IEICE TRANSACTIONS

on Electronics

DOI:10.1587/transele.2024REI0001

Publicized:2024/09/12

This advance publication article will be replaced by the finalized version after proofreading.

A PUBLICATION OF THE ELECTRONICS SOCIETY

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INVITED PAPER *Recent Progress in Electromagnetic Theory and Its Application*

Construction of A High Precision Analysis Method for Electromagnetic Fields and Its Application

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SUMMARY This paper describes the high-precision electromagnetic field analysis methods that the author has developed (point matching method considering edge condition and Modified Fourier series expansion method) and research on their applications. In addition, as a new application of periodic structures, it is discuss a new method for solving scattering problems involving arbitrarily shaped objects in inhomogeneous media.
key words: *Point matching method considering edge condition, Modified Fourier series expansion method, Inhomogeneous media, Electromagnetic wave, Multilayer division method.*

1. Introduction

In recent years, optical circuit devices have been developed for optical fiber communications, mobile phones, underground radars, photonic crystal waveguides, quantum cryptography, and other optical devices.In relevant computer simulation has become indispensable for device design, experiment interpolation, confirmation of experimental results, etc. [1]. In such a simulation technology, computational electromagnetics are essential [2]. There is a need for the computer simulation technology that has a wide range and is easy to evaluate and control errors.
Computational electromagnetism [3,4] is concerned with numerical simulation techniques for numerically solving Maxwell's equations, which are the basis of electromagnetic phenomena, and the electromagnetic field theory are closely related to its basis of the Institute of Electrical Engineers of Japan(IEEJ) and the Institute of Electronics, Information and Communication Engineers of Japan(IEICE) [5, 6].
In this paper, an author describes the high precision electromagnetic field analysis method which has developed for point matching method considering edge conditions, and improved Fourier series expansion method that can be applied to inhomogeneous media, metamaterials and related applications so on. In this application, it is also discuss a new method for analyzing electromagnetic wave scattering problems involving arbitrarily shaped objects with inhomogeneous media for transvers electric (TE) waves.

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2. High precision analysis method for electromagnetic fields

2.1 Point matching method considering edge condition

In the 1970s, the importance of numerical analysis methods for electromagnetic fields using computers was demonstrated [7,8], among which improvements were made to the point matching method (PMM) [9] (the method that satisfies boundary conditions using sample points). The modified point matching method (MPMM) [10] (the method for analyzing plane gratings in scattering problems using only conductors (or slits) as main points) was developed by the late Professor Toshio Hosono (as well as Fast Inversion of Laplace Transform FILT[11]) , and when the author entered graduate student in 1975, Assistant Professor (later Professor) Takashi Hinata was producing excellent research results on various problems which were published[12].In addition, we confirmed that MPMM[13] provides the same range of accuracy as Riemann-Hilbert method(RHM[14]).

In a conventional PMM[15] for the scattering problem of a rectangular cylinder with an edge, the convergence of the electromagnetic field for horizontally polarized waves (transvers magnetic (TM) waves)[17]is slower than that of vertically polarized waves (TE waves)[16]. Therefore, for both polarizations, we developed a new Point Matching Method (considering the edge points into regions where variables can be separated) in which the sample points do not include the edge points (determining the sample points and edge regions) [18, 19].

The application of the new PMM consists of the following three steps.

(Step 1): Divide the physical space into a finite number of regions such that it can be expanded using regular functions with a shape that can be locally variable-separated.

(Step 2): Approximate the electromagnetic field in each region using the finite terms of the complete system (mode function) in that region.

(Step 3): Select sample points on the boundary and perform boundary matching such that the coefficient matrix of the simultaneous linear equations that determines the expansion coefficients is not singular.

Here, the relationship between the number of expansion terms of the electromagnetic field and the sampling points is such that owing to the uniqueness of the solution to the boundary value problem, the number of expansion terms in the divided area is equal to the number of sampling points on the boundary surrounding the area, and the number of expansion terms on the boundary. We applied a point matching method that takes into account edge conditions to the strip conductor [20] and conducting rectangular cylinder [21], and obtained highly accurate results.

2.2 Scattering and guiding problems of inhomogeneous medium with dielectric gratings

In recent years, with the development of microfabrication technology, scattering and propagation problems in optical circuit devices such as optical fiber gratings and photonic crystal waveguides have attracted attention. Periodic structured media are widely applied in optical circuit devices, and their basic structures can generally be divided into the following types.

[A]Medium constant changes perpendicular to the wave propagation direction.

[B]Medium constant changes in the wave propagation direction.

[C]Medium constant changes in the direction of the slant angle γ.

Conventionally, the spatial harmonic expansion method [22] is widely used to analyze periodically structured media, however it has a narrow range of applications, and it is difficult to apply to arbitrary inhomogeneous media or shapes. Furthermore, even if the range of applications is wide, the propagation problem type in [C] requires highly accurate analysis of both polarizations using the same analytical method as the scattering problem.

For the problem involving scattering of TE waves due to a periodic structured medium type in [A], a wide range of dielectric constants can be used [23]. For TM waves, an improved Fourier series expansion method (the number of truncated modes of the electromagnetic field and the Fourier expansion of the medium) can be employed . By setting the number of terms, it has been applied to wave analysis of anti-reflection walls, absorption layers, chirped gratings [24], and plane lenses [25].

It can also have a wide range of dielectric constants for the scattering problem caused by the periodic structured medium types in [B] and use the eigenvalue equation of the MCK type [26], making error control easier and more applicable than that of conventional spatial harmonic expansion methods which developed a wide range and highly accurate analysis method .

On the other hand, the propagation problem required the development of an important new solution to the problem of optical waveguides (relief type gratings with an arbitrary shapes and arbitrary distribution of media).At that time, in September 1989, fortunately, I had the opportunity to go on sabbatical leave to the Massachusetts Institute of Technology (MIT) for Professor J.A. Kong on both scattering and guiding problems of relief gratings ,which he. had researched [27]. However there are no numerical results for type in [C]. I spent many days unable to apply it to the guiding problem [28]. Finally I came up with a good method on the multilayer inhomogeneous medium. The new method is constructed in which the number of dimensions of the characteristic equation to be solved is determined by the number of truncated modes of the electromagnetic field, rather than the total number of divisions, by dividing the structure into structures and converting the relational expression of the scattering coefficient into a matrix within each layer. Waveguide problems can now be analyzed with high accuracy using an algorithm with a calculation time comparable to that of scattering problems. After returning to Japan, the results obtained at MIT were jointly published with Professors Hosono and Kong in the English journal IEICE [29].

By using this new solution method in conjunction with the multilayer decomposition method, the shape and distribution of a heterogeneous medium that combines type in [A] and [B] can be applied to arbitrary problems [30], and because it is a high-precision analysis method, type in [C] can be easily applied to the problem of tilted gratings (all the layers have the same eigenvalue, only the eigenvector of the first layer is shifted), whereas the conventional method of calculating the eigenvalue is complex and is particularly suitable for propagation problems. This is a more practical and highly accurate analysis method [34,35] than that reported in [31-33] (which is difficult).

Additionally , our new method can also be easily applied to elliptical gratings [36], making it applicable to photonic crystal optical waveguides with various refractive index distributions.

2.3 Photonic crystal waveguide

Photonic crystals with a periodic permittivity distribution are known to have a blocking region (photonic bandgap) in a specific wavelength band that prevent the propagation of light waves (electromagnetic waves), therefor, they can be used in microscopic optical devices. Photonic crystal structures are attracting increasing attention because they are expected to be applied in photonic integrated circuits. In addition, because photonic crystal structure has a periodicity comparable to the wavelength of light, it is possible to reduce the amount of light by introducing a defect layer into the periodic structure. Depending on how the defect layers are arranged, they can be used in various optical circuit devices such as optical filters, optical resonators, and optical couplers [37]. Using analytical method, we established switching effects in photonic crystal optical waveguides [38-40] and electromagnetic field confinement techniques in defect structures [41-43]. Furthermore, we recently developed a new formulation of a point-matching method for solving scattering problems in mixed media (metals and inhomogeneous dielectrics) [44,45].

2.4 Metamaterials

In recent years, near-field problems such as metamaterials and surface plasmons, where both permittivity and permeability are negative, have attracted attention.

Conventionally, in the scattering problem of a singular point problem with a positive and negative permittivity distributions, it is difficult to analyze the case of a horizontally polarized incident (TM wave) and an oblique incident wave. In particular, the problem of an inflection point when the refractive index is positive or negative has not yet been solved [46].

However, the problems with this singularity cannot be solved using the improved Fourier series expansion method. Problems in which the permittivity is mixed between positive and negative are those for which difference equations such as the FDTD method cannot be applied, and energy absorption at the singularity is important. I considered extracting the energy absorption term from the wave equation as a logarithm (log term), and I was thinking of a wave equation that includes the logarithm, but I could not find a new solution, thus I added a loss term to the medium.While reducing the loss term, I calculated the eigenvalue to find the correct eigenvalue and, then substituted it into the wave equation to numerically find the eigenvector (mode function).
I tried it, but without success. It was later found that the solution to the wave equation in an inhomogeneous medium (excluding step distribution) consisting of positive and negative polarities is originally an equation that does not involve an energy absorption term. For the first time, we developed a numerical method to extrapolate both to a solution containing singularities while minimizing the loss [47].

2.5 Pulse response of dispersive media

In recent years, ground penetrating radar (GPR) has been used in a wide range of the exploration areas, including exploration of reinforced structures, metal detectors, buried object detection, mine detection, underground cavity exploration, and ruins/geological surveys. It has been used in engineering even in relatively shallow media of underground structures. This is an important research topic because accurate modeling of underground structures requires consideration of heterogeneity in addition to a single dispersive medium. The authors have previously solved the transient scattering problem of a structure with a dispersive medium [48] and a periodic arrangement of cavity regions considering underground heterogeneity using a combination of Fourier series expansion method and fast inverse Laplace transform [49]. The influence effects of the cavity width, thickness, and arbitrary cavity shape composed of multilayer media were investigated using the pulse response waveform [50]. We also analyzed the transient scattering problem of a strip conductor [51] or a structure in which an inclined cavity and a dispersive medium were arranged periodically and investigated the effects of the tilt angle and medium width [52,53].

3. Scattering analysis of arbitrarily shaped objects with inhomogeneous media

We analyze the scattering problem caused by a dielectric scattering cylinder in inhomogeneous media by an arbitrary shape as shown in Figure 1.The scattering problem caused by an arbitrarily shaped cylinder has also been analyzed using the atom method [54,55], but here we will use Figure 2(a) (For example, the analysis area of the scattering cylinder is uniform in the y direction, as in the case where the cylinders is composed of eight layers, and a solution within a periodic structure with period p is used in the z direction (r=p/2) .The permittivity and permeability in the outside region are assumed to be ε_0 and μ_0 .The time factor $\exp(-j\omega t)$ is suppressed throughout.

In the PMM, the mode expansion region, the number of truncation modes and the boundary conditions are important, so we will be described below.

$r \geq p/2$(*Outside region*)

In the formulation, when the TE wave (the electric field is only the y component in Figure 1) is assumed to be incident at $x < 0$ with the angle θ_0 ,

$$E_y^{(i)}(x,z) = E_0 e^{j(k_x x + k_z z)} , \qquad (1)$$

where, $k_x = k_0 \cos\theta_0$, $k_z = k_0 \sin\theta_0$.

The incident and scattered waves were expanded using cylindrical coordinates, with the number of sample points on the circumference equally spaced at 2N+1 (N represents the number of truncated modes of the electromagnetic field).

$$\theta_k = \frac{2\pi}{2N+1}k \ , \ k = 1 \sim (2N+1)$$

$$E_y^{(i)}(r,\theta_k) = E_0 \sum_{n=-N}^{N} J_n(k_0 r) e^{jn(\theta_k + \theta_0)} , \qquad (2)$$

$$E_y^{(s)}(r,\theta_k) = \sum_{n=-N}^{N} C_n H_n^{(1)}(k_0 r) e^{jn\theta_k} . \qquad (3)$$

Here, J_n and $H_n^{(1)}$ are Bessel and Hankel functions, respectively. C_n is unknown coefficients to be determined by boundary condition.

Using equation $\boldsymbol{H} = (\nabla \times \boldsymbol{E}) / (j\omega\mu_0)$, the magnetic fields are as follows.

$$H_\theta^{(i)}(r,\theta_k) = \frac{k_0}{j\omega\mu_0} E_0 \sum_{n=-N}^{N} J_n^{'}(k_0 r) e^{jn(\theta_k + \theta_0)} , \qquad (4)$$

$$H_\theta^{(i)}(r,\theta_k) = \frac{k_0}{j\omega\mu_0} \sum_{n=-N}^{N} C_n H_n^{(1)\prime}(k_0 r) e^{jn\theta_k} , \qquad (5)$$

$r < p/2$(*Inside region*)

In a periodic structure (Coordinate system of $x-z$), the inhomogeneous region is divided into multiple layers (Eight layers in the case of Figure 2(a)) and using the modified Fourier series expansion method in each layer of modulated index profile (Figure 2(b)) with width $W_i (i = 2 \sim 7)$ of medium[24], the electric field for each layer is given by the following equation. N is the same number of truncation modes in outside region.

$l = 1, 8$:(Vacuum layer)

$$E_y^{(l)}(x_k, z_k) = \sum_{n=-N}^{N} (a_n^{(l)} f_n^{(l)} + b_n^{(l)} g_n^{(l)}) \, , \quad (6)$$

where, $f_n^{(l)} = e^{j[k_n x_k + (k_z + \frac{2\pi n}{p}) z_k]}$, $g_n^{(l)} = e^{-j[k_n x_k - (k_z + \frac{2\pi n}{p}) z_k]}$,

$$k^2{}_n = k^2{}_0 - (k_z + \frac{2\pi n}{p})^2, \qquad x_k = r\sin\theta_k \ , \ z_k = r\cos\theta_k \, .$$

○ $l = 2 \sim 7$ (Inhomogeneous layer)

$$E_y^{(l)}(x_k, z_k) = \sum_{\nu=1}^{2N+1} (A_\nu^{(l)} s_\nu^{(l)} + B_\nu^{(l)} q_\nu^{(l)}) \, , \quad (7)$$

where, $s_\nu^{(l)} = e^{j(k_z z_k + h_\nu^{(l)} x_k)} \sum_{n=-N}^{N} {}^{(l)}u_n^{(\nu)} e^{j\frac{2\pi n}{p} z_k}$,

$$q_\nu^{(l)} = e^{j(k_z z_k - h_\nu^{(l)} x_k)} \sum_{n=-N}^{N} {}^{(l)}u_n^{(\nu)} e^{j\frac{2\pi n}{p} z_k} \, , x_k = r\sin\theta_k \ , \ z_k = r\cos\theta_k \, ,$$

and $h_\nu^{(l)}$, ${}^{(l)}u_n^{(\nu)}$ are eigen-value (Propagation constants) and eigen-mode (Modal functions) at inhomogeneous layers, respectively.

The magnetic field components are expressed as follows:

At

○ $l = 1, 8$ (Vacuum layer)

$$H_x^{(l)}(x_k, z_k) = -\frac{1}{\omega\mu_0} \sum_{n=-N}^{N} \left(k_z + \frac{2n\pi}{p} \right) (a_n^{(l)} f_n^{(l)} + b_n^{(l)} g_n^{(l)}) \, , \quad (8)$$

$$H_z^{(l)}(x_k, z_k) = \frac{1}{\omega\mu_0} \sum_{n=-N}^{N} k_n (a_n^{(l)} f_n^{(l)} - b_n^{(l)} g_n^{(l)}) \, , \quad (9)$$

$l = 2 \sim 7$ (Inhomogeneous layer)

$$H_x^{(l)}(x_k, z_k) = -\frac{1}{\omega\mu_0} \sum_{\gamma=1}^{2N+1} [A_\nu^{(l)} s'^{(l)}_\nu + B_\nu^{(l)} q'^{(l)}_\nu] \, , \quad (10)$$

$$H_z^{(l)}(x_k, z_k) = \frac{1}{\omega\mu_0} \sum_{\gamma=1}^{2N+1} [h_\gamma^{(l)} (A_\nu^{(l)} s_\nu^{(l)} - B_\nu^{(l)} q_\nu^{(l)})] \, , \quad (11)$$

where, $s'^{(l)}_\nu = e^{j\left(k_z z_k + h_\nu^{(l)} x_k\right)} \sum_{n=-N}^{N} {}^{(l)}u_n^{(\nu)} \left(k_z + \frac{2n\pi}{p} \right) e^{j\frac{2\pi n}{p} z_k}$,

$$q'^{(l)}_\nu = e^{j\left(k_z z_k - h_\nu^{(l)} x_k\right)} \sum_{n=-N}^{N} {}^{(l)}u_n^{(\nu)} \left(k_z + \frac{2n\pi}{p} \right) e^{j\frac{2\pi n}{p} z_k} \, .$$

θ_k *at* $r = p/2$ (*Boundary condition*)

The equation of the boundary condition for electric field

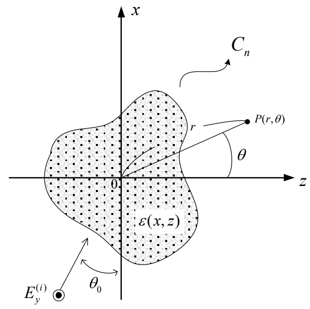
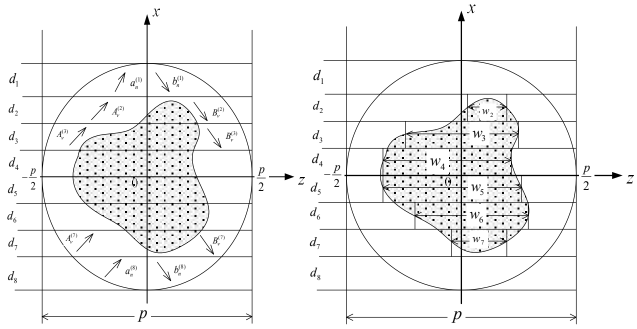


Figure 1 Coordinate system of scattering object



(a) (b)

Figure 2 (a) periodic structure (b) modulated step index of layers

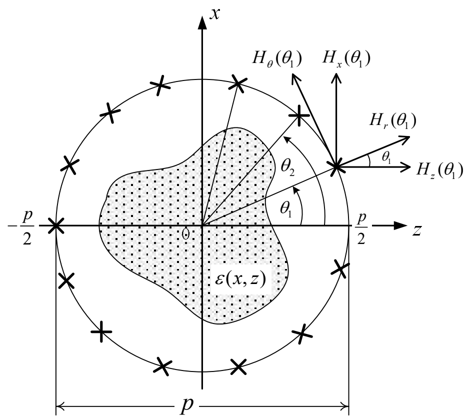


Figure 3 Matching points of the boundary at $r = p/2$ *and* θ_k

with component of y and magnetic field with component of θ at the $r = p/2$ *and* θ_k are (N=6 in the case of Figure 3) as follows:

$$\theta_k = \frac{2\pi}{2N+1} k \ , \ k = 1 \sim (2N+1)$$

○ $l = 1, 8$ (Vacuum layer)

$$E_y^{(i)}(r = p/2, \theta_k) + E_y^{(s)}(r = p/2, \theta_k) = E_y^{(l)}(x_k, z_k) \, , \quad (12)$$

$$H_\theta^{(i)}(r = p/2, \theta_k) + H_\theta^{(s)}(r = p/2, \theta_k) = H_x^{(l)}(x_k, z_k)\sin\theta_k - H_z^{(l)}(x_l, z_l)\cos\theta_k \, , \quad (13)$$

Using the orthogonality relation [10]

$$\sum_{k=1}^{2N+1} e^{j(n-m)\theta_k} = \begin{cases} 2N+1 \ ; n=m \\ 0 \quad\ ; n \neq m \end{cases}, \tag{14}$$

The formulation of C_m is as follows from Eqs. (12).

$$(2N+1)[J_m(k_0 p/2)e^{jm\theta_0} + C_m H_m^{(1)}(k_0 p/2) = \sum_{n=-N}^{N}(a_n^{(l)} F_{n,m}^{(l)} + b_n^{(l)} G_{n,m}^{(l)}) , \tag{15}$$

where, $F_{n,m}^{(l)} = \sum_{k=1}^{2N+1} f_n e^{-jm\theta_k}$, $G_{n,m}^{(l)} = \sum_{k=1}^{2N+1} g_n e^{-jm\theta_k}$,

$x_k = (k_0 p/2)\sin\theta_k$, $z_k = (k_0 p/2)\cos\theta_k$, and m=-N～N

From Eqs.(15), C_m is expressed as follows.

$$C_m = \frac{\sum_{n=-N}^{N}(a_n^{(l)} F_{n,m}^{(l)} + b_n^{(l)} G_{n,m}^{(l)}) - (2N+1)J_m(k_0 p/2)e^{jm\theta_0}}{(2N+1)H_m^{(1)}(k_0 p/2)}, \tag{16}$$

For the magnetic field of the component of θ at Eqs.(13) is expressed as follows:

$$-jk_0[\sum_{n=-N}^{N} J_n^{'}(k_0 p/2)e^{jn(\theta_l+\theta_0)} + \sum_{n=-N}^{N} C_n H_n^{'(1)}(k_0 p/2)e^{jn\theta_l}] = \left[\sum_{n=-N}^{N}\left(k_z + \frac{2n\pi}{p}\right)(a_n^{(l)} f_n^{(l)} + b_n^{(l)} g_n^{(l)})\sin\theta_l\right] + \sum_{n=-N}^{N} k_n (a_n^{(l)} f_n^{(l)} - b_n^{(l)} g_n^{(l)})\cos\theta_l \tag{17}$$

Substituting Eqs.(17) into Eqs.(16), and rearranging it into equations for $a_n^{(l)}$ and $b_n^{(l)}$,we yields

○ $l = 1,\ 8$ (Vacuum layer)

$$\sum_{n=-N}^{N} a_n^{(l)} F_n^{(l)} + \sum_{n=-N}^{N} b_n^{(l)} G_n^{(l)} = R_{(l)}^{(i)} , \tag{18}$$

where,

$$R_{(l)}^{(i)} = -jk_o \sum_{n=-N}^{N}[J_n^{'}(k_0 p/2) - \frac{J_n(k_0 p/2)H_n^{'(1)}(k_o p/2)}{H_n^{(1)}(k_o p/2)}]e^{jn(\theta_l+\theta_0)}$$

$$F_n^{(l)} = f_n^{(l)}\{\left(k_z + \frac{2n\pi}{p}\right)\sin\theta_l + k_n\cos\theta_l\} + jk_o \sum_{m=-N}^{N}\frac{F_{n,m}^{(l)} H_m^{(1)'}(k_o p/2)e^{jm\theta_l}}{(2N+1)H_m^{(1)}(k_o p/2)}$$

$$G_n^{(l)} = g_n^{(l)}\{\left(k_z + \frac{2n\pi}{p}\right)\sin\theta_l - k_n\cos\theta_l\} + jk_o \sum_{m=-N}^{N}\frac{G_{n,m}^{(l)} H_m^{(1)'}(k_o p/2)e^{jm\theta_l}}{(2N+1)H_m^{(1)}(k_o p/2)}$$

Similarly, for the ~~in the case of an~~ inhomogeneous layer($l = 2$～ 7), the equations for $A_\nu^{(l)}$ and $B_\nu^{(l)}$ can be obtained as follows.

○ $l = 2$～ 7 （Inhomogeneous layer）

$$\sum_{\nu=1}^{2N+1} A_\nu^{(l)} K_\nu^{(l)} + \sum_{n=1}^{2N+1} B_\nu^{(l)} L_\nu^{(l)} = T_{(l)}^{(i)} , \tag{19}$$

where,

$$T_{(l)}^{(i)} = -jk_o \sum_{n=-N}^{N}[J_n^{'}(k_0 p/2) - \frac{J_n(k_0 p/2)H_n^{'(1)}(k_o p/2)}{H_n^{(1)}(k_o p/2)}]e^{jn(\theta_l+\theta_0)}$$

$$K_\nu^{(l)} = \{s_\nu^{'(l)}\sin\theta_l + s_\nu^{(l)} h_\nu \cos\theta_l\} + jk_o \sum_{m=-N}^{N}\frac{F_{\nu,m}^{'(l)} H_m^{(1)'}(k_o p/2)e^{jm\theta_l}}{(2N+1)H_m^{(1)}(k_o p/2)}$$

$$L_\nu^{(l)} = \{q_\nu^{'(l)}\sin\theta_l - q_\nu^{(l)} h_\nu \cos\theta_l\} + jk_o \sum_{m=-N}^{N}\frac{G_{\nu,m}^{'(l)} H_m^{(1)'}(k_o p/2)e^{jm\theta_l}}{(2N+1)H_m^{(1)}(k_o p/2)}$$

$$F_{\nu,m}^{'(l)} = \sum_{k=1}^{2N+1} s_\nu e^{-jm\theta_k} , G_{\nu,m}^{'(l)} = \sum_{k=1}^{2N+1} q_\nu e^{-jm\theta_k}$$

For the unknown coefficients at Eqs.(18) and Eqs(19) in the inhomogeneous region, we can obtain the matrix relations between $(\mathbf{a}^{(l)}, \mathbf{b}^{(l)})$ $and, (\mathbf{A}^{(l)}, \mathbf{B}^{(l)})$ in the boundary condition at $x = -ld_\Delta$ (1～8) as follows [29].

$$\left[E_y^{(l)} = E_y^{(l+1)}\right]_{x=-ld_\Delta} , \left[H_z^{(l)} = H_z^{(l+1)}\right]_{x=-ld_\Delta}$$

$$\left.\begin{aligned} \begin{pmatrix}\mathbf{a}^{(1)}\\ \mathbf{b}^{(1)}\end{pmatrix} &= \begin{pmatrix}\mathbf{S}_1^{(1)} & \mathbf{S}_2^{(1)}\\ \mathbf{S}_3^{(1)} & \mathbf{S}_4^{(1)}\end{pmatrix}\begin{pmatrix}\mathbf{A}^{(2)}\\ \mathbf{B}^{(2)}\end{pmatrix} \\ \begin{pmatrix}\mathbf{A}^{(2)}\\ \mathbf{B}^{(2)}\end{pmatrix} &= \begin{pmatrix}\mathbf{S}_1^{(2)} & \mathbf{S}_2^{(2)}\\ \mathbf{S}_3^{(2)} & \mathbf{S}_4^{(2)}\end{pmatrix}\cdots\begin{pmatrix}\mathbf{S}_1^{(7)} & \mathbf{S}_2^{(7)}\\ \mathbf{S}_3^{(7)} & \mathbf{S}_4^{(7)}\end{pmatrix}\begin{pmatrix}\mathbf{A}^{(7)}\\ \mathbf{B}^{(7)}\end{pmatrix} \\ \begin{pmatrix}\mathbf{a}^{(8)}\\ \mathbf{b}^{(8)}\end{pmatrix} &= \begin{pmatrix}\mathbf{S}_1^{(8)} & \mathbf{S}_2^{(8)}\\ \mathbf{S}_3^{(8)} & \mathbf{S}_4^{(8)}\end{pmatrix}\begin{pmatrix}\mathbf{A}^{(7)}\\ \mathbf{B}^{(7)}\end{pmatrix} \end{aligned}\right\}, \tag{20}$$

where,

○ $l = 1,\ 8$ (vacuum layer)

$\mathbf{S}_k^{(l)} \triangleq [{}^{(l)}s_{n,\nu}^{(k)}]$; $k = 1 \sim 4,\ 1, 8$,

$$\left.\begin{aligned} {}^{(l)}s_{n,\nu}^{(1)} &\triangleq \frac{1}{2}[1 + u_{n,\nu}^{(l)}\ h_{n+N+1}^{(l+1)}/k_n]e^{-ih_\nu^{(l)}d_\Delta} , \\ {}^{(l)}s_{n,\nu}^{(2)} &\triangleq {}^{(l)}s_{n,\nu}^{(3)} \cdot e^{i\{h_{n+N+1}^{(l+1)} - h_\nu^{(l)}\}d_\Delta} , \\ {}^{(l)}s_{n,\nu}^{(3)} &\triangleq \frac{1}{2}[1 - u_{n,\nu}^{(l)}\ h_{n+N+1}^{(l+1)}/k_n], \\ {}^{(l)}s_{n,\nu}^{(4)} &\triangleq \frac{1}{2}[1 + u_{n,\nu}^{(l)}\ h_{n+N+1}^{(l+1)}/k_n]e^{ih_{n+N+1}^{(l+1)}d_\Delta} , \end{aligned}\right\}, \tag{21}$$

$\mathbf{U} \triangleq [u_{\nu,n}^{(l)}],\ n = -N,\cdots,0,\cdots,N.\ ,\ \nu = 1 \sim (2N+1).$

○ $l = 2$～ 7 （inhomogeneous layer）

$\mathbf{S}_k^{(l)} \triangleq [{}^{(l)}s_{n,\nu}^{(k)}]$; $k = 1 \sim 4,\ 2 \le l \le 7$,

$$\left.\begin{aligned} {}^{(l)}s_{n,\nu}^{(1)} &\triangleq \frac{1}{2}[v_{n,\nu}^{(l)} + u_{n,\nu}^{(l)}\ h_{n+N+1}^{(l+1)}/k_n]e^{-ih_\nu^{(l)}d_\Delta} , \\ {}^{(l)}s_{n,\nu}^{(2)} &\triangleq {}^{(l)}s_{n,\nu}^{(3)} \cdot e^{i\{h_{n+N+1}^{(l+1)} - h_\nu^{(l)}\}d_\Delta} , \\ {}^{(l)}s_{n,\nu}^{(3)} &\triangleq \frac{1}{2}[v_{n,\nu}^{(l)} - u_{n,\nu}^{(l)}\ h_{n+N+1}^{(l+1)}/k_n], \\ {}^{(l)}s_{n,\nu}^{(4)} &\triangleq \frac{1}{2}[v_{n,\nu}^{(l)} + u_{n,\nu}^{(l)}\ h_{n+N+1}^{(l+1)}/k_n]e^{ih_{n+N+1}^{(l+1)}d_\Delta} , \end{aligned}\right\}, \tag{22}$$

$\mathbf{V} \triangleq [v_{\nu,n}^{(l)}] = [\mathbf{U}^{(l)}]^{-1}[\mathbf{U}^{(l+1)}],\quad \mathbf{U} \triangleq [u_{\nu,n}^{(l)}]$

$n = -N,\cdots,0,\cdots,N.\ ,\ \nu = 1 \sim (2N+1).$

Using the Eqs.(20), the equations of $a_n^{(8)}$ and $b_n^{(8)}$,can be obtained from Eqs. (18) and Eqs.(19),however because there are 2(2N+1) unknown equations for the sample point (2N+1), it is necessary to construct auxiliary equations for

$(\mathbf{a}^{(l)},\mathbf{b}^{(l)})$ *and*, $(\mathbf{A}^{(l)},\mathbf{B}^{(l)})$ in each layers. Because C_m is same relation of each layers.

To do this, we apply Eqs.(16) to l=1 layer and l=8 layers, and create an auxiliary equation in which both sides are equal to C_m .

$$\sum_{n=-N}^{N}(a_n^{(l=1)}F_{n,m}^{(l=1)}+b_n^{(l=1)}G_{n,m}^{(l=1)})=\sum_{n=-N}^{N}(a_n^{(l=8)}F_{n,m}^{(l=8}+b_n^{(l=8)}G_{n,m}^{(l=8)}) \quad (23)$$

By converting Eqs. (23) into a relational expression between $(\mathbf{a}^{(1)},\mathbf{b}^{(1)})$ *and* $(\mathbf{a}^{(8)},\mathbf{b}^{(8)})$, we obtain

$$\begin{pmatrix}\mathbf{a}^{(1)}\\ \mathbf{b}^{(1)}\end{pmatrix}=\begin{pmatrix}\mathbf{S}_1^{(1)} & \mathbf{S}_2^{(1)}\\ \mathbf{S}_3^{(1)} & \mathbf{S}_4^{(1)}\end{pmatrix}\begin{pmatrix}\mathbf{S}_1^{(2)} & \mathbf{S}_2^{(2)}\\ \mathbf{S}_3^{(2)} & \mathbf{S}_4^{(2)}\end{pmatrix}\cdots$$
$$\cdots\begin{pmatrix}\mathbf{S}_1^{(7)} & \mathbf{S}_2^{(7)}\\ \mathbf{S}_3^{(7)} & \mathbf{S}_4^{(7)}\end{pmatrix}\begin{pmatrix}\mathbf{S}_1^{(8)} & \mathbf{S}_2^{(8)}\\ \mathbf{S}_3^{(8)} & \mathbf{S}_4^{(8)}\end{pmatrix}^{-1}\begin{pmatrix}\mathbf{a}^{(8)}\\ \mathbf{b}^{(8)}\end{pmatrix}. \quad (24)$$

By combining Eqs. (18),(19), and (23), $(\mathbf{a}^{(8)},\mathbf{b}^{(8)})$ can be obtained=.

Using the obtained $(\mathbf{a}^{(8)},\mathbf{b}^{(8)})$, the scattering coefficients C_m can be determined Eqs.(16) [56,57].

4.Conclusion

This paper, in ``Construction of a high precision electromagnetic field analysis method and its application", describes a point matching method that takes into account edge conditions, and the progress of analyzing electromagnetic wave scattering and guiding problems for inhomogeneous media. In particular, it was the new method to solve the problem of electromagnetic wave scattering from an inhomogeneous object using periodically solutions, and utilizes solutions in a homogeneous medium by combining the point matching method, multilayer division method, and improved Fourier series expansion method.

As a numerical analysis technique, it is desirable to use a simulation algorithm that allows for easy error evaluation and error control, which can be analyzed using existing computer libraries. Visualizing the simulation results will deepen the understanding of physical phenomena and have a great educational effect.

To achieve this, it is necessary to use not only information mathematics (algorithm) technology but also electromagnetism. I believe that learning basic skills such as network theory, function theory, and linear algebra is essential for creativity.

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