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Chaos and Synchronization - Potential Ingredients of Innovation in Analog Circuit Design?

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SUMMARY Recent years have seen a general resurgence of interest in analog signal processing and computing architectures. In addition, extensive theoretical and experimental literature on chaos and analog chaotic oscillators exists. One peculiarity of these circuits is the ability to generate, despite their structural simplicity, complex spatiotemporal patterns when several of them are brought towards synchronization via coupling mechanisms. While by no means a systematic survey, this paper provides a personal perspective on this area. After briefly covering design aspects and the synchronization phenomena that can arise, a selection of results exemplifying potential applications is presented, including in robot control, distributed sensing, reservoir computing, and data augmentation. Despite their interesting properties, the industrial applications of these circuits remain largely to be realized, seemingly due to a variety of technical and organizational factors including a paucity of design and optimization techniques. Some reflections are given regarding this situation, the potential relevance to discontinuous innovation in analog circuit design of chaotic oscillators taken both individually and as synchronized networks, and the factors holding back the transition to higher levels of technology readiness.

key words: *analog circuit design, analog computing, analog signal processing, bio-inspired robots, chaos, chaos synchronization, chaotic oscillator, data augmentation, distributed sensing, force field, hype cycle, innovation, neural systems, pattern generation, technology readiness*

1. Chaotic Dynamics: Unpredictable Yet Not Random

1.1 On the Notion of Deterministic Chaos

In colloquial speech, the word “chaos” is invariably associated with a lack of order, even confusion. This representation rests quite detached from the scientific definition, which entails dynamics endowed with a possibly very complex order, in principle free from any randomness [1]. The term originates from the ancient Greek χάος, which could be translated as abyss or immense empty space, but, far from signifying a sterile void, the reference was to the primeval disorder before all things came into being [2]. The generative potential implied in this ancestral association is more in line than the colloquial meaning with the ability of chaos and nonlinear dynamics to give rise to highly complex phenomena that are potentially relevant to electronic engineering in several ways [3]. This paper attempts to provide an overview of this topic from the perspective of potential inspirations for new analog circuit designs and applications.

In chaotic systems, small changes to the state variables are amplified exponentially over time. Consequently, therefore, the long-term evolution of the generated signal can be difficult, if not, in many practical cases, impossible to predict [4], [5]. Accordingly, classical chaos was discovered as an unintended consequence of approximation during keyboard entry of the initial conditions for numerically integrating a model of convection. A compelling summary was offered by Edward Lorenz, who asserted that in the presence of chaos “the present state completely or almost completely determines the future, but does not appear to do so” [6]. While de facto unpredictability juxtaposes randomness and chaos, as explained below, there is no stochastic content in chaos, and there are, in fact, fundamental differences from noise, which make it uniquely interesting [7], [8]. What is relevant to several potential applications considered in this work is not simply the near-unpredictability in itself, but rather the multitude of phenomena whose occurrence it enables.

1.2 An Example and Its Distinguishing Features

A paradigmatic example of chaos was discovered by Otto Rössler in the 1970s and consists of the system comprising the following three ordinary differential equations

$$\begin{cases} \dot{x}(t) = -y(t) - z(t) \\ \dot{y}(t) = x(t) + ay(t) \\ \dot{z}(t) = b + z(t)(x(t) - c)\,; \end{cases} \tag{1}$$

where a, b and c are constant parameters. It is evident that, by construction, the evolution of the variables $x(t)$, $y(t)$, and $z(t)$ is free of any randomness, and the system is structurally quite simple, with the only nonlinearity being the product $z(t)x(t)$ [9], [10]. These equations exemplify some essential requirements for chaos to arise in continuous-time autonomous dynamics, namely, the presence of at least three dimensions and at least one nonlinear term. A rather moving observation is that many of the distinguishing features of chaotic dynamics are universal, that is, appear in similar ways across systems that are structurally unrelated, for example, containing different types of nonlinearities, such as products, exponentials, saturation and threshold functions, even periodic functions [1], [3]–[5]. This is one of the aspects that makes chaotic dynamics potentially fertile for inspiring new ideas in analog circuit design, because similar phenomena

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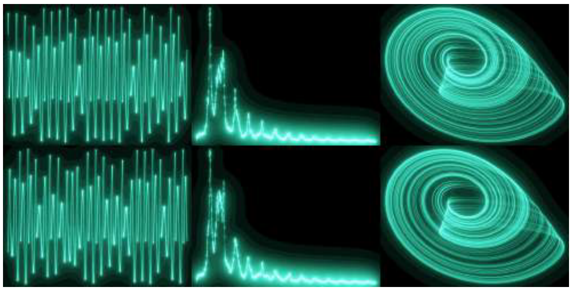

Fig. 1 Numerical simulations of the Rössler system, given parameter values $a = 0.3$, $b = 0.2$ and $c = 5.7$, repeated for initial conditions $x(0) = 6$, $y(0) = 0$ (top) and $x(0) = 0$, $y(0) = 6$ (bottom), $z(0) = 0$. Time-series of $x(t)$ (left), frequency spectrum (middle) and (x, y) Lissajous plot (right). Simulated analog oscilloscope plots. Axis scales: arbitrary units.

can be accessed through a multitude of distinct circuits [11].

The peculiar nature of chaotic dynamics can be illustrated by integrating the system in Eq. (1) starting from two different sets of initial conditions. As shown in Fig. 1, the generated signals are completely uncorrelated in the time domain, and they would eventually become so even for small differences in the initial conditions. However, the frequency spectra are similar, with a continuous baseline overlaid to some broad but distinguishable peaks. Thus far, this situation could be replicated trivially by a hypothetical noise source filtered to shape it according to a prescribed spectrum. However, when the relationship between the system variables (or, equivalently, the relationship between a measurement and the previous one at a suitable distance in the past) is visualized, a geometrical regularity becomes well-evident, in this case, having a spiral-like appearance. Clearly, this pattern, which stems from an underlying mathematical entity known as an attractor, or, more precisely, a strange attractor, could not be produced by noise; on the contrary, its geometrical features remain highly consistent even when the time-domain signals are entirely uncorrelated [1], [3]. The shapes generated by chaotic dynamics have considerable aspects of visual beauty resembling various natural objects, and, as stated, recur in similar ways across systems that are structurally entirely different [11]. Extensive literature exists regarding how to reconstruct attractors and measure the properties of chaotic dynamics in simulations as well as from experimental data, including fundamental parameters such as the Lyapunov exponents and fractal dimension [12].

1.3 Relevance to Engineering and Purpose of This Paper

Throughout the years, chaotic dynamics have been discovered across the most disparate physical and natural systems, from lasers, chemical reactions, and molecular dynamics, to neural activity, weather processes, population fluctuations in ecology, economics, and so on [1], [3], [4]. Therefore, chaos is an inextricable part of the fundamental fabric of the world around us. While today's analog and control system design techniques largely focus on linear systems (linear either by design or as an approximation of physical reality), the near totality of active electronic components and systems are, eventually, nonlinear. It is thus unsurprising that chaos has been found to occur spontaneously across diverse circuits including power converters, drive systems, frequency converters, radio amplifiers, and in general all situations involving active components and a feedback loop [13]–[15]. While chaos is frequently considered a severe form of instability that should be avoided, this paper aims to present an incomplete but hopefully rewarding personal perspective on the possibly overlooked potential of chaos, synchronization, and the related phenomena to become key ingredients of innovation in future analog circuit design. While attempts are made to provide a meaningful selection of authoritative references, solely for brevity and convenience several illustrative examples are drawn from the author's previous works.

2. Approaches to Designing Chaotic Oscillator Circuits

2.1 On the Issue of Circuit Synthesis

Unfortunately, the fact that chaos arises spontaneously across disparate situations does not imply that it is straightforward to design circuits generating chaotic signals. On the one hand, practically all semiconductor devices, including every type of diode and transistor, possess a nonlinear response that is, in principle, a suitable platform for chaos generation. But on the other hand, a profound difference between linear and nonlinear systems is that, for the latter, analytical solutions, even approximations, are precious exceptions. As a consequence, there is a scarcity of systematic synthesis techniques to design chaotic oscillators with prescribed features. It should therefore not be surprising that the first report of chaos in an electronic circuit, by van der Pol, was an incidental observation and for a long time not even recognized as such [16]. However, it would be incorrect to state that synthesis techniques are completely absent. For instance, semi-systematic procedures based on introducing diode-inductor and FET-capacitor composites into sinusoidal oscillators have been proposed [17]. Moreover the harmonic balance method, especially when combined with the more recent concept of the X-parameters, provides a powerful framework, though its use can be hindered by analytical difficulties [18], [19]. Additionally, algebraically simple systems based on jerk equations were proposed as a way of easily creating chaos generators using diodes and operational amplifiers [20]. However, the majority of the known chaotic circuits were not obtained using these methods.

2.2 Implementing Equation Systems

In principle, a straightforward way of obtaining a chaotic circuit is to electronically implement the algebraic relationships found in the ordinary differential equations that define a chaotic system. The variables can be represented as voltages or currents, and an operational amplifier is usually assigned to implement each equation. There is, in fact, a florid landscape of nonlinear equation systems having diverse dynamical and topological properties, some chaotic,

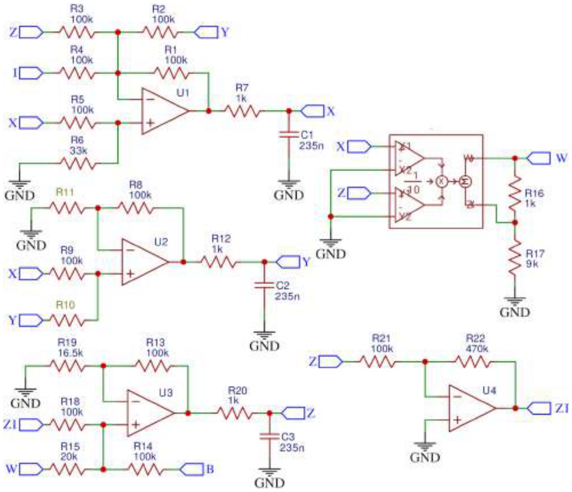

Fig. 2 Representative circuit implementation of the Rössler system, requiring several op-amps and an analog multiplier. Reprinted from Ref. [21], with permission from Elsevier.

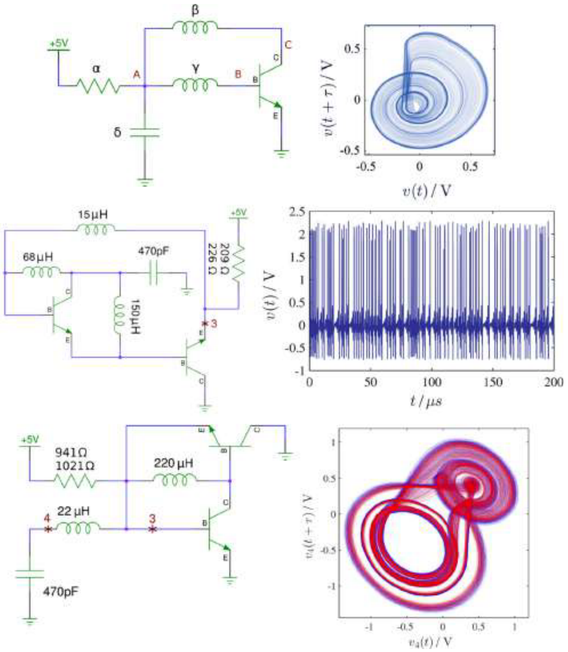

Fig. 3 Single- and dual-transistor atypical chaotic oscillators, known as Minati-Frasca circuits, generating a funnel-shaped attractor (top), spiking and bursting (middle), and a double-scroll attractor (bottom). Reprinted from Ref. [29], with the permission of AIP Publishing.

such as the Lorenz system, and some bidimensional but still capable of generating complex behaviors, such as the Lotka-Volterra equations, the Brusselator, and so on: this approach is very general and allows implementing virtually all of them electronically [11]. An example resulting circuit approximating Eq. (1) is shown in Fig. 2. The main limitation of this approach is evident, in that the minimalism of the initial equation system is lost, since the resulting circuit is rather large, involving multiple operational amplifiers, an analog multiplier, and a sizable number of passive elements. Such a circuit is poorly-suited for being optimized toward low-power and/or high-frequency applications, as well as for being implemented on an integrated circuit. This situation arises mainly because the fundamental circuit laws imply that an energy source is required to integrate each variable, and the basic mathematical and circuit nonlinearities are quite different. Yet, some chaotic oscillators were actually conceived to have a compact form both as equations and a circuit. That is the case for the theoretically simplest known chaotic circuit conceived by Leon Chua, and the hyperchaotic system introduced by Toshimichi Saito [22]–[24].

2.3 Circuit Variations and Serendipituous Discovery

Since the requirements for chaotic oscillation can be met by considerably smaller circuits, several reports have instead focused on circuits based on modifications of canonical topologies such as the Colpitts and Hartley oscillators. A comprehensive survey is beyond scope and can be found elsewhere, but, in general, two additions tend to be required [17]. One is sufficient nonlinearity: this can be introduced via an additional device such as a diode, or by operating the transistor in a large-signal regime where its own nonlinearity becomes significant. The other is an energy-storing element, typically an inductor, interacting with the nonlinearity to disturb the integration underlying sinusoidal signal generation in the original circuit. Such modifications have yielded a multitude of viable circuits, ranging from low-power to microwave-range oscillators [24], [25].

On the other hand, serendipity has also played an important role in the discovery of new circuit topologies able to sustain chaos, some oscillating autonomously and others requiring external excitation. These include several single-transistor circuits, for example, the Lindberg-Murali-Tamasevicius (LMT) circuit based on the principle of disturbance of integration, and other very small-size designs leveraging the junction capacitances or complex behaviors such as the Early effect [26]–[28]. In this context, a multitude of entirely new circuits, exemplified in Fig. 3 and generating diverse types of chaotic attractors, were found using large-scale computational searches. These involved describing each topology and the component values using a bit string, then applying a funnel-like approach to narrow down the potentially viable circuits based on circuit laws, before simulating and finally building them [29]. One consequence of this work was demonstrating that there are many possible chaotic circuits yet to be discovered, unrelated to the known ones having with different properties [30].

2.4 Integrated Circuit Realizations

While chaotic dynamics are deterministic (either entirely, in simulations, or predominantly, in experiments), the irregular fluctuations have for long represented an entropy source of interest towards high data-rate random number generation, and have also been leveraged toward solving particular problems where controllable turbulence is desirable, such as in regards to mixing actuators. Moreover, as further discussed

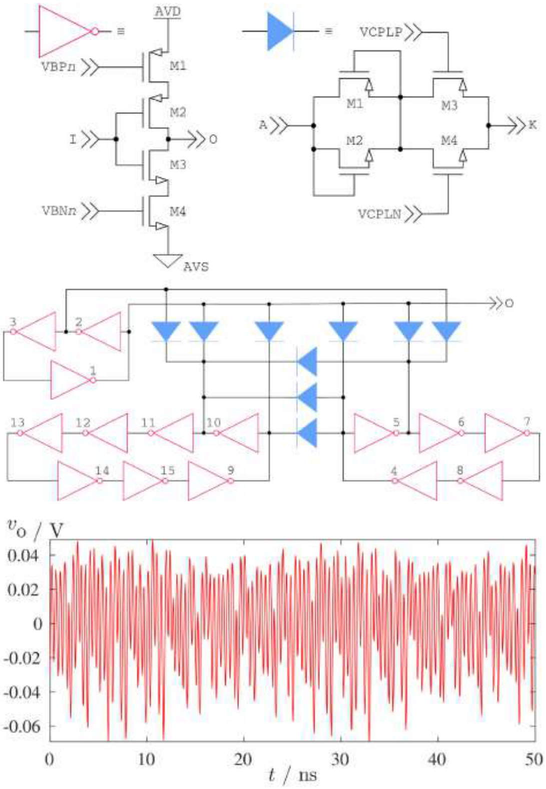

Fig. 4 A highly versatile CMOS chaos generator, not requiring any passive element, based on current starved inverters and diodes (top), coupled as interconnected 3/5/7-rings (middle), leading to broadband chaos (bottom). Reprinted from Ref. [37], with permission from Elsevier.

below, the broadband nature of chaos has made chaotic oscillators relevant to signal generation and modulation, for example in masking and in atypical modulation schemes seeking to securely encode a baseband signal [14], [31]–[33]. Particularly with these applications in mind, designs suitable for implementation as application-specific integrated circuits or elements thereof have been proposed throughout the years. While detailed reviews are found elsewhere, we note that several of them follow similar design principles as the circuits based on discrete elements described above and therefore require realizing on-chip reactive and resistive elements, which are area-consuming [34]–[36]. However, other designs more specifically tailored to integrated implementation were also proposed, including clocked architectures, and MOS-only circuits such as the one in Fig. 4, based on inverter rings having lengths of three, five, and seven, current-starved and cross-coupled using diodes, without any passive elements. Using this design, the generation of chaotic signals starting from a few tens of nanowatts of power consumption up to oscillation at several gigahertz was observed [37], [38].

2.5 Memristors and Summary

Thus far, we have omitted discussing circuits based on memristors. First postulated by Leon Chua over forty years ago through symmetry arguments, the memristor, a resistor endowed with memory, plays a crucial theoretical role as the “fourth circuit element” [39]. The study of memristor-based systems must be credited with nurturing a renewed enthusiasm towards the investigation of chaotic circuits, and evidence is accumulating on the role that this component can play a significant role in enriching the chaotic dynamics. Additionally, it provides a non-volatile state variable akin to a synapse, potentially disrupting the future of analog neural systems [40], [41]. At the same time, the majority of literature today is based on numerical simulations or hardware device emulators utilizing operational amplifiers, because obtaining and using physical devices to generate chaotic dynamics remains challenging, though it is possible [42]. Due to these reasons, the author believes that these efforts and potential applications are better reviewed separately [43].

In summary, the situation for chaotic oscillators appears different from other, more established areas of analog design for which extensive literature and systematic synthesis methods exist, such as amplifiers and phase-locked loops. Most works do not yet focus on engineering performance targets such as power consumption. However, the literature offers clear glimpses of the generality of these circuits and several possible directions toward obtaining them. The situation seems to be such that, for reasons related to the evolution of this field, the bulk of efforts have, so far, been invested in discovery and exploration, rather than optimization.

3. Synchronization and Pattern Generation

3.1 On the Peculiarity of Chaos Synchronization

In contemporary electronic engineering, the term synchronization is almost universally associated with the existence of a fixed phase relationship between two periodic timing signals. Across many applications, such a relationship needs to be controlled down to tight tolerance requirements, for example minimizing the phase noise of a source or aligning multiple clock domains with minimal jitter. When designing phase-locked loops, the locked and unlocked conditions are sharply counterposed in a binary sense [44]. Chaotic signals are profoundly different for two reasons. First, there are continuous and usually wide phase fluctuations. Second, in most cases, the signal is not conditioned to a fixed amplitude, and there are also irregular cycle-to-cycle amplitude variations. As a consequence, the concept of synchronization takes on a much more complex, and nuanced, meaning [45].

Let us consider two chaotic oscillators, for example, using the simplest circuit in Fig. 3. When realized as physical devices, there are unavoidable parametric mismatches due to tolerances, and there may even be intentional differences in the component values. As a consequence, entirely uncorrelated signals are generated. Let us now provide a means of energy exchange between them, for example, a variable resistor connected between the capacitors (other forms of coupling, such as capacitive or inductive, would be largely equivalent). As the value of this resistor is decreased from infinity, the energy exchange rate between the oscillators increases, in other words, the coupling is elevated. The route toward complete synchronization involves several phenomena. First, a fixed phase relationship is gradually established, initially with frequent slips and perhaps a lag, and eventually reaching a stable overlap. Because chaotic oscillations still preserve some elements of recurrence, what is observed is that the cycles, however irregular in duration, occur in unison. At this point, the fluctuations of the amplitudes are likely to still be uncorrelated. Second, as the coupling is elevated further, the amplitude fluctuations become increasingly coherent, that is, synchronization becomes apparent also in the signal envelope. Eventually, the two oscillators become completely synchronized, that is, their signals fully overlap. As stated, unlike periodic signals which have a sharp boundary between the locked and unlocked conditions, the regime between complete asynchrony and perfect synchronization, known as partial synchronization, is fuzzy and may encompass a wide range of coupling strengths [46]–[48].

3.2 On the Notion of Pattern Formation

One of the reasons why chaotic oscillators, and more generally nonlinear dynamics, are interesting is precisely the existence of this extended regime of partial synchronization because, within this region, the formation of spatial and spatiotemporal patterns can occur. This phenomenon, broadly known as morphogenesis from μορφή- (morphé, shape) and γένεσις (génesis, formation), has long fascinated generations of thinkers including Alan Turing. In particular, the puzzle is about how diverse and complex structures may spontaneously emerge from structureless, homogeneous situations, as beautifully exemplified by the concentric rings of the Belousov-Zhabotinskii chemical reaction [49]–[51]. A common way to study this phenomenon in an analog electronic context is to consider a ring of coupled oscillators: unlike other networks such as chains, lattices, rings, and so on, rings have a high degree of symmetry, because each node sees the same first, and second neighbors and so on. In the presence of the parametric tolerances and non-idealities that are ubiquitous in engineering, this symmetry in the structural couplings gets unavoidably broken via the dynamics [52].

3.3 Cluster Synchronization and Community Formation

A phenomenon that is particularly easy to observe is termed cluster synchronization. As exemplified in Fig. 5, it manifests itself through the formation of groups, or communities, of oscillators that become preferentially entrained with each other, even though the physical connections between them are of the same intensity as with the others. For low coupling strengths, clusters may simply consist of oscillator pairs. As the coupling becomes stronger, these clusters usually grow and can become more diverse in size, even nested. Eventually, they vanish as synchronization spreads across the entire network, and becomes simply a function of distance along the ring [53]. This phenomenon arises across diverse networks of nonlinear elements, such as social networks (readily experienced at large events where unfamiliar people first encounter), neural networks, and power distribution. Notably, it is far from a trivial reflection of how close the parameter values or the predominant frequencies are, rather reflecting a much more intricate underlying formation and destruction of synchronization manifolds as the system parameters are changed [48], [54]. Despite this complexity, analytical results allowing the prediction of cluster synchronization and the design of clusters into given configurations have been obtained [55], [56].

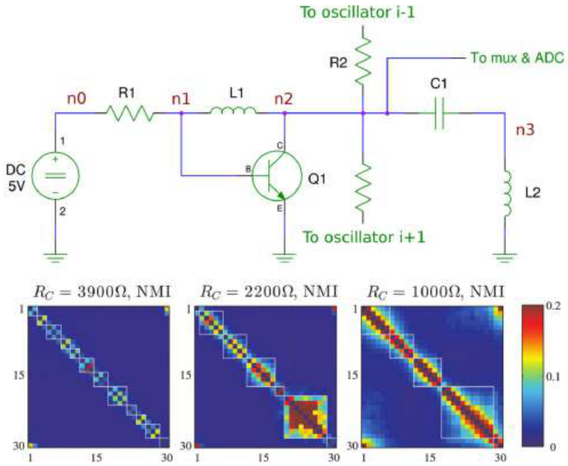

Fig. 5 Emergence of cluster synchronization, from single-transistor chaotic oscillators coupled as a ring (top), showing that well-separated communities (white boxes, color scale denotes synchronization level) appear at coupling resistor values intermediate (bottom, middle) between those so high that only some pairs of neighbors become entrained (bottom, left) and so low as to generate diffuse synchronization (bottom, right). Reprinted from Ref. [53], with the permission of AIP Publishing.

3.4 Remote Synchronization

Cluster synchronization "grows" along the structural connections like crystals along a string, but there is another, more exotic phenomenon that confers greater freedom: remote synchronization. The term refers to an ensemble of diverse mechanisms which allow structurally distant (that is, not directly connected) nodes to become more strongly synchronized than those that are directly coupled [48], [54]. A seminal observation was established in star networks, simulated numerically as well as realized using circuits realized using techniques similar to those shown in Fig. 2, where the hub node is strongly mismatched to the leaves (corresponding, respectively, to the core and periphery of the star). For intermediate coupling strengths, the oscillations of the leaves become phase-locked via a mechanism involving amplitude fluctuations in the hub, without synchronizing with it [57]. Other instances of remote synchronization have been described, generally involving star networks, and are thought to play a key role in conferring a certain flexibility for different synchronization patterns to play out from fixed structural connectivity, especially in the brain [58]. An electronic observation where this phenomenon leads to the emergence

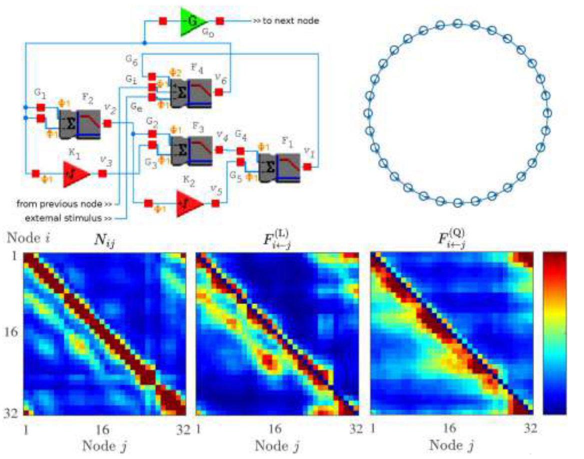

Fig. 6 Emergence of remote synchronization, in a ring of unidirectionally coupled oscillators realized using field-programmable analog arrays (top), showing that islands of distant oscillators (away from the diagonal) become preferentially synchronized (bottom, left); using increasingly general methods to detect synchronization, including nonlinear (bottom, middle) and non-parametric (bottom, right) ones, the routes of influence are revealed. Reprinted from Ref. [59], [60], with the permission of AIP Publishing.

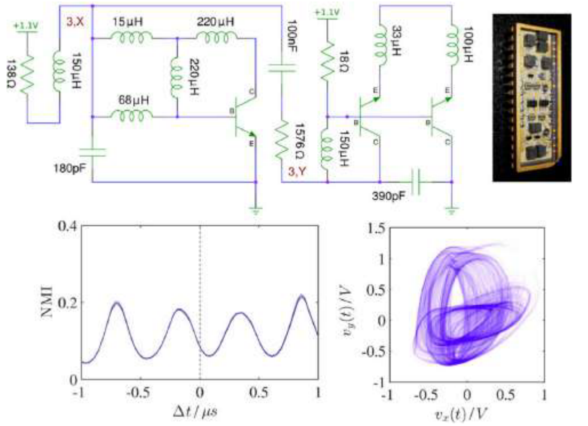

Fig. 7 Generalized synchronization, obtained by coupling two structurally-different single-transistor oscillators (top), leading to a complicated but clearly visible interdependence between their signals (bottom). Reprinted from Ref. [29], with the permission of AIP Publishing.

of complex patterns entails a ring of nodes based on field-programmable analog arrays coupled in a master-slave configuration. Given that each node acts, due to the nonlinearity, not only as an oscillator but also as a (de)modulator, intricate interplays between frequency sidebands arise. As shown in Fig. 6, these cause the formation of groups of non-adjacent synchronized nodes, effectively jumping connections. The effect could be explained by using nonlinear models to capture the relationships between all oscillator pairs, and related suppression effects [59], [60].

3.5 Generalized Synchronization

The etymology of the term synchronization, from συν- (syn, same) and χρόνος (chrónos, time), refers to temporal coincidence: it should therefore be no surprise that synchronization is normally considered in terms of phase coherence. Nevertheless, two chaotic oscillators can establish considerably more complex forms of statistical interdependence, such that they significantly influence each other, but this influence only becomes visible when a suitable representation is used, for example, through a time-lag reconstruction of each signal's past. This is named generalized synchronization. While it may arise in highly diverse scenarios, it is typically associated with the unidirectional coupling of parametrically or even structurally heterogeneous oscillators [48], [54], [61]–[63]. An example of the latter case, but with resistive bidirectional (also known as diffusive) coupling, is shown in Fig. 7. Because the dynamics are different, it is impossible to maintain phase locking, but even a Lissajous plot reveals an intricate, yet clearly visible, interdependence.

3.6 On the Pervasiveness of Synchronization Phenomena

The above is by no means a complete account of all observations of spatiotemporal pattern formation in chaotic systems, and other known phenomena include chimera states and swarming [48], [64]. Several fundamental considerations have been omitted, for instance regarding which phenomena require chaos or can be observed more generally for nonlinear oscillations (for example implementing electronically the Stuart-Landau equations), and which phenomena require parametric heterogeneity or can be observed in identical systems. As this brief survey targets hypothetical engineering applications of chaotic oscillators, those aspects are not discussed in detail [1], [3]–[5], [47], [48], [54]. The central point here is that, as for the fundamental properties of chaotic dynamics, such as the regularity and recurrence of strange attractors, the phenomena related to synchronization arise in similar ways across highly diverse scenarios and electronic circuits. They are, in particular, thought to play a key role in orchestrating neural activity in nervous systems, including the human brain, and some attempts are being made to juxtapose neuroscientific and electronic observations [65]. The striking richness of these phenomena makes it seem rather implausible that they would not contain anything of value to realize signal and pattern processing operations in the analog domain. However, they are, unavoidably, even harder to treat analytically than individual oscillators. Therefore, our ability to synthesize circuits and systems showing prescribed behaviors is rather limited, though, as mentioned above, some exceptions exist.

4. Examples of Potential Applications

4.1 Foreword

Complete mastery of the principles of functioning and rigorous verification of realization stand as deontological cornerstones of engineering responsibility. It may, therefore, seem surprising to even consider potential real-world applications of circuits and networks that we understand only in a rather incomplete manner. However, a broader view emphasizes virtues and sensitivity to context, which would include a duty to not be presumptuous or excessively risk averse with regards to new, as-yet to be understood, ideas [66]. The situation is not different from the approach taken towards the widely used neural network models, whose functioning remains weakly explainable [67]. Throughout the years, many possible applications have been proposed, the majority (but not the totality) of which apparently never advanced beyond the initial technology readiness levels, as discussed below.

4.2 Broadband Signal Generation and Modulation

The de facto unpredictability of chaotic signals initially fueled considerable interest towards hardware-level secure information transmission. However, due, in particular, to the limited immunity to attacks using auxiliary systems and issues related to noise and the need for tight parameter matching, purely analog chaos-based security has not yet gained a widespread advantage over digital techniques [32], [33], [68]. However, several algorithms and logic architectures for encryption leveraging diverse aspects of chaotic dynamics, including chaotic maps, have also been developed [69].

Further, the broadband but deterministic nature of chaotic signals offers some concrete advantages for spread spectrum signal generation, weak signal detection, and signal concealing. Applications related to optimizing RADAR and SONAR signals have been developed, though limited information is available in the public domain [70]–[74]. Considerable efforts were invested in obtaining nearly spectrally flat chaotic sources emitting over several microwave bands [75], [76]. A recent focus has been on ultra-wideband chaotic circuits for jamming and through-the-wall imaging [77].

4.3 Actuation and Computation

As mentioned previously, chaotic dynamics were also found to be a convenient basis for optimal solutions to some mechanical problems such as in mixing, and chaotic actuators found their way into some patents and eventually commercial applications, especially in Japan, though these did not yet evolve into market-disrupting applications. Examples include inducing chaos in the motion of water jet actuators in dishwasher nozzles, and reducing the defrosting time by 40% in household microwave ovens commercialized by Panasonic Corporation [14], [31], [78]–[80]. Another notable example was the invention of a chaotic logic gate, consisting of a compact chaotic circuit that could seamlessly morph between NOR, NAND, and XOR functionality based on applied control voltages, with considerable potential in obfuscation and defense against differential power and electromagnetic analysis attacks [81], [82]. A startup known as ChaoLogix was considered a rising star in this area, but was eventually acquired by the ARM Corporation, and no further public information about the fate of the technology is available [83]. However, some developments of this idea have continued to appear in the literature, for instance, a Boolean logic locking mechanism known as ChaoLock [84].

4.4 Other Applications and Focus on Synchronization

The vastness and fragmentation of literature on hypothetical and actual applications of chaotic dynamics implies that a comprehensive survey is beyond the scope of this work. Instead, after this brief summary, a selection of examples will be presented below, based for convenience on the author's previous works. While the majority of the existing applications focus on individual chaotic oscillators, here, particular emphasis is placed on systems based on the synchronization of chaos, as this is deemed to be a promising and less explored area. Before delving into examples, it should be mentioned that there are also other important applications of chaos theory, particularly in the analysis of experimental time series from natural sciences, particularly physiology, as well as in the generation of high-entropy patterns in encryption and optimization algorithms. As the focus of this paper is on circuit design, such literature is not covered, but excellent reviews can be found elsewhere [12], [85].

4.5 Realizing Physical "Toy Models"

Since, as stated, chaotic dynamics and the related synchronization phenomena have aspects of universality, meaning they occur similarly across vastly different systems, electronic chaotic circuits and networks hold considerable potential in creating "toy models". By toy model, we generally mean a highly simplified representation of a complex system, aiming to retain only a narrow set of key features, usually implemented in a stylized form, to facilitate manipulation and analysis [86]. Within this context, it is worth mentioning the relationship between numerical simulations and experiments. In many engineering applications, noise, tolerances, non-idealities of components, and other imperfections can lead to deviations from design targets. For nonlinear dynamical systems and networks, including natural ones, these aspects, which are challenging to simulate numerically, can have a more profound influence, changing qualitatively the overall behaviors that a system exhibits, such as the existence and stability of some synchronization states [52], [54]. Consequently, physical toy models can play a useful role in realistically exploring dynamics such as those unfolding in the brain, being vastly easier to manipulate than biological preparations. While this is not an industrial application, it is mentioned here due to its tangible downstream applications. For example, a central element of brain dynamics is the so-called default-mode network, comprising hub regions that are intensely interconnected and, consequently, are not only preferentially synchronized but also exhibit large-amplitude low-frequency activity fluctuations [87]. As shown in Fig. 8, using a ring network of single-transistor oscillators, to which long-range links were superimposed wiring up some regions as hubs, an analogous relationship between connectivity and dynamics is observed [88]. This observation supported further considerations about the functioning of brain hubs and their clinical relevance [89].

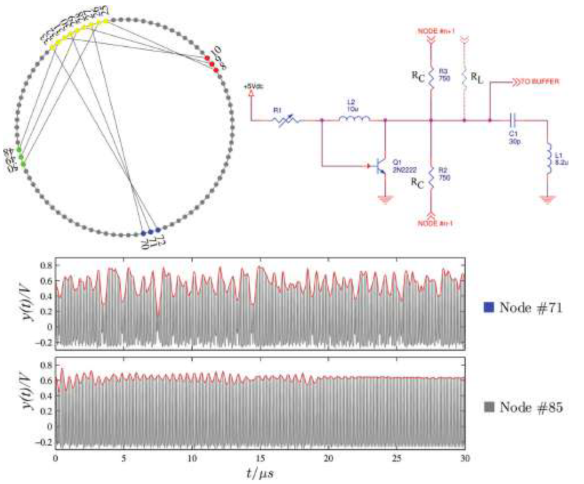

Fig. 8 Toy model of brain connectivity, consisting of a ring with four "hub" regions (top, left), each node consisting of a single-transistor chaotic oscillator (top, right). Within the hubs (yellow: principal, red, green, blue: secondary hubs), large-amplitude low-frequency fluctuations emerge (middle), whereas outside them, amplitude changes are small and noise-like (bottom). Reprinted from Ref. [88], with the permission of AIP Publishing.

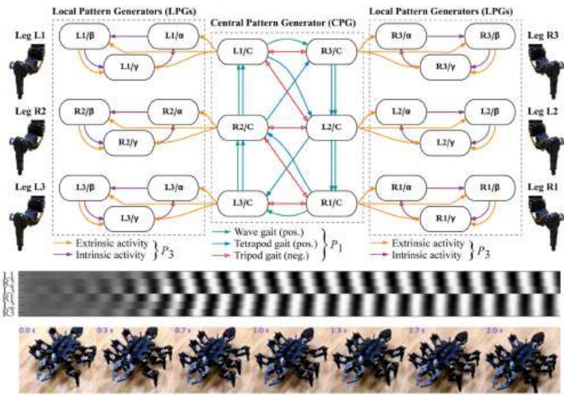

Fig. 9 Gait pattern and leg trajectory generator based on coupled field-programmable analog array oscillators (top), establishment of a gait pattern (middle) and corresponding still frames of insect-like robot (bottom). Reprinted from Ref. [91], with the permission of IEEE.

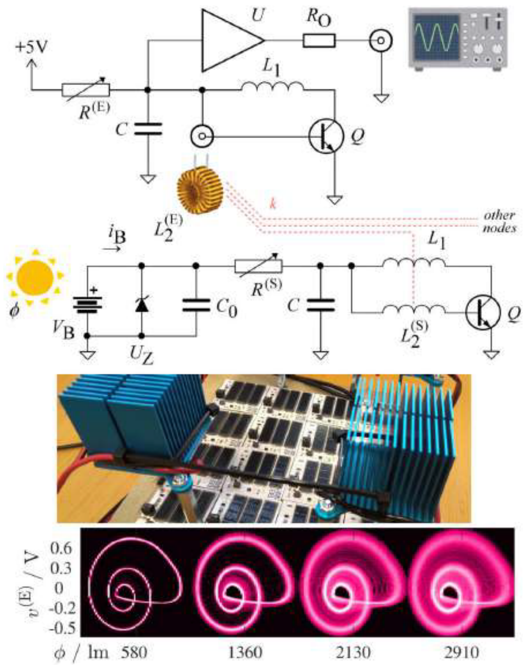

Fig. 10 Wireless distributed sensing, based on one "exciter" chaotic oscillator coupled wirelessly to a multitude of light-sensing nodes (top), experimental 4×4 array (middle), and response of the attractor to light intensity (bottom). Reprinted from Ref. [97], with the permission of the authors.

4.6 Motor Pattern Generation for Bio-Inspired Robots

Another area where tentative applications have been considered is robot control. On the one hand, the irregularity of chaotic dynamics represents a possible basis for trajectory generation mimicking exploratory behavior. On the other hand, the ability to generate regular spatiotemporal patterns allows for compact implementation of gait generators, as demonstrated in a wide range of experiments ranging from insect-like to swimming robots [90]. Here, the example of an 18-degree-of-freedom hexapod is considered. As visible in Fig. 9, the biological connectivity of neural populations in the insect brain realizing a central pattern generator could be replicated via six oscillators, realized using field-programmable analog arrays. The topology and strengths of their connections could be made to depend on a set of high-level parameters, allowing for a smooth transition and interplay between biologically-observed gaits, such as the wave and tripod gaits. Notably, through a hierarchical network of coupled oscillators, it is also possible to generate the full trajectories of all the leg actuators, bypassing the need for kinematic calculations [91]. The industrial potential of this hypothetical application is inherently linked to the future evolution of biologically-inspired robotics, and should not be dismissed, particularly when considering the potential for very low-power integrated analog realization of a complex robot controller and its potential integration with milli- and micrometer scale structures, of which a prime example is given by the insectoid robots from Festo Corporation [92], [93].

4.7 Distributed Sensing Using Wireless-Coupled Oscillators

When taking measurements, one is often concerned with obtaining an adequate reading of a physical quantity at a particular location. However, a trend across fields from agriculture to civil engineering involves making decisions also based on the statistical properties of a variable measured over an extended area, such as strain over a dam or moisture over a plot of land. The techniques concerned with these kinds of measurements have come to be known as distributed sensing. Since individually interrogating large populations of sensors is wasteful when the eventual purpose is calculating summary statistics over them, considerable efforts have been invested in developing distributed computing techniques to obtain such values by consensus, though having each node communicate directly with its neighbors [94]–[96].

While the known systems rely primarily on algorithms and digital links, recent work has shown the possibility of realizing distributed sensing through chaotic synchronization. As shown in Fig. 10, according to this approach each node is realized through a self-powered chaotic oscillator, modified to fulfill two requirements. First, one of the circuit parameters is directly coupled to the variable to be sensed, for example, to sense light, the supply voltage may be drawn from a photovoltaic source. Second, one of the circuit elements is modified so that it provides a mechanism for energy exchange with neighboring nodes, for instance through magnetic coupling via an inductor. Owing to the physical separation between sensors, couplings tend to be weak, but synchronization can be facilitated and readout can be implemented through a globally applied field, generated by an exciter chaotic oscillator whose dynamics become influenced by the variable to be sensed, for example, by its average [97]. While the approach is in its infancy and considerable challenges remain regarding long-distance couplings and spectral utilization (i.e., fractional bandwidth necessary to attain synchronization), the possibility of synchronizing single-transistor chaotic oscillators in the microwave L-band using narrow bandwidths has been shown [37].

4.8 Physical Reservoir Computing for Vector Classification

Deep-learning neural networks, particularly convolutional neural networks, have reached a level of performance in recognizing temporal and spatial patterns that was unimaginable a decade ago, however, one severe drawback is that the training process is extremely computationally demanding. A potential alternative approach that has gained increased interest is known as reservoir computing and involves leveraging a nonlinear dynamical system or network to project an input vector to an advantageous higher-dimensional space, where class separation can be attained using simpler classifiers that are easier to train. In particular, it has been shown that disparate physical systems can be used to realize reservoirs, in principle supporting classification at very low energy expenditure [98]–[100]. Within this framework, networks of chaotic oscillators, where the input vector is encoded through the system parameters or coupling strengths, and the output is extracted from the synchronization pattern, have an obvious relevance. As visible in Fig. 11, even small ensembles of dual-transistor spiking chaotic oscillators can meet the requirements for reservoir construction, and exhibit highly nontrivial responses such as competition between oscillator subpopulations [101]. Applied development efforts on these types of reservoirs appear particularly motivated considering the possibility of realizing low-power chaotic networks using CMOS circuits instead of discrete components, such as the oscillator provided in Fig. 4 [35], [37], [38].

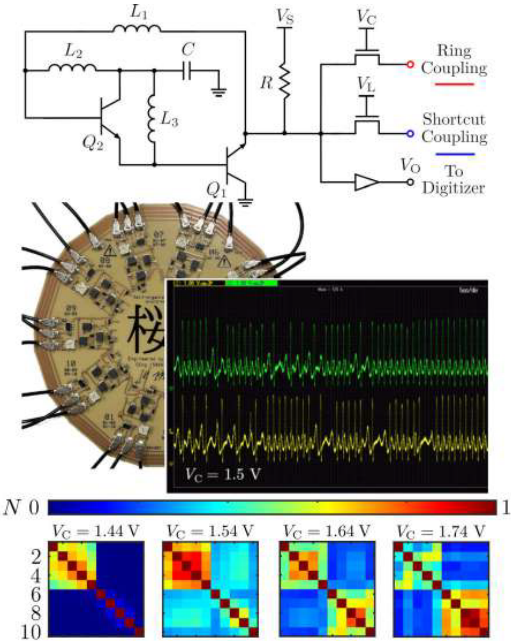

Fig. 11 Spiking oscillators (top), wired as a ring network with shortcuts, giving rise to spike synchronization (middle), and non-trivial pattern formation and competition effects (bottom), suitable for physical reservoir computing. Reprinted from Ref. [101], with permission from Elsevier.

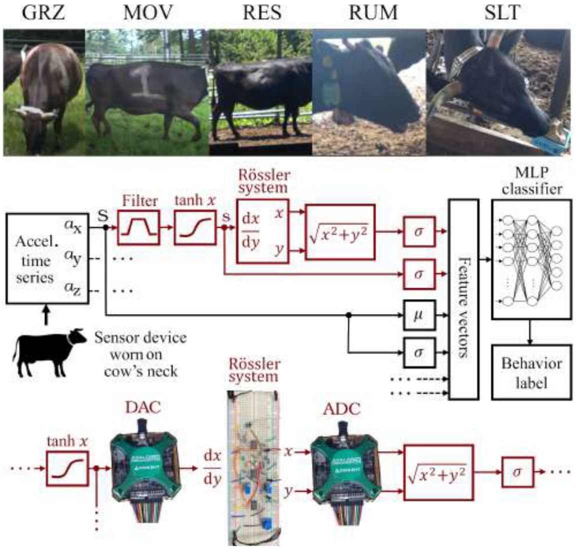

Fig. 12 Time series data augmentation aiming to enhance the classification of cattle behaviors based on accelerometer data (top), using a Rössler system as nonlinear feature transformer (middle), and its hardware realization (bottom). Reprinted from Ref. [21], with permission from Elsevier.

4.9 In- or Near-Sensor Time Series Data Augmentation

Another challenge with training convolutional neural networks is that they preferentially learn linear features, hindering the classification of complex time series such as those generated by natural systems, for instance, animal behavior [102], [103]. However, nonlinear and especially chaotic oscillators can exhibit non-trivial responses when externally perturbed with unrelated signals, and, in doing so, can effectively realize transformers that project nonlinear into linear features [1], [3]–[5]. As shown in Fig. 12, a hypothetical application of this approach involves feeding low-dimensional sensor data, such as the signals emitted by a triaxial accelerometer, into one or several chaotic systems, and using their responses to increase the dimensionality of the data input to a classifier. In essence, this realizes a form of data augmentation, which can considerably increase classification accuracy [21]. This approach, too, appears worthy of applied development efforts, particularly in light of the possibility of augmenting the data in the analog domain at low power expenditure, within the sensor package itself [104].

5. Engineering Challenges and Innovation Paradigms

5.1 On the Current Situation in Analog Circuit Innovation

At present, innovation in circuit design is shaped by several forces, stemming from an ever-increasing penetration of technology in society and an insatiable hunger for data, amidst tightening environmental constraints. On the one hand, the Internet of Things paradigm implies a greater diffusion of sensor and control edge nodes, reaching greenfield and brownfield application scenarios that impose challenging requirements related to connectivity and power supply availability. In some cases, new paradigms involving a tight integration between the cybernetic and physical worlds are being envisioned, such as the use of smart dust sensor networks. Simultaneously, the explosion of applications based on artificial intelligence and big data is fueling an endless growth in the density, size, and power consumption of data centers. On the other hand, the room for process innovations to provide the answers to these challenges has become increasingly narrow, and it is clear that much more is needed than advancements at the level of device manufacturing. Compared to a decade ago, this situation is leading to a thirst for architectural innovation, including unconventional solutions, and increased turbulence due to the interplay and competition between different approaches [105]–[107].

A topic of intense debate is the potential role of radically innovative analog circuits in addressing these challenges. An increased number of researchers deem analog technology as an enabler of lower power consumption and higher integration at both edge and data center levels, as popularized in a recent article in the journal “Wired” [108]–[111]. The capability of VLSI analog circuits to leverage physical phenomena to realize complex computations with inconspicuous but inventive circuits is well-known, and was extensively demonstrated as a basis for sensor data processing three decades ago [112]. However, in the meantime, the flexibility, reliability, and forgiving nature of digital processing keep pushing towards transitioning more functions, such as signal conditioning, to the digital domain [113].

In an inspiring recent commentary on this journal, Chris Mangelsdorf put forward four forces that are causing a loss of creativity and innovation in analog integrated circuit design: process evolution, risk aversion, digitally assisted analog, and corporate culture. In brief, the current state of analog design is such that the level of integration, tool complexity, and tape-out cost have reached levels so high that the introduction of radical, discontinuous innovations is heavily discouraged [114]. There is also a paradoxical drawback of excessive focus on total quality management which, while being credited with guiding countless corporations towards excellence, especially in Japan and the USA, nowadays risks acting as a force hindering breakthrough innovation, undoubtedly including circuit design [115]. Looking back at countless fields, radical innovations have played an irreplaceable role in resetting the product life cycle curve and injecting turbulence allowing the exploration of previously unthinkable concepts. This is exemplified by the evolution of printing technologies which was, coincidentally, modeled using elements from chaos theory [116]. At the dawn of every parameter comparison table being diligently targeted by optimization-focused engineers, there once was a first, crude and highly imperfect version of a new idea.

5.2 On the Hypothetical Contributions of Chaotic Circuits

So, what is the potential role, if any, of chaotic circuits supporting analog innovation in this scenario? For optimization-focused and performance-pressed corporate engineers and managers, attempting to embrace different and incomplete design principles such as those required for considering chaotic circuits, may sound like a heresy. The uncertainty and risk associated with investing in development efforts on such circuits are vastly higher than those related to more known and established forms of analog signal processing. Yet, already from the condensed and necessarily incomplete summary that could be given in this paper, it should be apparent that chaotic circuits also have many unique properties that plausibly offer very high potential rewards toward concrete applications, probably higher than some more established ideas of analog computing. They can be structurally minimalist yet feature a rich phenomenology, they can be realized through diverse technologies and have led to promising though preliminary results towards a range of applications from sensing to signal analysis. While, albeit with notable exceptions, chaos-based designs have not yet made it beyond the first stages of technology readiness, there are reasons to believe they may contain substantial latent innovation potential able to initiate new product life cycles. What appears to be missing is an innovation framework and organizational environment that allows this potential to be expressed [117]–[119]. Below, some arguments in support of this speculation are offered. It should also be mentioned that, compared to areas such as photonics and nanotechnology, the experimental study of nonlinear dynamics, chaos, and related phenomena is highly economically accessible. Significant discoveries can still be made using inexpensive electronic apparatuses, and this has contributed significantly to the development of highly internationally-regarded research communities in

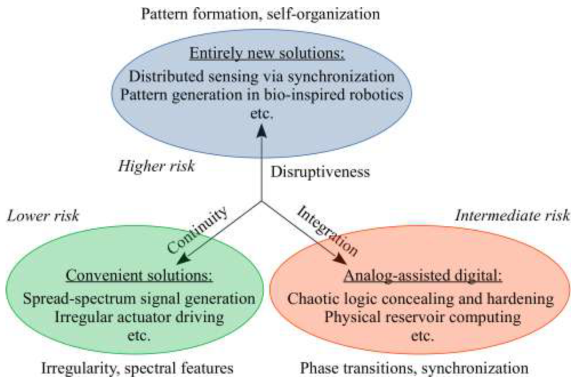

Fig. 13 Conceptual model of three hypothetical paradigms for the contribution of chaotic circuits in the analog design innovation process, maximizing disruptiveness (blue), integration (orange), and continuity (green).

low- and middle-income countries [120].

As shown in Fig. 13, in the author's opinion, three hypothetical innovation paradigms that could be put forward.

First, using chaotic signals as convenient solutions to existing and well-understood engineering problems. This paradigm maximizes continuity, minimizing risk, yet leverages only the most superficial properties of chaotic dynamics, that is, their irregularity and spectral content. One example is the usage of chaotic oscillators to generate broadband or spread-spectrum signals in a way that is controlled and parsimonious from a circuit perspective. Another, as reviewed above, is using chaos as a source of turbulence to drive actuators involved in particular processes such as mixing, where laminar flows should be avoided.

Second, using chaotic networks and systems as a way to assist and enhance preexisting digital architectures, which may seem as paradoxical when juxtaposed with the transition towards digitally-assisted analog advocated by some. This is arguably the most balanced paradigm, retaining a connection with existing products and solutions but aiming to leverage some deeper properties of chaotic dynamics to solve specific, well-defined problems. These properties are more closely related to the features that distinguish chaos from noise, and include phase transitions and synchronization phenomena. One example is the use of chaos to conceal logic activity, as was described above. Other examples involve using chaotic systems as preprocessors for machine learning, either in the form of physical reservoir networks or isolated systems used for in-sensor data augmentation.

Third, using high-level emergent properties, such as pattern formation via self-organization, to create entirely new solution concepts, emphasizing disruptiveness and potential gain. One example here is the idea of implementing distributed sensing that leverages collective dynamics to coalesce measurements, and another is the notion of analog-level realization of bio-inspired robot behaviors.

While this representation is hypothetical and tentative, it hopefully serves to make the general point that chaotic circuits and networks could enter mainstream design engineering using different routes, associated with diversified levels of discontinuity and risk.

5.3 On Some Engineering Issues Hindering Adoption

Having surveyed some cultural and organizational aspects, we now turn to four important technical considerations.

First, as mentioned, compared to other areas of analog circuit design, we are relatively bereft of formal synthesis and design methodologies when it comes to chaotic circuits, taken both individually and as networks [121]. This is a severe obstacle to widespread adoption, because reliance on serendipity poses high risks when designing toward specific targets, and the potential hemorrhage of time associated with empirically searching for suitable parameters compounds this risk. Short of major theoretical breakthroughs, the situation is unlikely to change, as there are profound mathematical differences in the tractability of linear and nonlinear systems. For the latter, the generative potential and the lack of closed-form solutions are two sides of the same coin [1], [3]–[5]. Yet, the literature is now vast enough that suitable circuit and network blueprints to take as starting points could be found for many situations, and the in-principle applicability has been described across diverse areas.

Second, because they have for mostly been considered to an academic topic, chaotic circuits have not yet received the large-scale optimization and integration efforts that have been invested in almost all other circuits, such as amplifiers and timing devices. A significant part of the literature is based on realizations using discrete components, which are outdated technologies from a commercial perspective. For this reason, they have a reputation for being power-inefficient, but it can be argued that there is a serious imbalance of evidence; on the contrary, some of the works surveyed in this paper demonstrated integrated realization with low-power operation across many frequency decades, from kilohertz to gigahertz. Of the optimization techniques well-known to those skilled in integrated circuit design and found in standard textbooks, none are inherently inapplicable to these circuits, the matter rather being one of purpose and focus [122]. In fact, subthreshold operation enhances the nonlinear behavior of transistors and would be particularly suitable for low-power chaos generation [123].

Third, even engineers who have a clear understanding of the difference between chaos and noise may be tempted to assume that there is an insurmountable sensitivity to random parameter variations and the like. It is not so, and it must be clarified that the dependence on initial conditions and parameter values are conceptually different. Though even small differences in initial conditions may produce uncorrelated time series, their statistical properties, reflecting the geometry of the underlying attractor, will be similar. Although there may be regions in the parameter space where discontinuous transitions and bifurcation cascades occur, chaotic circuits generally also possess large regions within which the impact of the parameter values on the qualitative features of the dynamics is gradual, not unlike other analog circuits [24], [52], [54]. In general, chaotic circuits and networks have yet

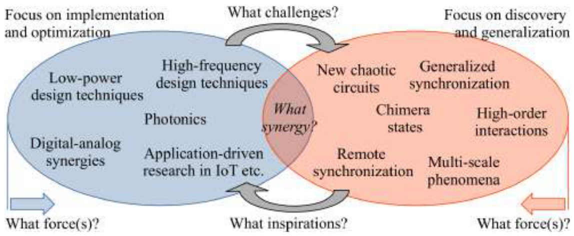

Fig. 14 Conceptual model of the relationship between the existing efforts around general analog circuit development (blue) and basic research around chaos, synchronization and related phenomena (orange), alongside possible interactions and forces. Today, there appears to be limited synergy.

to receive even a small fraction of the attention that other circuits have enjoyed in terms of process-voltage-temperature sensitivity analysis and optimization. Which circuits and oscillation modes could be more or less sensitive, and how to mitigate the parametric tolerances affecting particular elements, remains largely to be analyzed. However, notably, at least two designs of PVT-robust integrated chaotic oscillators have been reported [124], [125].

Fourth, simulation is not a trivial matter, neither for discrete nor for integrated circuit realizations. Industry-standard SPICE engines, even in the most advanced implementations, turn out to have a fair performance in predicting which circuits are potentially chaotic but fall well short of accurately representing the detailed effects of the circuit parameters (e.g., when chaos arises). Different software implementations, such as `LTSpice` and `ngspice` may give different results, and, in some cases, even the order of lines in the netlist may influence the results. The impact of component non-idealities such as inductor self-resonances and capacitor losses may be non-negligible, and, particularly when networks of heterogeneous oscillators are simulated, issues related to stiffness can arise [29], [126], [127]. While this scenario is daunting, the situation is not very different from the troubles encountered when handling high-frequency and even worse microwave circuits decades ago, which required and were successfully addressed by developing tailored simulation tools [128]. The study of these heavily nonlinear and numerically challenging entities never reached a scale warranting consideration in terms of how to improve circuit-level simulation algorithms, and there is no reason to exclude the possibility of attaining substantial accuracy improvements.

5.4 Research and Development Needing Attractive Forces

Why is it, then, that decades after the first discoveries in this area, we have yet to see any disruptive industrial application? As a florid literature developed, this question was asked repeatedly [85], [129], [130]. In an attempt to further represent the current situation, a conceptual model such as the one shown in Fig. 14 may be put forward. According to it, there are two domains, which could be loosely associated, on the one hand, with the targets of engineers focusing on implementation and optimization across the main application areas involving analog circuits, and, on the other hand, with the mindsets of physicists and other scientists who are rewarded by uncovering the fundamental phenomena and speculating on the hypothetical applications of the circuits.

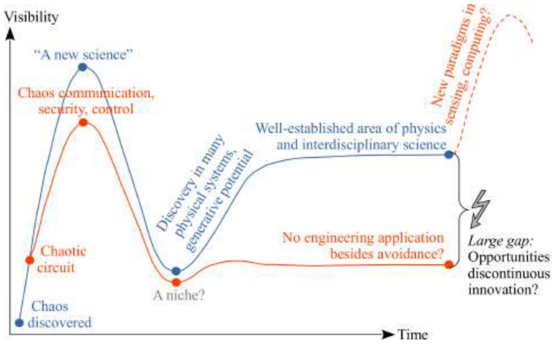

Fig. 15 Hype cycle-based conceptualization of the evolution of fundamental research (blue) and applied engineering studies and development (orange) around chaos and chaotic circuits, and the hypothetical future.

The problem seems to be that these two worlds have evolved largely independently. As elsewhere in technological innovation, the issue can be expressed in terms of organizational force fields: regarding the first domain, there has been no propulsion toward understanding, incorporating, and improving the results obtained in the second domain, partly due to the above-mentioned aversion to risk and due to technical factors [131]. At the same time, the theoretical study of these circuits and phenomena has evolved organically throughout academic circles and journals, reaching a sort of self-sufficiency, wherein a constant stream of scientifically rigorous results can be obtained even without the sheer effort associated with convincing industrial partners to invest. This perspective is, unavoidably, highly simplified since there have been examples to the contrary such as with regards to chaotic logic gates. However, the author's take is that, as a grand average, this describes the state of the field.

A way of addressing this situation is through fostering inbound open innovation in the industry, creating organizational structures tasked with bringing in early-stage analog technologies, including those based on chaos [132]. Another way involves more explicitly seeking to generate attractive forces. First, what concrete technical challenges could the first domain offer to the second as a rewarding testing ground for new theoretical advancements? An interesting precedent is found in the regular brain-computer interface competitions, and some of the applications mentioned above surveyed in this paper could be taken as targets [133]. Second, what inspirations could the existing literature on chaotic circuits offer to use in new ways tools and technologies from the electronics industry? The potential role of X-parameters, initially developed for radio amplifier linearization, in chaotic circuit synthesis and analysis is a prime example [19].

5.5 From Hype to Sleeping Beauty?

A different account of the situation is encapsulated in the hype cycle graph visible in Fig. 15, according to which the

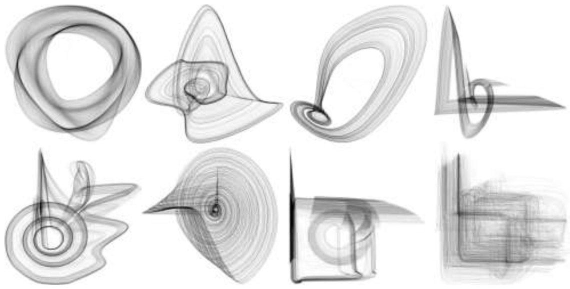

Fig. 16 Eight examples of chaotic attractors reconstructed from oscilloscope signals of single- or dual-transistor oscillators. Data from Ref. [29].

situation could be summarized as follows [134]. Throughout the decades following its discovery, chaos and the associated phenomena followed an inverted parabola of visibility, firstly reaching a point of acclaim as the harbinger of a new scientific paradigm popularized in James Gleick's well-known book, then declining due to factors including lack of tangible applications, competition from other fields, and even academic journal controversies [135]. It can be said that, to a decent approximation, the visibility of works investigating chaotic electronic circuits and associated ideas followed a similar trend, eventually becoming a well-respected but perhaps relatively narrow area of physics and engineering. This is by undoubtedly a simplification since the history of nonlinear science across diverse fields is remarkably varied [136]. But the argument being put forward is that, over the last two decades, there appears to have been a sort of forking. On the one hand, the discovery of consistent phenomena across different systems, such as synchronization behaviors, fueled a resurgence of scientific interest [48]. On the other hand, no enticing engineering applications have gained visibility, though it would be incorrect to state that they do not exist entirely, as exemplified by ultra-wideband innovations [77].

In conclusion, are chaotic circuits a sort of sleeping beauty technology waiting to be (re)discovered by engineers and capable of injecting valuable innovation in analog design [137]? Or are they destined to almost remain confined to the curiosity of those who cherish the unquestionable beauty of their phenomenology, exemplified in Fig. 16 [138]? A typical answer from middle-aged engineers who lived through the hype period is that these are old ideas from decades ago, and if they did not lead to a breakthrough then, they are unlikely to do so now. That is precisely the same answer that one would have received about neural networks a couple of decades ago, before a convergence of theoretical breakthroughs, technological advances and commercial factors led to their rediscovery and explosion. Several disruptive technologies have experienced cyclical winters, and a deep winter does not predict the absence of spring [139].

In the author's opinion, a rigorous answer to this question cannot be given at this stage. However, it seems that no research area has produced such a rich body of theoretical and experimental results, including detailed proof-of-concept works towards specific applications, yet remained so detached from, and largely unknown to, those who develop solutions in the industry. Therefore, the likelihood that it contains many latent opportunities appears to be rather high, and it seems reasonable to at least trigger three actions.

First, conduct rigorous technology assessments of the diverse hypothetical applications considered throughout the literature, aiming to understand, in a far deeper and more rigorous way than was possible in this work, whether there are consistent factors that have prevented reaching high technology readiness levels, and to what extent such factors are technical as opposed to cultural and organizational.

Second, create organizational environments suitable for inbound open innovation in analog circuit design, allowing, among other threads, to also vigorously explore the potential applications of chaotic circuits. An aspect of this effort could be creating libraries of reusable chaotic cells geared towards applications, such as low-power or high-frequency signal generation, and in-sensor time series augmentation.

Third, foster awareness and education, since only a minority of graduates have a working awareness of these circuits and a feeling for their potential, and most of them do not venture beyond first- and second-order solutions. Others have realized this, and developed kits and curricula that could be leveraged to help the concepts from this area percolate through to the lab benches of mainstream developers, perhaps also through enticing maker communities, that have a remarkable amplification capability [140], [141].

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