection PGMBC, wherein all of the protected receivers are fully protected (zero receive power). Finally, the general PGMBC, wherein some receivers are fully protected, and some receivers are partially protected. Results will show that when the receive constraints are active, sum-capacity may exist at only a single achievable point which is obtained through a particular receiver ordering and its corresponding optimum transmit covariance. Investigations will also show the potential gains in spectral efficiency of having full CSI knowledge of the channel to protected receivers rather than only knowing the average power of their channels.

Capacity-achieving precoding for the PGMBC has been proposed [10]. This method requires convex optimization of the MAC dual problem and a generalization of the

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MAC-to-BC covariance transformations developed in [6]. This method requires a number of iterative min-max subgradient optimization or interior-point optimization steps and MAC-to-BC covariance transformations for all encoding-decoding orders to obtain the optimum precoding solution. Such optimization can be prohibitively complex in some practical applications.

In this paper, precoding algorithms for the PGMBC are proposed, where simpler and well-known multiuser precoding techniques can be performed over a modified broadcast channel, which is referred to as the "protection-implied BC." In the protection-implied BC, the receiver protection constraints have been implied in the channel, which means that by satisfying the transmit power constraints on the protection implied channel, the receiver protection constraints are guaranteed to be met. To arrive at the protection-implied solution, a protection matrix is calculated from the nullspace of the protected users and their maximum allowed receive power. The product of the channel and the protection matrix becomes the protection-implied channel. Any standard multi-user MIMO precoding scheme (e.g. LQ-DPC Zero-forcing) may then be performed on the protection-implied channel. Such an approach avoids the use of rigorous convex optimization and allows for intuitive precoder design. Simulations suggest that LQ-DPC zero forcing on the protection-implied channel can achieve good spectral efficiency especially at low receive power limits.

In this paper, it will be shown that when the SINR-matching duality dirty paper coding (DPC) precoding [6] is used as the precoding method on the protection-implied channel, capacity under full protection (zero receive power) and near-sum-capacity under partial protection constraints (limited non-zero receive power) are achieved.

The paper organization is as follows. First, the transmission model is introduced in Sect. 2, followed by a discussion of its applications in Sect. 3. Next, the capacity region is discussed in Sect. 4. Then, protection-implied precoding is elaborated in Sect. 5 followed by an investigation of its sensitivity to channel estimation error in Sect. 6. Finally, an example scenario is presented in Sect. 7, followed by the conclusions in Sect. 8.

2. Transmission Model of the PGMBC

In the succeeding, $\|\theta\|$, $|\mathbf{\Theta}|$, $\mathbf{\Theta}^{-1}$, $\mathbf{\Theta}^{\dagger}$, $\mathbf{\Theta}^{T}$, $\mathbf{\Theta}^{H}$, $\mathrm{Tr}(\mathbf{\Theta})$, $\mathrm{cols}(\mathbf{\Theta})$, and $\mathcal{R}(\mathbf{\Theta})$ denote the 2-norm, determinant, inverse, pseudo-inverse, transpose, conjugate transpose, trace, no. of columns, and range of $\mathbf{\Theta}$ respectively. $\mathbf{I}_N$ is the $N \times N$ identity matrix and $\mathrm{diag}(\lambda_i, \ldots, \lambda_j)$ is a diagonal matrix with $\lambda_i, \ldots, \lambda_j$ entries. $\mathrm{blockdiag}(\mathbf{\Theta}_i, \ldots, \mathbf{\Theta}_j)$ is a block-diagonal matrix with entries $\mathbf{\Theta}_i, \ldots, \mathbf{\Theta}_j$. $\mathbf{\Theta} \geq 0$ means that $\mathbf{\Theta}$ is positive semidefinite. $\mathbf{g} \sim \mathcal{CN}(\theta, \mathbf{\Theta})$ is a complex Gaussian random vector with mean of θ at each element and covariance $\mathbf{\Theta}$. $(\mathbf{\Theta})_{m:n,j:k}$ are the elements at the mth–nth row and jth–kth column. $\bar{\mathbf{\Theta}}$ represents the orthonormal vectors which span the nullspace of $\mathbf{\Theta}^H$. Finally, $\mathbf{\Theta}_{1,\ldots,N}$ denotes the matrix set $\{\mathbf{\Theta}_1, \ldots, \mathbf{\Theta}_N\}$ and $\mathbf{\Theta}(k,k)$ refers to the k'th square block

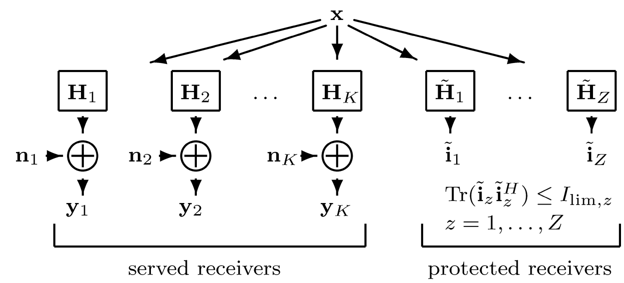

Fig. 1 System model of the protected Gaussian MIMO BC.

along the diagonal of $\mathbf{\Theta}$.

Consider a GMBC with an N-antenna transmitter and K served receivers with $M_1, \ldots, M_K$ antennas as shown in Fig. 1. The transmitter sends private independent information streams $\mathbf{d} = [\mathbf{d}_1^T, \ldots, \mathbf{d}_K^T]^T$, where $\mathbf{d}_k \sim \mathcal{CN}(0, \mathbf{I}_{M_k})$ for receiver k. Let $\mathbf{x} \in \mathfrak{C}^{N\times 1}$ be the input and let $\mathbf{H}_k \in \mathfrak{C}^{M_k \times N}$ be the channel of receiver k. Noise at receiver k is $\mathbf{n}_k \sim \mathcal{CN}(0, \mathbf{I}_{M_k}) \in \mathfrak{C}^{M_k\times 1}$. $\mathbf{y}_k \in \mathfrak{C}^{M_k\times 1}$ is the kth received signal,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{y} \triangleq \left[\mathbf{y}_1^T, \cdots, \mathbf{y}_K^T\right]^T$, $\mathbf{n} \triangleq \left[\mathbf{n}_1^T, \cdots, \mathbf{n}_K^T\right]^T$, and $\mathbf{H} \triangleq \left[\mathbf{H}_1^T, \cdots, \mathbf{H}_K^T\right]^T$ are the quasi-static channels known at the transmitter. The covariance of the input is $\mathbf{\Omega} \triangleq \mathbb{E}[\mathbf{x}\mathbf{x}^H]$. The input component containing the symbols for receiver k is $\mathbf{x}_k = \mathbf{T}_k\mathbf{d}_k$ where $\mathbf{T}_k \in \mathfrak{C}^{N\times M_k}$. The total transmit weighting matrix is $\mathbf{T} = [\mathbf{T}_1, \ldots, \mathbf{T}_K]$. Equivalently,

$$\mathbf{\Omega}_k \triangleq \mathbb{E}[\mathbf{T}_k\mathbf{d}_k\mathbf{d}_k^H\mathbf{T}_k^H] = \mathbf{T}_k\mathbf{T}_k^H \tag{2}$$

$$\mathbf{\Omega} \triangleq \mathbb{E}[\mathbf{T}\mathbf{d}\mathbf{d}^H\mathbf{T}^H] \triangleq \mathbf{T}\mathbf{T}^H. \tag{3}$$

A positive semidefinite constraint is imposed on $\mathbf{\Omega}$. Furthermore, linear constraints on $\mathbf{\Omega}$ are imposed,

$$\mathrm{Tr}(\mathbf{\Omega}\mathbf{S}_1) \leq 1, \ \ldots, \ \mathrm{Tr}(\mathbf{\Omega}\mathbf{S}_L) \leq 1, \tag{4}$$

where there are L constraints and $\mathbf{S}_l$, $l = 1, \ldots, L$, are $N \times N$ positive semidefinite matrices. For example, under a sum-power constraint, $\mathbf{S}_l = \mathbf{S}_{\mathrm{SP}} \triangleq P_{\mathbf{\Omega}}^{-1}\mathbf{I}_N$. Or under per-antenna-power constraints, $\mathbf{S}_l = \mathbf{S}_{\mathrm{PAP},n}$ are single-entry matrices, with

$$(\mathbf{S}_{\mathrm{PAP},n})_{n,n} \triangleq P_{\mathbf{\Omega},n}^{-1}, \ n = 1, \ldots, N. \tag{5}$$

In the PGMBC, receivers may be classified as either *protected*, where receive power is limited, or *unprotected*, where the receive power is not limited. Receivers are also classified as either *served* or *unserved*, where private independent information is sent to the served receivers, which may be protected or unprotected. No information is sent to unserved receivers. At the protected receivers, power limits are imposed on the received signal $\tilde{\mathbf{i}}_z \in \mathfrak{C}^{\tilde{M}_k\times 1}$,

$$\mathrm{Tr}(\tilde{\mathbf{H}}_z\mathbf{\Omega}\tilde{\mathbf{H}}_z^H) \leq I_{\mathrm{lim},z} \quad z = 1, \ldots, Z, \tag{6}$$

where $I_{\mathrm{lim},z} \geq 0$ is the receive power limit at the zth receiver. If $I_{\mathrm{lim},z} = 0$, then *full protection* transmission to the receiver is performed. Otherwise, *partial protection* transmission is performed. The set of served receivers and the set of protected receivers can intersect, i.e. a served receiver may also be protected. When the protected receivers are different from the served receivers, the receive power limits may be considered as *interference limits*.

For the partially protected receivers, (6) is converted to a constraint on $\mathbf{\Omega}$,

$$\mathbf{S}_l = \mathbf{S}_{\text{Protect},z} \triangleq \tilde{\mathbf{H}}_z^H \tilde{\mathbf{H}}_z / I_{\text{lim},z} \quad I_{\text{lim},z} > 0. \tag{7}$$

Meanwhile, for the fully protected receivers, the constraint is

$$\text{Tr}(\tilde{\mathbf{H}}_z \mathbf{\Omega} \tilde{\mathbf{H}}_z^H) = 0, \;\; z = 1, \ldots, Z. \tag{8}$$

3. Applications of the PGMBC

Receive power protection for the broadcast channel is applicable to many scenarios, as illustrated in Fig. 2. The protected receivers may be non-destination receivers in a wireless distributed network. In the top subfigure, Tx A and Tx B are simultaneously transmiting, and are serving Rxs C and D respectively. To limit co-channel interference, Tx A protects Rx D, and Tx B protects Rx C. Txs A and B may represent mesh nodes which simultaneously access the shared channel to transmit to nodes indicated by C and D. Alternatively, A and B may be base stations which exchange scheduling and channel information to perform coordinated beamforming. Under coordinated beamforming, Rx C is an in-cell terminal served by Tx A, while Rx D is an other-cell terminal served by Tx B.

The base stations may even exchange channel information to act as a single distributed transmitter that can perform nulls for spectrum sharing. For example, the protected receiver may be an earth station receiver operating in the same frequency band as that of the cellular network. This approach of performing cooperative nulls was investigated in [11], [12] wherein under ideal channel information and delay conditions, significant improvement on user terminal spectral efficiencies were achieved compared to conventional spectrum sharing.

Receive power may be limited to receivers of other systems, as in cognitive radio. Protection may also be performed in order to prevent eavesdropping of secret information (*eavesdropping protection*). Even if the eavesdropping receiver has perfect knowledge of the demodulation and decoding, by limiting its receive power, secrecy is achieved.

Service with receive power protection may be applied to a receiver under several circumstances. The first case is for *service sharing*. As illustrated in the second subfigure of Fig. 2, Rx I may be simultaneously receiving from Txs F and G on the same channel. For example, consider the case wherein if the receive signal powers at I induced by Tx F and G are close to each other, both signals can be properly detected by I, such as in CDMA reception. However, the pathlosses from F and G may be different. Therefore,

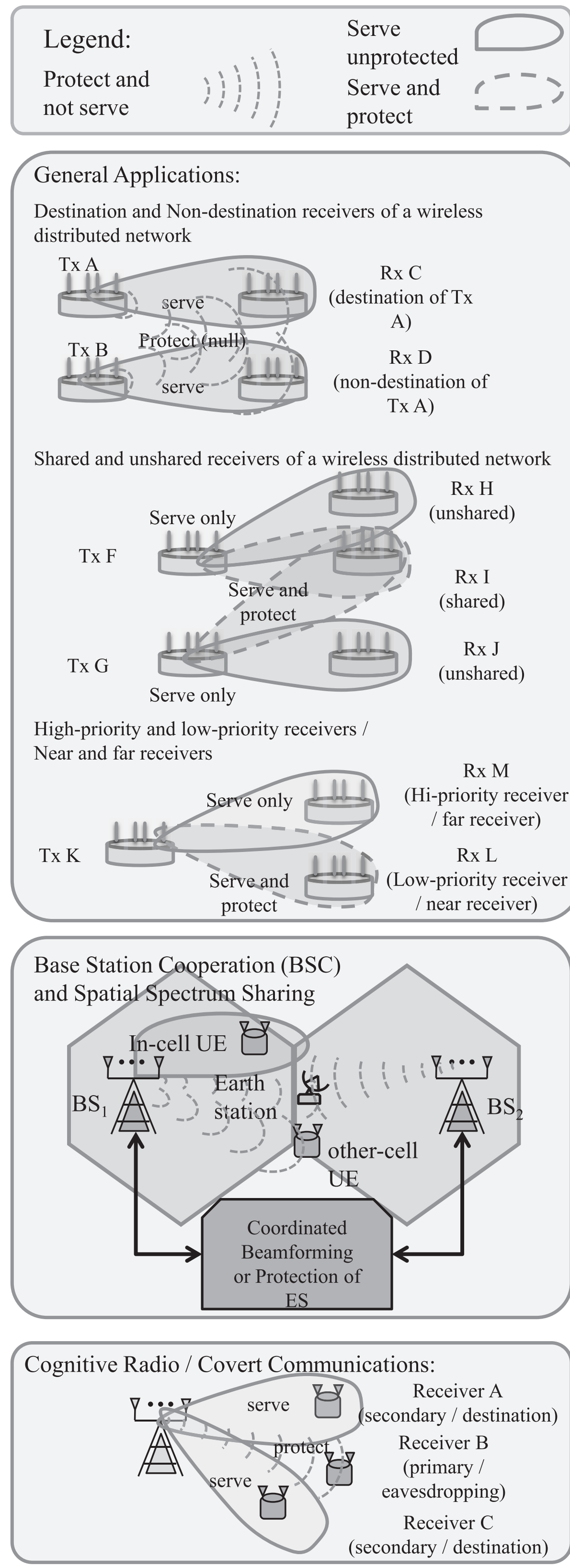

Fig. 2 Application examples of Gaussian MIMO broadcast channel with protection constraints.

receive power protection are done by Txs F and G to balance the receive powers on I. Traditionally, Tx power must be reduced by either F and G. But if both are also serving other Rxs (H and J), reducing the Tx powers would affect the throughputs of H and J. Therefore, the receive power protection approach may be applied in order to simultaneously perform the receive power balance and maximize the throughputs.

The second case where service with receive power protection may be applied is the case wherein simultaneous transmission needs to be performed to two or more receivers, but there is a prioritization of throughput. For example, Rx M is high priority while Rx L is low priority. Since L is low priority, it only requires a smaller receive signal power to achieve the smaller target throughput. If the Tx power of K is reduced, then high throughput cannot be achieved on M. But through the model, throughput can be maximized while reducing the overall transmit power and deprioritizing node L. The third case is when a Rx is too close to the Tx while another is quite far. In this scenario, Tx K is "shouting" on L but M is barely within coverage. L may be too close so that the received signal saturates and yields poor detection. If the Tx power is simply reduced to reduce the saturation, M loses to be in coverage. Through the model, the receiver powers can be adjusted to balance the receive powers to provide the required throughput.

4. Capacity Region of the PGMBC

In the succeeding, we will discuss the capacity regions of special cases of the PGMBC. The first is the partial protection GMBC, where $I_{\lim,z} > 0,\ \ z = 1,\dots,Z$. The second is full protection GMBC, where $I_{\lim,z} = 0,\ \ z = 1,\dots,Z$. The third is the general PGMBC in which some of the users may be partially protected and some are fully protected.

4.1 Partial Protection GMBC

Let $\mathbf{R} = [R_1\ \ R_2\ \ \dots\ \ R_K]^T$ be a given rate vector. Weingarten [9] showed that for any input constraint such that $\mathbf{\Omega}$ lies in a compact set of positive semidefinite matrices, every achievable rate vector in a Gaussian MIMO broadcast channel can be obtained via dirty paper coding (DPC). Under partial protection GMBC, the constraints (4) satisfy this property. Hence, the capacity region of partial protection GMBC corresponds to the multiantenna DPC achievable region in which the following rate is achievable for served receiver k:

$$R^{\mathrm{B}}_{\pi(k)} = \log \frac{\left|\mathbf{I} + \mathbf{H}_{\pi(k)} \left(\sum_{i \le k} \mathbf{\Omega}_{\pi(i)}\right) \mathbf{H}^H_{\pi(k)}\right|}{\left|\mathbf{I} + \mathbf{H}_{\pi(k)} \left(\sum_{i < k} \mathbf{\Omega}_{\pi(i)}\right) \mathbf{H}^H_{\pi(k)}\right|}, \tag{9}$$

where $\mathbf{H}_{\pi(k)}$ is the channel to receiver k for a receiver reordering of the $\mathbf{H}$ denoted by $\pi(k)$, and where receiver $\pi(K)$ is coded first, receiver $\pi(K-1)$ is coded 2nd and so forth.

Let $C_{\mathrm{B}}(\mathbf{H}_{1,\dots,K}, \mathbf{S}_1, \dots, \mathbf{S}_L)$ denote the capacity region of the partial protection GMBC with the channel $\mathbf{H}$ under the set of transmit antenna and partial protection constraints specified by $\mathbf{S}_1, \dots, \mathbf{S}_L$. The notation $\mathbf{H}_{1,\dots,K}$ implies that the channel of each link, N, and $M_k\ \ k = 1, \dots, K$ are specified. In the succeeding, the number of antenna elements at each receiver will be implied and we shall omit the subscript notation $1, \dots, K$ for simplicity. Each rate vector in the capacity region is obtained using DPC by forming the union of all rate vectors using (9) over all Hermitian covariance matrices $\mathbf{\Omega}_1, \dots, \mathbf{\Omega}_K$ over all $K!$ receiver permutations which satisfy the constraints.

4.1.1 Sum Capacity of the Partial Protection GMBC

The sum capacity, denoted by R^{B}_C is the maximum sum of rates of a rate vector within this capacity region. This bound was generalized by Yu [7] for MIMO with arbitrary linear constraints on $\mathbf{\Omega}$, of which the partial protection GMBC is a case. Utilizing this bound for the partial protection GMBC, R^{B}_C is equal to the saddle point of the following problem:

$$\max_{\mathbf{\Omega}_{\mathrm{S}}} \min_{\mathbf{\Omega}_{\mathrm{N}}} \quad \log \frac{|\mathbf{H}\mathbf{\Omega}_{\mathrm{S}}\mathbf{H}^H + \mathbf{\Omega}_{\mathrm{N}}|}{|\mathbf{\Omega}_{\mathrm{N}}|} \tag{10}$$

$$\text{subject to} \quad \mathrm{Tr}(\mathbf{\Omega}_{\mathrm{S}}\mathbf{S}_1) \le 1 \tag{11}$$

$$\vdots$$

$$\mathrm{Tr}(\mathbf{\Omega}_{\mathrm{S}}\mathbf{S}_L) \le 1 \tag{12}$$

$$\mathbf{\Omega}_{\mathrm{N}}(k,k) = \mathbf{I}_{M_k},\ \ k = 1, \dots, K \tag{13}$$

$$\mathbf{\Omega}_{\mathrm{S}} \ge 0, \mathbf{\Omega}_{\mathrm{N}} \ge 0. \tag{14}$$

where $\mathbf{\Omega}_{\mathrm{S}}$ and $\mathbf{\Omega}_{\mathrm{N}}$ are the transmit and noise covariances respectively.

4.1.2 Capacity Region of the Partial Protection GMBC

The capacity region of the partial protection GMBC coincides with the capacity region of its dual multiple access channel (MAC). Letting $\gamma_1, \dots, \gamma_L$ be dual variables of the GMBC problem (10)–(14) and $\mathbf{S} \triangleq \sum_{l=1}^{L} \gamma_l \mathbf{S}_l$, the boundary points of the dual MAC capacity region are the user capacities from the covariances at the saddle points of the following problem:

$$\min_{\mathbf{S}} \max_{\mathbf{\Pi}} \quad \sum_{k=1}^{K} \mu_k \log \left| \frac{\sum_{i=k}^{K} \mathbf{H}_i^H \mathbf{\Pi}_i \mathbf{H}_i + \mathbf{S}}{\sum_{i=k+1}^{K} \mathbf{H}_i^H \mathbf{\Pi}_i \mathbf{H}_i + \mathbf{S}} \right| \tag{15}$$

$$\text{subject to} \quad \mathrm{Tr}(\mathbf{\Pi}) \le 1 \tag{16}$$

$$\mathrm{Tr}(\mathbf{S}\mathbf{\Omega}_{\mathrm{S}}) \le 1 \tag{17}$$

$$\mathbf{S} \ge 0, \mathbf{\Pi} \ge 0, \mathbf{\Pi} \text{ block diagonal}, \tag{18}$$

where $0 \le \mu_1 \le \dots \le \mu_K$, and $\sum_{k=1}^{K} \mu_k = 1$, are user-rate weights which determine each boundary point. $\mathbf{\Pi}_i$ is the transmit covariance of the ith user and $\mathbf{\Pi} = \mathrm{blockdiag}(\mathbf{\Pi}_1, \dots, \mathbf{\Pi}_K)$. If there were only transmit power constraints, $\mathbf{S}$ would be diagonal, and $\mathbf{\Omega}_{\mathrm{S}}$ could be easily eliminated. However, in general, such as in PGMBC, $\mathbf{S}$ is non-diagonal. Therefore $\mathbf{\Omega}_{\mathrm{S}}$ is eliminated by performing a maximization over $\mathbf{\Pi}$ and utilizing the Karush-Kuhn-Tucker

(KKT) condition:

$$\begin{aligned}\boldsymbol{\Omega}_{\mathrm{S}} =&\mu_K \mathbf{S}^{-1} - \mu_1 \left(\sum_{k=1}^{K} \mathbf{H}_k^H \boldsymbol{\Pi}_k \mathbf{H}_k + \mathbf{S} \right)^{-1} \\ &+ \sum_{k=2}^{K} (\mu_{k-1} - \mu_k) \left(\sum_{i=k}^{K} \mathbf{H}_i^H \boldsymbol{\Pi}_i \mathbf{H}_i + \mathbf{S} \right)^{-1}.\end{aligned}$$

After substitution, a minimization over $\gamma_1, \ldots, \gamma_L$ is performed. The alternating maximization and minimization is repeated until the saddle point is reached.

4.2 Full Protection GMBC

Performing singular value decomposition (SVD) on $\tilde{\mathbf{H}}$,

$$\tilde{\mathbf{H}} \triangleq \boldsymbol{\Psi}\boldsymbol{\Lambda}\ddot{\mathbf{V}}^H \qquad \ddot{\mathbf{V}} \triangleq [\bar{\mathbf{V}}\ \mathbf{V}], \tag{19}$$

where $\bar{\mathbf{V}}$ is a thin matrix of orthonormal vectors which spans $\tilde{\mathbf{H}}$ and $\mathbf{V}$ is a thin matrix of orthonormal vectors which spans the nullspace of $\tilde{\mathbf{H}}$. Under the full protection PGMBC, $\mathbf{T}$ is constrained to be in the nullspace of $\tilde{\mathbf{H}}$. Hence using $\mathbf{V}$, we create a transmit matrix

$$\mathbf{T} = \mathbf{V}\mathbf{C} \quad \mathbf{C} \in \mathbb{C}^{\mathrm{cols}(\mathbf{V})\times n} \quad (n > 0), \tag{20}$$

which guarantees $\tilde{\mathbf{H}}\mathbf{T}\mathbf{d} = \mathbf{0}$. Here, $\mathbf{V}$ is the *protection matrix* and $\mathbf{C}$ is the *user precoding matrix*. $\mathbf{C}$ is decomposed into $\mathbf{C} = [\mathbf{C}_1, \ldots, \mathbf{C}_K]$, and $\mathbf{x}_k$ can be expressed as $\mathbf{x}_k = \mathbf{V}\mathbf{c}_k$, where $\mathbf{c}_k \triangleq \mathbf{C}_k\mathbf{d}_k$. Its covariance is

$$\boldsymbol{\Omega}_k = \mathbb{E}[\mathbf{V}\mathbf{C}_k\mathbf{d}_k\mathbf{d}_k^H\mathbf{C}_k^H\mathbf{V}^H] = \mathbf{V}\boldsymbol{\Sigma}_k\mathbf{V}^H, \tag{21}$$

where $\boldsymbol{\Sigma}_k \triangleq \mathbb{E}[\mathbf{C}_k\mathbf{d}_k\mathbf{d}_k^H\mathbf{C}_k^H] \geq 0$. Assuming a certain user-ordering, (9) becomes

$$R_k^{\mathrm{B}} = \log \frac{\left|\mathbf{I} + \mathbf{H}_k\mathbf{V}\left(\sum_{i\leq k}\boldsymbol{\Sigma}_i\right)\mathbf{V}^H\mathbf{H}_k^H\right|}{\left|\mathbf{I} + \mathbf{H}_k\mathbf{V}\left(\sum_{i<k}\boldsymbol{\Sigma}_i\right)\mathbf{V}^H\mathbf{H}_k^H\right|}, \tag{22}$$

where $\mathbf{H}_k\mathbf{V}$ is an equivalent *protection-implied channel*, and $\boldsymbol{\Sigma}_k$ is the covariance of the signal component for the protection-implied channel.

DPC yields independent covariances $\boldsymbol{\Sigma}_k$. Therefore

$$\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_1 + \ldots + \boldsymbol{\Sigma}_K, \tag{23}$$

$$\boldsymbol{\Omega} = \mathbf{V}\boldsymbol{\Sigma}\mathbf{V}^H. \tag{24}$$

4.2.1 Sum Capacity of the Full Protection GMBC

The full protection constraints may be regarded as additional $\sum_{z=1}^{Z} \tilde{M}_z$ linear equality constraints on the general BC optimization problem (10)–(14). In optimization, a problem with linear equality constraints can be expressed as an equivalent problem where the equality constraints are eliminated [13]. Via

$$\boldsymbol{\Omega}_{\mathrm{S}} = \mathbf{V}\boldsymbol{\Sigma}_{\mathrm{S}}\mathbf{V}^H, \tag{25}$$

the full protection constraints are eliminated to solve the sum capacity R_C^{B}. The full protection problem is in the same form as (10)–(14), where the channel $\mathbf{H}$ becomes $\mathbf{HV}$ and the constraint matrices on $\boldsymbol{\Sigma}_{\mathrm{S}}$ are $\mathbf{V}^H\mathbf{S}_l\mathbf{V}$, $l = 1, \ldots, L$. In this equivalent problem, $\mathbf{S}_1, \ldots, \mathbf{S}_L$ are the transmit constraints $\mathbf{S}_{\mathrm{SP}}$ or $\mathbf{S}_{\mathrm{PAP}}$.

4.2.2 Capacity Region of the Full Protection GMBC

The capacity region of the full protection GMBC is denoted by $\mathcal{C}_{\mathrm{B}}(\mathbf{H}, \mathbf{S}_1, \ldots, \mathbf{S}_L|\mathbf{V})$ where $\mathbf{V}$ spans the null space as defined in (19) and $\mathbf{S}_1, \ldots, \mathbf{S}_L$ are the transmit antenna constraints. Its boundary points are obtained from saddle points of the MAC dual (15)–(18) via (25).

4.3 General PGMBC

The capacity region of the general PGMBC where some of the receivers have full protection and some have partial protection is obtained by: First, obtaining the protection matrix $\mathbf{V}$ corresponding to the fully protected receivers. Then, solving the partial-protection GMBC problem under the protection-implied channel $\mathbf{HV}$. In this equivalent problem, the channel of the kth partially-protected receiver becomes $\mathbf{H}_k\mathbf{V}$.

Under a sum-power transmitter constraint,

$$I_{\mathrm{lim},z} \geq P_{\boldsymbol{\Omega}}\mathrm{Tr}(\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z) \quad z = 1, \ldots, Z. \tag{26}$$

When (26) is met, the interference constraints remain inactive. Consequently, the capacity region of partial protection coincides with that of no protection.

On the otherhand, when $I_{\mathrm{lim},z} \to 0^+$, $z = 1, \ldots, Z$, the problem can be simplified to a full protection problem where,

$$\lim_{I_{\mathrm{lim},z}\to 0^+} R_C^{\mathrm{B}} = R_{C,\mathrm{FP}}^{\mathrm{B}}, \tag{27}$$

where $R_{C,\mathrm{FP}}^{\mathrm{B}}$ is the sum capacity under full protection.

4.4 Gap to Sum Capacity of Having Only Channel Power Information

In some of the applications described in Sect. 3, transmit power back-off (TPBO) has been widely applied, where the transmitter only has knowledge of the channel power to each protected receiver [14]. In TPBO, the transmit power is reduced to guarantee that the receiver protection constraints are met, where

$$P_{\boldsymbol{\Omega}}^{\mathrm{backoff}} \triangleq \min\left(P_{\boldsymbol{\Omega}}, \min_{z=1,\ldots,Z} I_{\mathrm{lim},z}/\mathrm{Tr}(\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z)\right). \tag{28}$$

The achievable region of TPBO is the capacity region of the GMBC with sum-power constraint $P_{\boldsymbol{\Omega}}^{\mathrm{backoff}}$. This represents the achievable region when the transmitter only has knowledge of the channel power of an unserved protected receiver. By finding the capacity region of the PGMBC, we also find the rate gap between TPBO and capacity-achieving transmission schemes. Therefore, by appropriately modeling the scenarios in Sect. 3 as PGMBCs, we find the potential gains of having complete CSI of the channel to the protected receivers as opposed to having only knowledge of their channel powers.

The sum rate gap of TPBO to sum capacity is defined as $R^{\text{B,Gap}}_{\text{TPBO}} = R^{\text{B}}_{C} - R^{\text{B}}_{\text{TPBO}}$. Obviously, when $P^{\text{backoff}}_{\mathbf{\Omega}} = P_{\mathbf{\Omega}}$, the protection constraints are inactive, the TPBO achievable region and PGMBC capacity region and GMBC capacity region coincide, and there are no rate gaps. However, when $P^{\text{backoff}}_{\mathbf{\Omega}}/P_{\mathbf{\Omega}}$ is very small, TPBO leads also to very small achievable spectral efficiencies compared to the capacity region because of large power reduction. As $I_{\lim,z} \to 0^+$, $P^{\text{backoff}}_{\mathbf{\Omega}} \to 0$ and $R^{\text{B}}_{\text{TPBO}} \to 0$. Therefore, using (27), we have the rate gap of TPBO to sum capacity at full protection:

$$\lim_{I_{\lim,z}\to 0^+} R^{\text{B,Gap}}_{\text{TPBO}} = R^{\text{B}}_{C,\text{FP}} \quad z = 1, \ldots, Z. \tag{29}$$

Because the spectral efficiency of TPBO approaches zero under full protection, this means that non-zero capacity at full protection is only achieved when a precoding algorithm utilizes both phase information in addition to amplitude information of the channel to protected receivers. Obviously, though it may not achieve the sum capacity, a precoder which utilizes the phase information has a sum-rate with infinite ratio over the sum rate of TPBO.

5. Precoding for the PGMBC

As mentioned in the introduction, capacity-achieving precoding has been achieved through the methods in [10], in which weighted-sum-rate optimization jointly considers receive power level constraints and transmit power level constraints. Because convex optimization is involved, there is a larger requirement of operations to ensure that the receive power level constraints are met. The protection-implied precoding approach allows direct control of the receive power level to the protected users, while imposing a limit on the transmission power, without the need for convex optimization procedures. Therefore, less number of operations can be performed. Further discussion on the complexity are found later in this section.

The achievable regions of protection implied precoding will be introduced. It will be shown that its achievable region is the capacity region under full-protection, and does not reach the capacity region under partial-protection. The achievable region is obtained through solving for the capacity-achieving user precoding over the protection implied channel.

Solving for the capacity-achieving precoding on the protection-implied channel involves convex optimization, but this analysis is performed in this chapter to illustrate the optimum information rates of this new precoding scheme. A protection-implied multiple access channel (MAC) will be introduced, and the optimization is performed through this MAC to solve for the achievable regions.

5.1 The Protection-Implied Multiple Access Channel

A protection-implied MAC is a MAC whose channel from the kth user is given by $(\mathbf{H}_k\mathbf{Q})^H$, where $\mathbf{Q}$ is a general protection matrix. Under a sum-power assump)tion at the BC, via minimax duality we can assume $\mathbf{s} \sim \mathcal{CN}(0, \mathbf{I})$ without loss of generality. Hence, the rate of receiver k for any set of MAC transmit covariances $\mathbf{\Pi}_1, \ldots, \mathbf{\Pi}_K$ and successive decoding where receiver 1 is decoded first, receiver 2 second, and so on, is

$$R^{\text{M}}_k = \log \frac{\left|\mathbf{I} + \sum_{i=k}^{K}\left(\mathbf{Q}^H\mathbf{H}_i^H\mathbf{\Pi}_i\mathbf{H}_i\mathbf{Q}\right)\right|}{\left|\mathbf{I} + \sum_{i=k+1}^{K}\left(\mathbf{Q}^H\mathbf{H}_i^H\mathbf{\Pi}_i\mathbf{H}_i\mathbf{Q}\right)\right|}, \tag{30}$$

with transmit power $P_i = \text{Tr}(\mathbf{\Pi_i})$. For a sum-power constraint $P_{\mathbf{\Pi}} = \sum_{i=1}^{K} P_{\mathbf{\Pi},i}$, the capacity region is

$$\mathcal{C}_{\text{M}}((\mathbf{HQ})^H, P_{\mathbf{\Pi}}^{-1}\mathbf{I}) \triangleq \bigcup_{\text{Tr}(\mathbf{\Pi}P_{\mathbf{\Pi}}^{-1}\mathbf{I})\leq 1} (R_1, \ldots, R_K): \\ \sum_{k=1}^{K} R_k \leq \log\left|\mathbf{I} + \sum_{k=1}^{K}\mathbf{Q}^H\mathbf{H}_k^H\mathbf{\Pi}_k\mathbf{H}_k\mathbf{Q}\right|. \tag{31}$$

5.2 Protection-Implied Precoding for the Full Protection GMBC

The sum-power constraint of the protection-implied BC is applied on $\mathbf{\Sigma}$. However, for the protection-constrained BC, the sum-power constraint is applied on $\mathbf{\Omega}$. Nevertheless, the following theorem holds.

Theorem 1: $\mathcal{C}_{\text{B}}(\mathbf{H}, P_{\mathbf{\Omega}}^{-1}\mathbf{I}|\mathbf{V}) = \mathcal{C}_{\text{M}}(\mathbf{V}^H\mathbf{H}^H, P_{\mathbf{\Omega}}^{-1}\mathbf{I})$.

Proof: Via (23)–(24), and (A· 2), for full protection,

$$P_{\mathbf{\Omega}} \geq \sum_{k=1}^{K}\text{Tr}(\mathbf{\Omega}_k) = \sum_{k=1}^{K}\text{Tr}(\mathbf{\Sigma}_k), \ (\mathbf{Q} = \mathbf{V}). \tag{32}$$

Therefore, the full-protection-implied channel solution satisfies the constraints of the BC, and $\mathcal{C}_{\text{B}}(\mathbf{HV}, P_{\mathbf{\Omega}}^{-1}\mathbf{I}) = \mathcal{C}_{\text{B}}(\mathbf{H}, P_{\mathbf{\Omega}}^{-1}\mathbf{I}|\mathbf{V})$. SINR-matching duality states that every achievable rate vector in a sum-power BC is achievable in its dual MAC [6]. Treating the protection-implied MAC $\mathbf{H}_{\text{Q}}^H$ as a general MAC, $\mathcal{C}_{\text{B}}(\mathbf{H}, P_{\mathbf{\Omega}}^{-1}\mathbf{I}|\mathbf{V}) = \mathcal{C}_{\text{M}}(\mathbf{V}^H\mathbf{H}^H, P_{\mathbf{\Omega}}^{-1}\mathbf{I})$, which was to be proven. □

By utilizing Theorem 1, we arrive at capacity-achieving precoding under full protection using SINR-matching duality user precoding.

Sum-power Full Protection Protection-implied Precoding:

1. Evaluate $\mathbf{V}$ (19) and set-up the protection-implied channel.
2. Perform multiuser precoding on the protection implied channel $\mathbf{HV}$. For SINR-matching duality precoding, formulate and optimize the protection-implied MAC (31). Perform the MAC-to-BC transformations [6] to obtain $\mathbf{\Sigma}$. Transform $\mathbf{\Sigma}$ into $\mathbf{C}$.

3. Repeat step 2 for other receiver permutations. Select the best $\boldsymbol{\Omega}$ (e.g. highest sum capacity) among the receiver permutations.

5.3 Protection-Implied Precoding for the Partial Protection GMBC

The SINR duality transform can be used for sum-power partial protection transmission by designing the protection matrix to set an upperbound on the protection and then optimizing its protection-implied BC. The partial protection (PP) matrix is

$$\mathbf{Q} = \mathbf{Q}_{\mathrm{PP}} \triangleq \sigma\epsilon\mathbf{V}\mathbf{V}^H + \sigma\delta\mathbf{I}, \qquad \text{(PP matrix)} \tag{33}$$

where $0 \leq \delta \leq 1$ controls the maximum protection level, and σ and $\epsilon \geq 0$ are set for $\mathbf{T}$ to satisfy power constraints. Proper selection of δ, σ and ϵ are discussed in Sect. 5.3.3. It will be shown in section 5.3.4 that under sum-power precoding, $\epsilon = 1 - \delta$, which results in

$$\mathbf{Q} = \sigma[(1-\delta)\mathbf{V}\mathbf{V}^H + \delta\mathbf{I}], \qquad \text{(sum-power PP matrix)} \tag{34}$$

The following theorems show the properties of special cases of SINR-matching precoding.

5.3.1 Full Protection Special Case

To show near-sum-capacity of the partial protection precoding obtained through SINR duality, first we consider the full protection special case of (33) where, $\delta = 0$ and $\sigma = 1$. This sets up the protection-implied channel, $\mathbf{H}_{\mathbf{Q}} = \mathbf{H}\mathbf{V}\mathbf{V}^H$.

Theorem 2: $\mathcal{C}_{\mathrm{B}}(\mathbf{H}\mathbf{V}\mathbf{V}^H, P_{\boldsymbol{\Omega}}^{-1}\mathbf{I}) = \mathcal{C}_{\mathrm{B}}(\mathbf{H}, P_{\boldsymbol{\Omega}}^{-1}\mathbf{I}|\mathbf{V})$.

Proof: Since $\mathbf{V}\mathbf{V}^H$ is hermitian, the covariance matrix of the protection-constrained BC is $\boldsymbol{\Omega} = \mathbf{V}\mathbf{V}^H\boldsymbol{\Sigma}\mathbf{V}\mathbf{V}^H$. By setting $\boldsymbol{\Sigma} = \mathbf{V}\bar{\boldsymbol{\Sigma}}\mathbf{V}^H$, we have $\boldsymbol{\Omega} = \mathbf{V}\bar{\boldsymbol{\Sigma}}\mathbf{V}^H$. Via (A·2), $\mathrm{Tr}(\boldsymbol{\Sigma}) = \mathrm{Tr}(\bar{\boldsymbol{\Sigma}})$. Therefore, every rate vector of the protection-implied BC $\mathbf{H}\mathbf{V}$ can be achieved by the protection-implied BC $\mathbf{H}\mathbf{V}\mathbf{V}^H$, and

$$\mathcal{C}_{\mathrm{B}}(\mathbf{H}\mathbf{V}\mathbf{V}^H, P_{\boldsymbol{\Omega}}^{-1}\mathbf{I}) = \mathcal{C}_{\mathrm{B}}(\mathbf{H}\mathbf{V}, P_{\bar{\boldsymbol{\Sigma}}}^{-1}\mathbf{I} = P_{\boldsymbol{\Sigma}}^{-1}\mathbf{I}). \tag{35}$$

Via Theorem 1, $\mathcal{C}_{\mathrm{B}}(\mathbf{H}\mathbf{V}\mathbf{V}^H, P_{\boldsymbol{\Omega}}^{-1}\mathbf{I}) = \mathcal{C}_{\mathrm{B}}(\mathbf{H}, P_{\boldsymbol{\Omega}}^{-1}\mathbf{I}|\mathbf{V})$. □

Theorem 3: $\mathcal{C}_{\mathrm{M}}(\mathbf{V}\mathbf{V}^H\mathbf{H}^H, P_{\boldsymbol{\Omega}}^{-1}\mathbf{I}) \subseteq \mathcal{C}_{\mathrm{B}}(\mathbf{H}, P_{\boldsymbol{\Omega}}^{-1}\mathbf{I}|\mathbf{V})$.

Proof: The SINR-duality transform on the protected $\mathbf{H}_{\mathbf{Q}} = \mathbf{H}\mathbf{V}\mathbf{V}^H$ gives $\sum_{k=1}^{K}\mathrm{Tr}(\boldsymbol{\Pi}_k) = \sum_{k=1}^{K}\mathrm{Tr}(\boldsymbol{\Sigma}_k)$ for any $\mathbf{Q}$. Expanding this via (A·3),

$$\begin{aligned}\sum_{k=1}^{K}\mathrm{Tr}(\boldsymbol{\Pi}_k) &= \sum_{k=1}^{K}\mathrm{Tr}(\mathbf{V}\mathbf{V}^H\boldsymbol{\Sigma}_k\mathbf{V}\mathbf{V}^H + \bar{\mathbf{V}}\bar{\mathbf{V}}^H\boldsymbol{\Sigma}_k\bar{\mathbf{V}}\bar{\mathbf{V}}^H)\\ &= \sum_{k=1}^{K}\mathrm{Tr}(\boldsymbol{\Omega}_k) + \sum_{k=1}^{K}\mathrm{Tr}(\bar{\mathbf{V}}\bar{\mathbf{V}}^H\boldsymbol{\Sigma}_k\bar{\mathbf{V}}\bar{\mathbf{V}}^H).\end{aligned} \tag{36}$$

The power terms $\sum_{k=1}^{K}\mathrm{Tr}(\bar{\mathbf{V}}\bar{\mathbf{V}}^H\boldsymbol{\Sigma}_k\bar{\mathbf{V}}\bar{\mathbf{V}}^H)$ represent the power of $\boldsymbol{\Sigma}_k$ allocated in the nullspace of $\mathbf{H}\mathbf{V}\mathbf{V}^H$. Since it is allocated in the nullspace, it does not contribute to capacity of $\mathbf{H}\mathbf{V}\mathbf{V}^H$. These terms represent the additional required power for every rate vector in the MAC which means that it cannot achieve the BC capacity region.

Theorem 4: The sum capacity points of the sum-power protected BC ($\mathbf{H}$) with full protection constraints defined from $\mathbf{V}$ are achievable in the protection-implied MAC with protection-implied channel $\mathbf{V}\mathbf{V}^H\mathbf{H}^H$ with the same sum-power.

Proof: For each receiver of the protection-implied BC $\mathbf{H}_{\mathbf{Q}} = \mathbf{H}\mathbf{V}\mathbf{V}^H$, Eq. (22) can be expressed as

$$R_k^{\mathrm{B}} = \log\left|\mathbf{I} + \mathbf{A}_k^{-1/2}\mathbf{H}_k\mathbf{V}\mathbf{V}^H\boldsymbol{\Sigma}_k\mathbf{V}\mathbf{V}^H\mathbf{H}_k^H\mathbf{A}_k^{-1/2}\right|, \tag{37}$$

where $\mathbf{A}_k \triangleq \mathbf{I} + \mathbf{H}_k\mathbf{Q}\left(\sum_{i<k}\boldsymbol{\Sigma}_i\right)\mathbf{Q}^H\mathbf{H}_k^H$.

Expanding $\boldsymbol{\Sigma}_k$ by (A·4),

$$R_k^{\mathrm{B}} = \log\left|\mathbf{I} + \mathbf{A}_k^{-1/2}\mathbf{H}_k\boldsymbol{\Theta}_k\mathbf{H}_k^H\mathbf{A}_k^{-1/2}\right|, \tag{38}$$

where $\boldsymbol{\Theta} \in \mathcal{R}(\mathbf{V})$. The $\bar{\boldsymbol{\Theta}} \in \mathcal{R}(\bar{\mathbf{V}})$ component does not provide capacity and is wasted power. Therefore at sum capacity points, all power is given to $\mathcal{R}(\mathbf{V})$; $\boldsymbol{\Sigma}_k = \mathbf{V}\mathbf{V}^H\boldsymbol{\Theta}_k\mathbf{V}\mathbf{V}^H$, and $\sum_{k=1}^{K}\mathrm{Tr}(\bar{\mathbf{V}}\bar{\mathbf{V}}^H\boldsymbol{\Sigma}_k\bar{\mathbf{V}}\bar{\mathbf{V}}^H) = 0$. Eq. (36) then becomes $\sum_{k=1}^{K}\mathrm{Tr}(\boldsymbol{\Pi}_k) = \sum_{k=1}^{K}\mathrm{Tr}(\boldsymbol{\Omega}_k)$. □

5.3.2 No Protection Special Case

In the no protection (NP) case, $\sigma = \delta = 1$ and $\mathbf{H}_{\mathbf{Q}} = \mathbf{H}$. The SINR-matching duality, which was originally derived in [6] for this case, holds.

5.3.3 General Sum-Power Partial Protection Case

The full protection and no protection cases are extreme points of the general partial protection case. Theorem 4 and Sect. 5.3.2 show that the BC and MAC achieve the same sum capacity in the two extreme cases. Under general partial protection ($0 < \delta < 1$), the protection-implied MAC and BC do not exhibit duality, and the power penalty between the MAC and BC powers at sum capacity points generally increases as δ veers from 0 and 1. This can be partially addressed by selecting $\sigma \geq 1$ as a factor for $\mathbf{Q}$ or by readjusting δ and ϵ and redoing the MAC optimization and transforms. By carefully selecting δ, σ, and ϵ, the constraints can be more closely approached. In addition, the sum-rate is asymptotic to sum-capacity as $I_{\mathrm{lim},z} \to 0$ and $I_{\mathrm{lim},z} \to P_{\boldsymbol{\Omega}}\mathrm{Tr}(\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z)$, as shown earlier. Therefore, near-sum-capacity precoders can be achieved, as verified through a simulation example in Sect. 7.

5.3.4 Selection of δ, ϵ and σ under Sum-Power Constraints

From (6) and (24) we have the following constraint equation for each protected element,

$$\mathrm{Tr}(\mathbf{Q}\boldsymbol{\Sigma}\mathbf{Q}^H\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z) \leq I_{\mathrm{lim},z}. \tag{39}$$

σ is used to for power normalization after MAC optimization and covariance transformation. Initially, $\sigma_{\mathrm{init}} = 1$ which sets a temporary $\mathbf{Q} = \epsilon\mathbf{V}\mathbf{V}^H + \delta\mathbf{I}$. Expanding (39),

$$\mathrm{Tr}((\epsilon^2\mathbf{V}\mathbf{V}^H\mathbf{\Sigma}\mathbf{V}\mathbf{V}^H + 2\delta\epsilon\mathbf{\Sigma}\mathbf{V}\mathbf{V}^H + \delta^2\mathbf{\Sigma})\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z) \leq I_{\mathrm{lim},z}, \tag{40}$$

since $\mathbf{Q}$ is hermitian symmetric. Because $\mathbf{V}$ is orthogonal to $\tilde{\mathbf{H}}_z$, $\delta^2\mathrm{Tr}(\mathbf{\Sigma}\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z) \leq I_{\mathrm{lim},z}$. Since both $\mathbf{\Sigma}$ and $\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z$ are positive semidefinite, we use the result of [15] to adjust the limit, $\delta^2\mathrm{Tr}(\mathbf{\Sigma}\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z) \leq \delta^2\mathrm{Tr}(\mathbf{\Sigma})\mathrm{Tr}(\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z) \leq I_{\mathrm{lim},z}$. From this, we obtain a δ which guarantees that partial protection constraints for all protected elements are met,

$$\delta = \min_{z=1,\ldots,Z} \sqrt{I_{\mathrm{lim},z}/\mathrm{Tr}(\mathbf{\Sigma})\mathrm{Tr}(\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z)}. \tag{41}$$

Next, ϵ is chosen to guarantee $\mathrm{Tr}(\mathbf{\Omega}) \leq \mathrm{Tr}(\mathbf{\Sigma})$ so that the power constraint $P_{\mathbf{\Omega}}$ is not exceeded by $\mathrm{Tr}(\mathbf{\Omega})$ when $\mathrm{Tr}(\mathbf{\Sigma}) = P_{\mathbf{\Omega}}$. When ϵ is chosen for this condition, (41) gives a lower bound on δ under both sum-power and per-antenna-power constraints,

$$\delta_{\mathrm{bnd}} \triangleq \min\left(1, \min_{z=1,\ldots,Z} \sqrt{I_{\mathrm{lim},z}/P_{\mathbf{\Omega}}\mathrm{Tr}(\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z)}\right) \tag{42}$$

where $\delta_{\mathrm{bnd}} = 1$ is set as the upper limit in case protection is not required. To meet both the protection and power constraints through ϵ, we use (24) and (A$\cdot$1), to arrive at the condition $\mathrm{Tr}\left(\mathbf{C}\mathbf{C}^H\left((\epsilon+\delta)^2\mathbf{V}\mathbf{V}^H + \delta^2\bar{\mathbf{V}}\bar{\mathbf{V}}^H\right)\right) \leq \mathrm{Tr}(\mathbf{C}\mathbf{C}^H)$. Since $\delta \leq 1$, the maximum coefficient of $\bar{\mathbf{V}}\bar{\mathbf{V}}^H$ is 1. Via (A$\cdot$1), the inequality is always met when the maximum coefficient of $\mathbf{V}\mathbf{V}^H$ is also 1, which leads to $(\epsilon+\delta)^2 \leq 1$. Solving for ϵ under sum-power,

$$\epsilon_{\mathrm{SP}} \triangleq 1 - \delta. \quad \text{(sum-power)} \tag{43}$$

Once δ is selected, $\mathbf{Q}_{\mathrm{tmp}} = (1-\delta)\mathbf{V}\mathbf{V}^H + \delta\mathbf{I}$ is set. The MAC is optimized and the covariance transform is performed to form temporary BC covariance matrices $\mathbf{\Sigma}_{\mathrm{tmp}}$ and $\mathbf{\Omega}_{\mathrm{tmp}}$. Since $\mathrm{Tr}(\mathbf{\Omega}_{\mathrm{tmp}}) \leq \mathrm{Tr}(\mathbf{\Sigma}_{\mathrm{tmp}})$ by a selection of $\epsilon = 1-\delta$, in general $\mathrm{Tr}(\mathbf{\Omega}_{\mathrm{tmp}}) \leq P_{\mathbf{\Omega}}$, and sum capacity is not achieved. By selecting a proper σ, the power of $\mathbf{\Omega} = \sigma^2\mathbf{\Omega}_{\mathrm{tmp}}$ is maximized, thereby pushing the achieved rates closer to capacity. For sum-power closed-form (SPCF) solutions,

$$\sigma_{\mathrm{SPCF}} \triangleq \min\left(\sqrt{\frac{P_{\mathbf{\Omega}}}{\mathrm{Tr}(\mathbf{\Omega}_{\mathrm{tmp}})}}, \min_{\substack{z=1,\\ \ldots,Z}} \sqrt{\frac{I_{\mathrm{lim},z}}{\mathrm{Tr}(\mathbf{\Omega}_{\mathrm{tmp}}\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z)}}\right).$$

5.3.5 Iterative Protection Descent

δ_{bnd}, and its respective σ_{SPCF} (43) ensure that protection constraints are met. However, depending on the receiver channels, the achieved received powers may be too low compared to constraints. To reach closer to capacity, an iterative method to find a near-optimum δ can be done. In the Iterative Protection Descent (IPD) algorithm, $\delta_{\mathrm{init}} = 1$, and steps 1 to 3 of the sum-power Partial Protection Precoding are performed. If protection constraints are not met by $\mathbf{\Omega}$, step 4 is done. The updated δ for the next iteration is,

$$\delta_{\mathrm{new}} = \max\left(\delta_{\mathrm{bnd}}, \kappa\delta_{\mathrm{old}}\left(\min_z \frac{I_{\mathrm{lim},z}}{\mathrm{Tr}(\mathbf{\Omega}_{\mathrm{tmp}}\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z)}\right)^{\eta}\right), \tag{44}$$

where $\kappa < 1$, and $\eta < 0.5$. Setting $\kappa < 1$ forces δ to make at least a geometric descent in value. It helps avoid asymptotic or cyclic evolution of δ as δ approaches the capacity-achieving δ; Parameter η controls the speed of interference level descent.

At the next iteration, new values are computed for σ_{SPIPD},

$$\sigma_{\mathrm{SPIPD}} \triangleq \sqrt{P_{\mathbf{\Omega}}/\mathrm{Tr}(\mathbf{\Omega}_{\mathrm{tmp}})}. \tag{45}$$

Utilizing Eqs. (42) and (43)–(45) we arrive at a partial protection precoding scheme:

Sum-power Partial Protection Protection-implied Precoding:

1. Assign $\sigma_{\mathrm{init}} = 1$, and select the initial δ to set-up $\mathbf{Q}$. For SPCF, $\delta = \delta_{\mathrm{bnd}}$. For IPD, $\delta_{\mathrm{init}} = 1$.
2. Perform multi-user precoding on the protection-implied channel $\mathbf{HQ}$. For SINR-matching duality user precoding, optimize the protection-implied MAC with ($\mathbf{H}_{\mathbf{Q}} = \mathbf{HQ}$); perform the MAC-to-BC transformations [6]; obtain $\mathbf{\Omega}_{\mathrm{tmp}}$ via (24).
3. Normalize $\mathbf{\Omega}_{\mathrm{tmp}}$ by σ^2_{SPCF} or $\sigma^2_{\mathrm{SPIPD}}$ to obtain $\mathbf{\Omega}$.
4. IPD Algorithm — A: If the protection constraints are not satisfied, reset $\sigma_{\mathrm{init}} = 1$, adjust δ (44) and σ_{SPIPD} (45). Repeat steps 2 to 4. Terminate when $\delta_{\mathrm{new}} \leq \delta_{\mathrm{bnd}}$. B: But if but the achieved rate increases compared to the rate of the previous δ, continue to decrease δ until a maximum rate is reached.
5. Repeat steps 1 to 4 for the other receiver permutations. Select the best $\mathbf{\Omega}$ (e.g. maximum sum-rate) among the permutations.

Under SINR-matching precoding, the achievable rates of $\mathbf{H}_{\mathbf{Q}}$ increases as δ increases due to the increasing eigenvalues for higher δ. However, under sub-optimum precoding, the achievable rate for a higher δ is not necessarily higher. Therefore, Step 4B is additionally performed to search for the best rates from lower δ values.

The sub-optimum algorithms can be further refined to yield better spectral efficiencies depending on the user precoding method used. For example, under per-antenna power constraints, each candidate ϵ is taken as the greater among two quadratic solutions, and ϵ_{PAP} is

$$\epsilon_{\mathrm{PAP}} \triangleq \min_{n=1,\ldots,N}\left(-\beta_n + \sqrt{\beta_n^2 - 4\alpha_n\chi_n}\right)\Big/2\alpha_n, \tag{46}$$

where a complex-valued ϵ_{PAP} cannot be chosen. Eq. (46) guarantees that the power constraints are met when a sum-power constraint equal to the total of the per-antenna constraints is imposed on $\mathbf{\Sigma}$.

5.4 Computational Complexity of Precoding for the PGMBC

Each minimization or maximization step to solve for optimum precoding vectors for the PGMBC can be considered as a determinant maximization program with linear matrix inequality constraints [16]. Having N as the number of transmit antennas, the complexity of the Newton step using the "schoolbook" method is given by $O(N^3)$ which comes from the complexity of calculating the Hessian. $O(N^3)$ is also the complexity to obtain the nullspace vectors $\mathbf{V}$ using "schoolbook" Gauss-Jordan elimination. The complexity of obtaining $\mathbf{Q}$, and DPC-ZF for $\mathbf{C}$ are the same. Recent implementations have reduced the complexity of these algorithms to $O(N^{2.376})$ using group-theoretic algorithms [17].

Under the PGMBC, in general, the variables $\boldsymbol{\Omega}$ and the protection constraint matrices $\mathbf{S}$ are dense. Hence, further complexity reduction is not applicable. Therefore, the added complexity of achieving the optimum precoders compared to protection-implied precoding is found in the total number of Newton steps. The total number of Newton steps is $O(\sqrt{N}\log(1/\iota))$ under fixed-reduction method, and $O(\log(1/\iota))$ with predictor steps, where ι is the required accuracy for convergence [16].

In practice, $O(\log(1/\iota)) \approx 30$ for a single minimization or maximization step [13]. If iterative maximization and minimization is performed, then the total number of required becomes $O(\log^2(1/\iota))$. However, joint maximization-minimzation interior-point algorithm developed by Yu in [8] has a complexity of $O(\log(1/\iota))$. Therefore, with the use of an interior-point algorithm with predictor steps, protection-implied algorithms and capacity-achieving schemes have the same order of complexity, though capacity-achieving schemes may involve much more calculations, which include Hessian calculations and MAC-to-BC covariance transformation. Of course, the actual disparity in calculation time and memory requirements depends on the specific implementation.

6. Sensitivity of Protection-Implied Precoding to Channel Estimation Error

In this section, we investigate the sensitivity of protection-implied precoding on achieving a receive power level to channel estimation error. For simplicity, the analysis is conducted for the case of sum-power transmitter constraints and a single protected receiver with a single antenna element. The upper limit of the receive power level is

$$I_{\max,z} = P_{\boldsymbol{\Omega}}\mathrm{Tr}(\tilde{\mathbf{H}}_z^H\tilde{\mathbf{H}}_z). \tag{47}$$

The analysis for its sensitivity begins with the definition for the estimate of the channel to the protected receiver, $\hat{\tilde{\mathbf{H}}} \triangleq \tilde{\mathbf{H}} + \mathring{\tilde{\mathbf{H}}}$ where $\mathring{\tilde{\mathbf{H}}}$ is the estimation error of the channel to the protected receiver.

Letting $\hat{\bar{\mathbf{V}}}$ and $\hat{\mathbf{V}}$ denote basis vector and nullspace vector of the channel estimate respectively, from (A·4) we have the following identities:

$$\begin{aligned}\hat{\bar{\mathbf{V}}}\hat{\bar{\mathbf{V}}}^H =& \mathbf{V}\mathbf{V}^H\hat{\bar{\mathbf{V}}}\hat{\bar{\mathbf{V}}}^H\mathbf{V}\mathbf{V}^H + \bar{\mathbf{V}}\bar{\mathbf{V}}^H\hat{\bar{\mathbf{V}}}\hat{\bar{\mathbf{V}}}^H\bar{\mathbf{V}}\bar{\mathbf{V}}^H \\ =& \|\mathbf{V}^H\hat{\bar{\mathbf{V}}}\|^2\mathbf{V}\mathbf{V}^H + \|\bar{\mathbf{V}}^H\hat{\bar{\mathbf{V}}}\|^2. \end{aligned} \tag{48}$$

From (A·1),

$$\begin{aligned}\hat{\mathbf{V}}\hat{\mathbf{V}}^H =& \mathbf{I} - \|\mathbf{V}^H\hat{\bar{\mathbf{V}}}\|^2\mathbf{V}\mathbf{V}^H - \|\bar{\mathbf{V}}^H\hat{\bar{\mathbf{V}}}\|^2 \\ =& \left(1 - \|\mathbf{V}^H\hat{\bar{\mathbf{V}}}\|^2\right)\mathbf{V}\mathbf{V}^H + \left(1 - \|\bar{\mathbf{V}}^H\hat{\bar{\mathbf{V}}}\|^2\right)\bar{\mathbf{V}}\bar{\mathbf{V}}^H \end{aligned} \tag{49}$$

Under channel estimation error, instead of (34) the following partial protection matrix is formed under $\sigma = 1$,

$$\hat{\mathbf{Q}} = \sigma[(1-\delta)\hat{\mathbf{V}}\hat{\mathbf{V}}^H + \delta\mathbf{I}], \tag{50}$$

and the receive power at the protected receiver is

$$\mathbf{I} = \mathrm{Tr}(\tilde{\mathbf{H}}\hat{\mathbf{Q}}\boldsymbol{\Sigma}\hat{\mathbf{Q}}^H\tilde{\mathbf{H}}^H). \tag{51}$$

The same analysis as in Sect. 5.3.4 can be done, wherein the sum terms with $\mathbf{V}\mathbf{V}^H$ are eliminated, since $\mathbf{V}$ is orthogonal to $\tilde{\mathbf{H}}$. This results to

$$\begin{aligned} I =& \delta^2\mathrm{Tr}(\boldsymbol{\Sigma}\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}) + (1-\delta)^2\Big(1 \\ &- \|\bar{\mathbf{V}}^H\hat{\bar{\mathbf{V}}}\|^2\Big)^2\mathrm{Tr}\left(\boldsymbol{\Sigma}\bar{\mathbf{V}}\bar{\mathbf{V}}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\bar{\mathbf{V}}\bar{\mathbf{V}}^H\right) \\ \geq& \delta^2 I_{\max} + (1-\delta)^2\left(1 - \|\bar{\mathbf{V}}^H\hat{\bar{\mathbf{V}}}\|^2\right)^2 I_{\max}\mathrm{Tr}(\bar{\mathbf{V}}\bar{\mathbf{V}}^H) \end{aligned} \tag{52}$$

where (52) arises from the trace inequality result of [15]. Note that Tr($\boldsymbol{\Sigma}$ has been replaced by $P_{\boldsymbol{\Omega}}$. Therefore, when the sum power constraint is met and $\sigma = 1$, any user precoding matrix $\boldsymbol{\Sigma}$ on the protection-implied channel does not affect the upper limit, and any variation of $\boldsymbol{\Sigma}$ due to channel estimation error to any receiver (protected or unprotected; served or unserved) does not affect the receive power level I upper bound.

By selecting $\delta = \delta_{\mathrm{bnd}}$, the lower bound is preserved for a target I_{lim}. From (42) and after some derivation,

$$\begin{aligned} I \geq& \min(I_{\max}, I_{\mathrm{lim}}) + \left(1 - \|\bar{\mathbf{V}}^H\hat{\bar{\mathbf{V}}}\|^2\right)^2\Big(I_{\max} \\ &+ \min(I_{\max}, I_{\mathrm{lim}}) - \min(2I_{\max}, 2\sqrt{I_{\max}I_{\mathrm{lim}}})\Big) \end{aligned} \tag{53}$$

Using the definition of basis vectors and the following inner-product relation

$$\|\bar{\mathbf{V}}^H\hat{\bar{\mathbf{V}}}\|^2 = \frac{\|\tilde{\mathbf{H}}(\tilde{\mathbf{H}} + \mathring{\tilde{\mathbf{H}}})^H\|^2}{\|\tilde{\mathbf{H}}\|\|\tilde{\mathbf{H}} + \mathring{\tilde{\mathbf{H}}}\|}, \tag{54}$$

we have the receive power in terms of the channel and its estimation error,

$$\begin{aligned} I \geq& \min(I_{\max}, I_{\mathrm{lim}}) + \left(1 - \frac{\|\tilde{\mathbf{H}}(\tilde{\mathbf{H}} + \mathring{\tilde{\mathbf{H}}})^H\|^2}{\|\tilde{\mathbf{H}}\|\|\tilde{\mathbf{H}} + \mathring{\tilde{\mathbf{H}}}\|}\right)^2\Big(I_{\max} \\ &+ \min(I_{\max}, I_{\mathrm{lim}}) - \min(2I_{\max}, 2\sqrt{I_{\max}I_{\mathrm{lim}}})\Big). \end{aligned} \tag{55}$$

Note that this is an upper-limit prior to power normalization. Therefore, if power normalization is not performed, i.e. $\sigma = 1$, the receive power upper limit is guaranteed.

In the special case of full protection,

$$I|_{I_{\mathrm{lim}}=0} \geq \left(1 - \frac{\|\tilde{\mathbf{H}}(\tilde{\mathbf{H}} + \mathring{\tilde{\mathbf{H}}})^H\|^2}{\|\tilde{\mathbf{H}}\|\|\tilde{\mathbf{H}} + \mathring{\tilde{\mathbf{H}}}\|}\right)^2 I_{\max}. \tag{56}$$

In (55) we see that the increase in receive power over the target depends not only on the magnitude of the estimation error, but also only on the orthogonality of the estimation error $\mathring{\tilde{\mathbf{H}}}$ to the channel $\tilde{\mathbf{H}}$. If the error subspace is the same as the channel subspace, regardless of the error magnitude, there is zero additional receive power. We also find a merit to protection-implied precoding — Through (55), a target receive power can be achieved even with estimation errors, provided that sufficient estimation accuracy to the protected receivers is ensured.

7. Simulation Example

Consider the PGMBC with served channels $\mathbf{H}_1 = [0.5\ \ 2]$, $\mathbf{H}_2 = [2\ \ 0.5]$ and protected channel $\tilde{\mathbf{H}}_1 = [1.1\ \ 0.9]$, $P_{\mathbf{\Omega}} = 1$. Hence, $N = 2$, $K = 2$ and $M_1 = M_2 = \tilde{M}_1 = 1$. The default IPD parameter values are $\kappa = 0.99$, $\eta = 0.1, 0.25$. Since $\tilde{\mathbf{H}}_1 \neq \mathbf{H}_1, \mathbf{H}_2$, we view the received signal at $\tilde{\mathbf{H}}_1$ as interference. The CVX modeling language (ver. 1.2) [18] was used to obtain the solutions.

The sum capacity curve for the PGMBC example is drawn in Fig. 3. It is observed that the sum capacity of partial protection approaches that of full protection as $I_{\text{lim},1} \to 0$, and approach that of no protection as $I_{\text{lim},1} \to P_{\mathbf{\Omega}}\text{Tr}(\tilde{\mathbf{H}}_z^H \tilde{\mathbf{H}}_z) = 2.02$. When $I_{\text{lim},1} \geq 2.02$, (26) is met and the sum capacity curve flattens to that of no protection. The capacity gain of partial protection over full protection increases as the interference limit approaches the interference level yielded by transmission without protection constraints.

The capacity regions for the PGMBC example are shown in Fig. 5. The sum capacity points match those shown in Fig. 3. For this example, the interference constraints remain active for the three interference limit cases ($I_{\text{lim},1} = 0.01, 0.1, 1$). Consequently, sum capacity is achieved at a single point in each case. When $I_{\text{lim},1} \geq 2.02$, the capacity region of partial protection coincides with that of no protection.

Since the desired user channels are symmetric with respect to each other, the no protection region is symmetric across the $R_1 = R_2$ axis. However, the protection constraints introduces increasing asymmetry of the capacity region as the interference limit decreases. Since the protected channel is more correlated to that of the 2nd user, its capacity decreases more sharply compared to the 1st user. For this example, $\mathbf{V}$ is a vector. Therefore, under full protection, the antenna weights are forced to be along $\mathbf{V}$, with no degree of freedom except power normalization to meet the power constraint. Consequently, the capacity region becomes triangular since the boundaries are achieved by time sharing, and maximum sum-rate is achieved by allocating all the power to the 1st user.

The maximum sum-rate and achievable region of TPBO are plotted in Figs. 3 and 5 respectively. The plots show that excellent relative gains in spectral efficiency are achieved by capacity-achieving schemes over TPBO especially at very low interference limits ($P_{\mathbf{\Omega}}^{\text{backoff}}/P_{\mathbf{\Omega}} \ll 1$). The achievable regions of TPBO are symmetric, as opposed to the asymmetry of the capacity regions, especially at low interference limits. This shows that TPBO penalizes less those receivers whose channel is more correlated to that of the protected channel.

The maximum sum rates for some protection-implied SINR-matching precoding are drawn in Fig. 3. The sum ca-

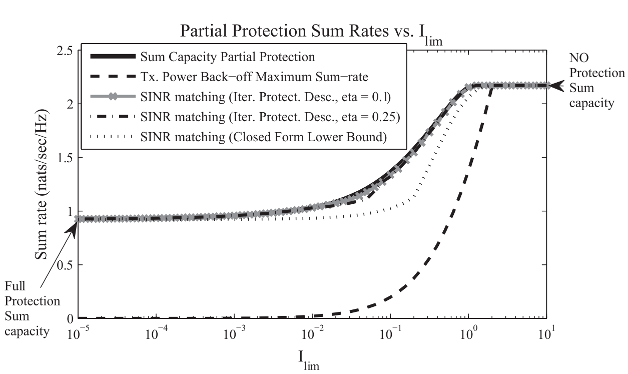

Fig. 3 Simulation example: Maximum sum rates of TPBO and protection-implied precoding vs. Interference power limit.

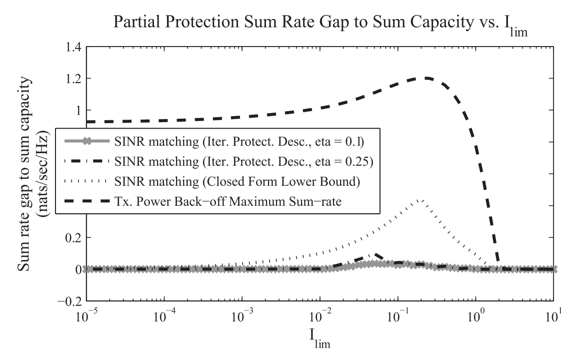

Fig. 4 Simulation example: Sum rate gap to sum capacity of TPBO and protection-implied precoding.

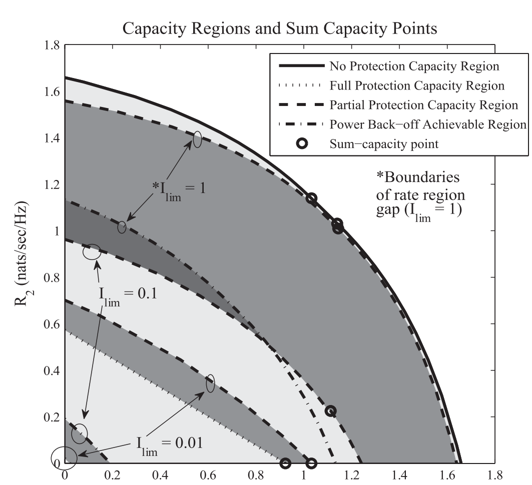

Fig. 5 Simulation example: Capacity regions and sum capacity points under sum-power constraints. The circles and the segments between the circles within each curve represent the sum capacity points.

pacity curve is also drawn for comparison where the same result for no protection is obtained through the methods in [5], [6]. It is observed that the maximum sum rate curves of SINR-matching partial protection precoding approach that of full protection as $I_{\text{lim},1} \to 0$, and approach that of no protection as $I_{\text{lim},1} \to 2.02$ (26). This is because their sum-rates are capacity-achieving at the extreme values, where $\delta = 0$ and $\delta = 1$, which was proven in Theorem 4 and Sect. 5.3.2. The interference limit is never met under the power constraint when $I_{\text{lim},1} > 2.02$ since the limited transmit power prevents the high interference level. Furthermore, the sum rate of IPD is very close to that of the sum capacity, especially as the interference limit approaches the extremes, which shows that this scheme allows near-optimality.

It is also observed in Fig. 3 that the difference of sum rates between IPD and the Closed-Form Lower Bound becomes more evident as the interference limit moves away from the extremes, since their δ values may vary. In addition, by reducing the protection descent parameter from $\eta = 0.25$ to $\eta = 0.1$, the sum capacity is more closely approached by IPD, because higher δ achieves higher capacity, and more careful descent of δ is performed using a lower η. However, this gain in capacity under lower η is with a penalty of an increase in the average number of descent iterations.

In Fig. 4, the gap of the sum rate to sum capacity are plotted. The gaps remained low throughout all values of I_{lim} under iterative protection descent, and higher for closed form lower bound. Under both protection-implied precoding schemes, as $I_{\text{lim}} \to 0$, the gap disappeared. However, under TPBO, the gap achieved the sum capacity at zero interference limit. This gap of TPBO was not monotonically decreasing but exhibited a peak at around 1 decade below $I_{\max}$ (around $I_{\text{lim}} = 0.3$). These results confirm the analysis in Sects. 4.4 and 5.3 — under full-protection, only by having phase knowledge in addition to channel power knowledge, the gap to capacity can be kept arbitrarily low. However, as $I_{\text{lim}} \to I_{\max}$, the gaps to capacity disappear.

The achievable regions of protection-implied, precoding schemes are shown in Fig. 6. Under the no protection and full protection SINR-matching schemes, the achievable region is the capacity region, which includes the sum capacity line segment. Their maximum sum rate points add to their respective sum capacities, which match those shown in Fig. 7. Under partial protection SINR-matching, the maximum sum rates are single points on each curve. This shows that in general, the maximum sum-rates are not line segments, but obtained using a particular user ordering. Under the IPD, the induced interference level depends on the receiver ordering and the descent iterations. Results show that the IPD achievable region is able to reach very close to the capacity region.

The performance of LQ DPC Zero-forcing [19] for the protection-implied channel **HQ** using IPD are also shown in in Fig. 7. The stream power allocation was chosen in order to achieve the maximum capacity under the constraints. It is observed that at low interference limits, LQ DPC Zero-

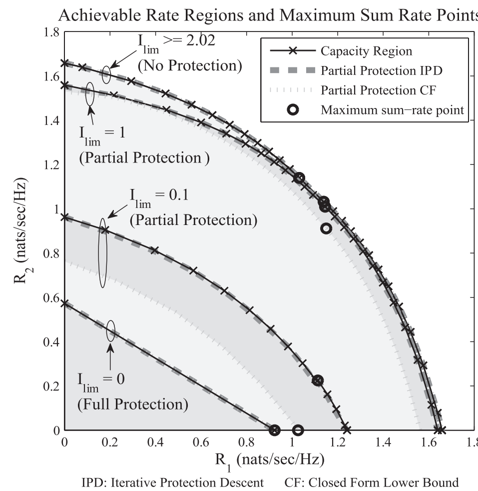

Fig. 6 Simulation example: Achievable rate regions and maximum sum rates of SINR-matching protection-implied precoding. The circles and the segments between the circles within each curve under no protection represent maximum sum rates. $\eta = 0.1$.

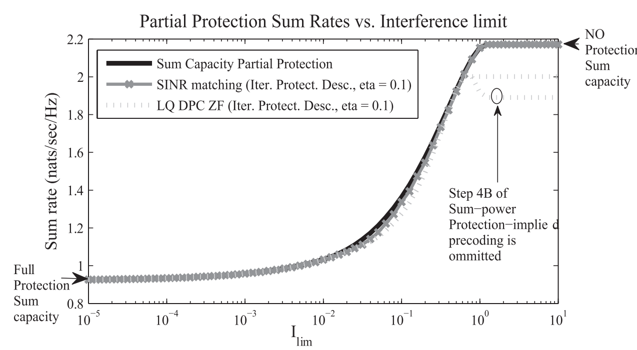

Fig. 7 Simulation example: Maximum sum rates of protection-implied precoding (LQ DPC Zero-forcing) vs. Interference power limit.

forcing performs almost as well as SINR-matching under IPD until around $I_{\text{lim}} \approx 0.6$, in which case it plateaus. At these points, $\delta = 1$ and there is no protection. It is known that the capacity difference of LQ DPC Zero-forcing and optimum DPC is not negligible at moderate SNR values [19]. The sum rate of the sum-power protection-implied BC precoding algorithm where step 4B is ommitted is also shown. It is observed that for this case, the sum rate of partial protection at $I_{\text{lim}} \approx 0.6$ is better than that of no protection. It shows that that a lower value of δ gave a higher achievable rate, which is possible for sub-optimal user precoding. Higher sum-rates were achieved through step 4B.

The convergence properties of the IPD algorithm is shown in Fig. 8. Results show that convergence is achieved within a few iterations for all cases. Moreover, the δ at convergence is significantly higher than δ_{bnd} for all cases, which allows higher capacity over using δ_{bnd} since more energy can be allocated in the null space of the protected channel. As expected, convergence is achieved faster using $\eta = 0.25$. However, $\eta = 0.1$ achieves better sum-rates because it is more careful in its descent. It is observed that the ratio

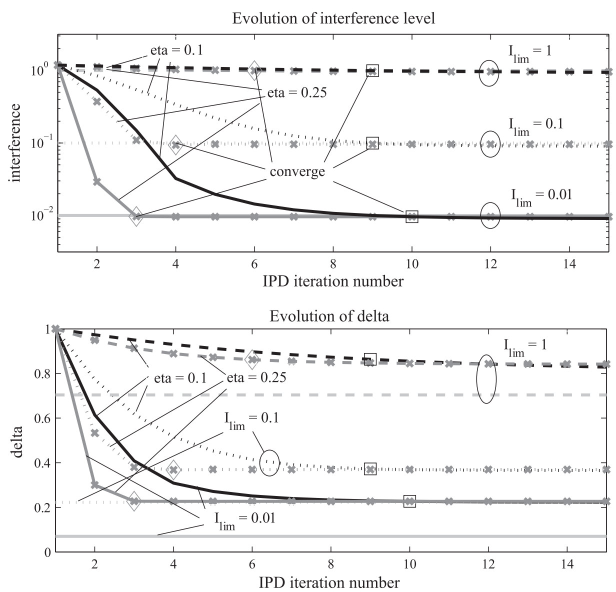

Fig. 8 Simulation example: Evolution of iterative protection descent algorithm. SINR-matching duality precoding is used for user precoding.

of convergence speeds between $\eta = 0.1$ and $\eta = 0.25$ increases with decreasing interference limit. Proper design of η is therefore dependent on the interference limit and the desired gap from capacity.

8. Conclusion

In this paper, we investigated the Gaussian MIMO broadcast channel with receive power protection constraints in addition to transmit power constraints. Application examples for the PGMBC were given and its sum-capacity and capacity region was discussed. The potential spectral efficiency gain of having complete CSI of the channel to protected users, over having only channel power information. The rate gap was derived by deriving the achievable region of transmit power back-off and comparing this with the capacity region. Results have shown that excellent gains of capacity-achieving schemes over transmit power back-off are achieved especially at low receive power limits. As the protection limit approaches zero, the throughput when having only channel power knowledge to the protected receiver approaches zero, and its gap to sum capacity becomes the sum capacity itself. On the otherhand, when the protection limit is high enough such that there is no transmit power back-off, the gap obviously becomes zero.

From this result, we find that feedback of full CSI of the protected channel is very desirable to maximize spectral efficiency of future applications. For example, in spectrum sharing and cognitive radio, this means that coordinated CSI feedback or interference signal feedback from the primary receivers to the secondary transmitter results in significant spectral efficiency gains. However, in practice, this feedback may be difficult to attain.

Capacity-achieving precoding for the PGMBC has been proposed in [10], where Lagrangian duality and MAC-to-BC transformations are used. The protection-implied precoding approach does not achieve the capacity region under partial protection. However, it allows good performance with less computational cost by using established user-precoding schemes on the protection-implied channel. Moreover, it can be easily extended to per-antenna power constraints and the maximum receive power level can be directly controlled by using a single parameter.

Protection-implied precoder design is more intuitive compared to the weighted-sum-rate optimization for two reasons. First, the protection-implied matrix $\mathbf{Q}$ is designed without regard to the channel to the served users $\mathbf{H}$. Second, the user precoding matrix $\mathbf{C}$ on the equivalent channel $\mathbf{HQ}$ is designed without regard to protection constraints. These cannot be done in weighted-sum-rate optimization, since the transmit power and receive protection constraints must be jointly considered in the optimization. For example, under protection implied precoding, since the design of the protection matrix does not consider $\mathbf{H}$, the receive protection can be guaranteed even with channel state estimation (CSE) errors on $\mathbf{H}$. This was illustrated for the single-user single-receive antenna case. However, the same protection isn't necessarily guaranteed using weighted-sum-rate optimization since the CSE error on $\mathbf{H}$ affects the result of each optimization iteration.

Capacity-achieving precoding through generalized SINR-matching duality proposed in [10] requires convex-optimization operations and MAC-to-BC transformations, which may be prohibitively complex in many applications. The protection-implied precoder design does not require convex-optimization operations, which means that it can be less computationally complex. In the case where the original SINR-matching duality based precoding in [6] is used to obtain the user precoding matrix $\mathbf{C}$, sum-capacity is achieved under full-protection and near-sum-capacity can be achieved under partial protection through protection-implied precoding.

There are numerous applications for the PGMBC, as discussed in Sect. 3. For example, partial-protection precoding has been applied to multi-cell coordinated beamforming in cellular systems. The conventional approach to coordinated beamforming was to perform full protection. Full protection does not take advantage of the relative receive powers of the serving and interfering cell. More recently optimum coordinated beamforming was achieved by maximizing the signal-to-leakage-plus-noise ratio (Max-SLNR) [20]. Max-SLNR maximizes the system capacity, but involves convex optimization steps. In [21] the partial protection approach was utilized to avoid convex optimization while providing better performance than full protection.

The proposed algorithms assume perfect channel state information at the transmitter and perfect synchronization, which are impossible in practice. Nevertheless, these algorithms produce useful upperbounds for a broad class of applications. In general, CSE is more difficult for receivers under protection constraints, which in turn prevents accurate protection. This CSE-vs-Protection paradox is an issue that must be addressed in future work.

References

[1] H. Sato, "An outer bound to the capacity region of broadcast channels (corresp.)," IEEE Trans. Inf. Theory, vol.IT-24, no.3, pp.374–377, May 1978.
[2] G. Caire and S. Shamai, "On achievable rates in a multi-antenna broadcast downlink," Monticello, IL, Oct. 2001.
[3] M. Costa, "Writing on dirty paper (corresp.)," IEEE Trans. Inf. Theory, vol.IT-29, no.3, pp.439–441, May 1983.
[4] P. Viswanath and D. Tse, "Sum capacity of the multiple antenna gaussian broadcast channel," Information Theory, 2002. Proc. 2002 IEEE International Symposium on, p.497, 2002.
[5] W. Yu and J. Cioffi, "Sum capacity of Gaussian vector broadcast channels," IEEE Trans. Inf. Theory, vol.50, no.9, pp.1875–1892, Sept. 2004.
[6] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," IEEE Trans. Inf. Theory, vol.49, no.10, pp.2658–2668, Oct. 2003.
[7] W. Yu, "Uplink-downlink duality via minimax duality," IEEE Trans. Inf. Theory, vol.52, no.2, pp.361–374, Feb. 2006.
[8] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," IEEE Trans. Signal Process., vol.55, no.6, pp.2646–2660, June 2007.
[9] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," IEEE Trans. Inf. Theory, vol.52, no.9, pp.3936–3964, Sept. 2006.
[10] L. Zhang, Y. Xin, and Y.C. Liang, "Weighted sum rate optimization for cognitive radio MIMO broadcast channels," Proc. IEEE Int. Conf. Commun. (ICC'08), pp.3679–3683, May 2008.
[11] K. Sakaguchi, "Base station cooperation MIMO for spatial spectrum sharing," IEICE Technical Report, SR2007-103, March 2008.
[12] I. Garcia, D. Chanin, K. Sakaguchi, K. Araki, T. Sugawara, T. Sotoyama, and T. Fukagawa, "Base station cooperation null-beamforming for spectrum sharing between next generation cellular and fixed satellite systems," IEICE Technical Report, SR2007-90, March 2008.
[13] S. Boyd and L. Vandenberghe, Convex Optimization, Cambridge University Press, Cambridge, United Kingdom, 2004.
[14] S. Srinivasa and S. Jafar, "Soft sensing and optimal power control for cognitive radio," Proc. IEEE Global Telecommun. Conf. (GLOBECOM'07), pp.1380–1384, Nov. 2007.
[15] X. Yang, X. Yang, and K. Teo, "A matrix trace inequality," J. Math. Anal. Appl., vol.263, no.1, pp.327–331, 2001.
[16] L. Vandenberghe, S. Boyd, and S. Wu, "Determinant maximization with linear matrix inequality constraints," SIAM J. Matrix Anal. Appl., vol.19, no.2, pp.1691–1706, July 1998.
[17] D. Coppersmith and S. Winograd, "Matrix multiplication via arithmetic progressions," J. Symbolic Computation, vol.9, no.3, pp.251–280, March 1990.
[18] M. Grant, S. Boyd, and Y. Ye, "CVX: Matlab Software for Disciplined Convex Programming (software package) URL: http://www.stanford.edu/boyd/cvx/," Tech. Rep., Stanford University, Feb. 2009.
[19] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," IEEE Trans. Inf. Theory, vol.49, no.7, pp.1691–1706, July 2003.
[20] L. Venturino, N. Prasad, and X. Wang, "Coordinated linear beamforming in downlink multi-cell wireless networks," IEEE Trans. Wirel. Commun., vol.9, no.4, pp.1451–1461, April 2010.
[21] I. Garcia, K. Sakaguchi, and K. Araki, "Partial protection coordinated beamforming for LTE-Advanced," IEICE Technical Report, SR2009-278, March 2010.

Appendix: Thin Matrices of Orthonormal Vectors

Given any $\ddot{\mathbf{\Theta}} \geq 0$, $\ddot{\mathbf{V}} = [\bar{\mathbf{V}}\ \mathbf{V}]$ as defined in (19), $\mathbf{\Theta} \triangleq \mathbf{V}\ddot{\mathbf{\Theta}}\mathbf{V}^H$, and $\bar{\mathbf{\Theta}} \triangleq \bar{\mathbf{V}}\ddot{\mathbf{\Theta}}\bar{\mathbf{V}}^H$,

$$\mathbf{I} = \mathbf{V}\mathbf{V}^H + \bar{\mathbf{V}}\bar{\mathbf{V}}^H \tag{A·1}$$

$$\mathrm{Tr}(\mathbf{\Theta}) = \mathrm{Tr}(\mathbf{V}\mathbf{\Theta}\mathbf{V}^H) \geq 0 \tag{A·2}$$

$$\mathrm{Tr}(\mathbf{\Theta}) = \mathrm{Tr}(\mathbf{V}\mathbf{V}^H\mathbf{\Theta}\mathbf{V}\mathbf{V}^H) + \mathrm{Tr}(\bar{\mathbf{V}}\bar{\mathbf{V}}^H\mathbf{\Theta}\bar{\mathbf{V}}\bar{\mathbf{V}}^H) \geq 0 \tag{A·3}$$

$$\ddot{\mathbf{\Theta}} = \mathbf{V}\mathbf{V}^H\mathbf{\Theta}\mathbf{V}\mathbf{V}^H + \bar{\mathbf{V}}\bar{\mathbf{V}}^H\bar{\mathbf{\Theta}}\bar{\mathbf{V}}\bar{\mathbf{V}}^H \tag{A·4}$$

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