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Scattering of Electromagnetic Waves by Inhomogeneous Dielectric Gratings Loaded with Conducting Strips —Matrix Formulation of Point Matching Method—

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SUMMARY We have proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings loaded with parallel perfectly conducting strips using the combination of improved Fourier series expansion method and point matching method. Numerical results are given for the transmission and scattering characteristics for TE and TM cases.

key words: *inhomogeneous dielectric, conducting strips, point matching method*

1. Introduction

Recently, the refractive index can be easily controlled by desigining periodic structures such as optoelectronic devices, photonic bandgap crystals, frequency selective devices, and other applications by the development of manufacturing technology of optical devices. Thus, the scattering and guiding problems of the inhomogeneous gratings have been of considerable interest, and many analytical and numerical methods which are applicable to the dielectric gratings having an arbitrarily periodic structures with combination of dielectric and/or metallic materials [1]–[5]. However, analysis of the dielectric grating [2]–[5] with metallic materials is only homogeneous type except [1]. Tamia et al. [1] proposed the modal transmission-line theory of multilayered grating structure with inhomogeneous region, in which the metallic region is not perfectly conducting type. For the inhomogeneous region, we have also analyzed the scattering and guiding problems by utilizing an improved Fourier series expansion method [6], and multilayer method [7], [8] and, in the perfectly conducting strip, we also analyzed the scattering problems by utilizing Point Matching Method [9], [10].

Although the Point Matching Method has wide range of applicability for the scattering problems with the perfectly conducting strips in inhomogeneous media, the order of the matrix size for the simultaneous equation depends on the number of strip layers because of the convergence domain of Point Matching Method [9].

In this paper, we proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with conducting strips [11]–[14] using the combination of improved Fourier series expansion method and Point Matching Method. The order of the matrix size for the simultaneous equation depends on the modal truncation number, but it does not depend on number of strip layers using matrix formulations.

Numerical results are given for the transmitted scattered characteristics for the case of periodically loaded with parallel perfectly conducting strips for TE and TM cases. The effects of the inhomogeneous dielectric gratings compared with that of the slanted angle of the perfectly conducting strips on the transmitted power are discussed. Our approach also can treat periodic configurations having arbitrary combinations of dielectric, metallic, and perfectly conducting components [19].

2. Method of Analysis

We consider inhomogeneous dielectric gratings loaded with parallel perfectly conducting strips shown in Fig. 1. The grating is uniform in the y-direction and the permittivity $\varepsilon(x, y)$ is an arbitrary periodic function of z.

The permeability μ is assumed to be μ_0 in the free space. The time dependence $\exp(-i\omega t)$ is suppressed throughout.

2.1 TE Wave

When the TE wave (the electric field has only the y-component)

$$E_y^{(i)} = e^{ik_0(z\sin\theta_0 - x\cos\theta_0)}, \quad k_0 \triangleq \omega\sqrt{\varepsilon_0\mu_0}, \tag{1}$$

is assumed to be incident from $x < 0$ at the angle θ_0, the electric fields in the regions S_1 $(x \leq 0)$, and S_3 $(x \geq D)$ are expressed [10] as

$\underline{S_1\ (x \leq 0)}$:

$$E_y^{(1)} = E_y^{(i)} + e^{ik_0 z\sin\theta_0}\sum_{n=-N}^{N} a_n e^{i(-k_n^{(1)}x + 2\pi nz/p)} \tag{2}$$

$\underline{S_3\ (x \geq D)}$:

$$E_y^{(3)} = e^{ik_0 z\sin\theta_0}\sum_{n=-N}^{N} b_n e^{i\{k_n^{(3)}(x-D) + 2\pi nz/p\}} \tag{3}$$

$$H_z^{(j)} = \{i\omega\mu_0\}^{-1}\partial E_y^{(j)}\big/\partial x,\ (j = 1,\ 3),$$

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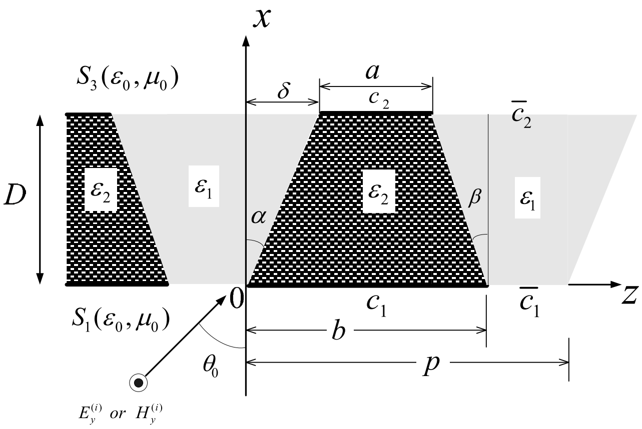


Fig. 1 Structure of inhomogeneous dielectric gratings loaded with conducting strips.

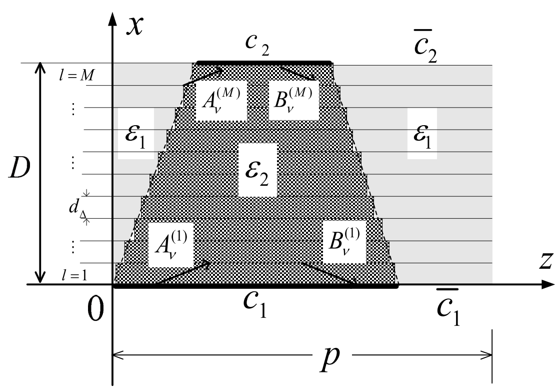


Fig. 2 Approximated inhomogeneous layers.

where $k_n{}^{(j)} \triangleq \sqrt{k_0\varepsilon_j/\varepsilon_0 - (2\pi n/p + k_0 \sin\theta_0)^2}$ is the propagation constants in the x direction and k_0 is the wave number, and ε_0 is the permittivity in the free space.

The inhomogeneous layer is approximated by a stratified layers of modulated index profile with d_Δ shown in Fig. 2 and taking each layer as the modulated dielectric gratings $\varepsilon_2^{(l)}(z)$. The electromagnetic fields are expanded appropriately by a finite Fourier series as follows:

$\underline{S_2\ (0 < x < D)}$:

$$E_y^{(2,l)} = \sum_{\nu=1}^{2N+1}\left[A_\nu^{(l)} e^{ih_\nu^{(l)} x} + B_\nu^{(1)} e^{-ih_\nu^{(l)}(x-d_\Delta)}\right] f_\nu^{(l)}(z)$$
$$H_z^{(2,j)} = \{i\omega\mu_0\}^{-1}\partial E_y^{(2,j)}\big/\partial x, \tag{4}$$
$$f_\nu^{(l)}(z) \triangleq e^{ik_0\sin\theta_0 z}\sum_{m=-N}^{N} u_m^{(\nu,l)} e^{i\,2\pi m z/p};\ l = 1 \sim \mathrm{M},$$

where $A_\nu^{(1\sim M)}$, $B_\nu^{(1\sim M)}$. a_n and b_n are unknown coefficients to be determined from boundary conditions. $h_\nu^{(l)}$ and $u_n^{(\nu,l)}$ are the propagation constant and eigenvectors, respectively.

It is satisfy the following eigenvalue equation in regard to $h_\nu^{(l)}$ [6].

$$\mathbf{\Lambda}^{(l)}\mathbf{U}^{(l)} = \left\{h_\nu^{(l)}\right\}^2\mathbf{U}^{(l)} \tag{5}$$

where,

$$\mathbf{U}^{(l)} \triangleq \left[u_n^{(\nu,l)}\right] = \left[u_{-\mathrm{N}}^{(\nu,l)}, \cdots\ u_0^{(\nu,l)}, \cdots\ u_N^{(\nu,l)}\right]^T,\ T\text{: transpose},$$
$$\mathbf{\Lambda}^{(l)} \triangleq \left[\alpha_{n,m}^{(l)}\right],\ a_{n,m}^{(l)} \triangleq k_0^2\xi_{n,m} - (2\pi n/p + k_0\sin\theta_0)^2,$$
$$\xi_{n,m}^{(l)} \triangleq \frac{1}{p}\int_0^p \left\{\frac{\varepsilon_2^{(l)}(z)}{\varepsilon_0}\right\} e^{i\,2\pi(n-m)z/p}dz,$$
$$m,\ n = (-N, \cdots, 0, \cdots, N)$$

In the inhomogeneous regions, we obtain the relationship between, $\mathbf{A}^{(1)}$, $\mathbf{B}^{(1)}$ in the first layer and, $\mathbf{A}^{(M)}$, $\mathbf{B}^{(M)}$ in the end of layer using boundary condition at $x = -ld_\Delta$ $(l = 1 \sim M-1)$.

$$\left[E_y^{(2,l)} = E_y^{(2,l+1)}\right]_{x=-ld_\Delta},\ \left[H_z^{(2,l)} = H_z^{(2,l+1)}\right]_{x=-ld_\Delta} \tag{6}$$

In general, the matrixes $\mathbf{\Lambda}$ of in Eq. (5) are not Hermitian and the orthogonality relations for $\{u^{(\nu,l)}\}$ are not expected.

Therefore substitution of Eq. (4) into Eq. (6) yields, using the orthogonality properties of $\{e^{i\,2\pi nz/p}\}$, the following matrix equations [8]:

$$\begin{pmatrix}\mathbf{A}^{(1)}\\ \mathbf{B}^{(1)}\end{pmatrix} = \begin{pmatrix}\mathbf{S}_1^{(1)} & \mathbf{S}_2^{(1)}\\ \mathbf{S}_3^{(1)} & \mathbf{S}_4^{(1)}\end{pmatrix}\begin{pmatrix}\mathbf{S}_1^{(2)} & \mathbf{S}_2^{(2)}\\ \mathbf{S}_3^{(2)} & \mathbf{S}_4^{(2)}\end{pmatrix}\cdots$$
$$\cdots\begin{pmatrix}\mathbf{S}_1^{(M)} & \mathbf{S}_2^{(M)}\\ \mathbf{S}_3^{(M)} & \mathbf{S}_4^{(M)}\end{pmatrix}\begin{pmatrix}\mathbf{A}^{(M)}\\ \mathbf{B}^{(M)}\end{pmatrix} = \begin{pmatrix}\mathbf{S}_1 & \mathbf{S}_2\\ \mathbf{S}_3 & \mathbf{S}_4\end{pmatrix}\begin{pmatrix}\mathbf{A}^{(M)}\\ \mathbf{B}^{(M)}\end{pmatrix}, \tag{7}$$
$$\text{where } \mathbf{S}_k^{(l)} \triangleq \left[{}^{(l)}s_{n,\nu}^{(k)}\right],\ k = 1 \sim 4,\ l = 1 \sim M$$
$${}^{(l)}s_{n,\nu}^{(1)} \triangleq v_{n,\nu}^{(l)}\left[1 + h_{n+N+1}^{(l+1)}/h_\nu^{(l)}\right]e^{-ih_\nu^{(l)}d_\Delta}/2,$$
$${}^{(l)}s_{n,\nu}^{(2)} \triangleq {}^{(l)}s_{n,\nu}^{(3)}\cdot e^{i\{h_{n+N+1}^{(l)} - h_\nu^{(l)}\}d_\Delta},$$
$${}^{(l)}s_{n,\nu}^{(3)} \triangleq v_{n,\nu}^{(l)}\left[1 - h_{n+N+1}^{(l+1)}/h_\nu^{(l)}\right]/2,$$
$${}^{(l)}s_{n,\nu}^{(4)} \triangleq v_{n,\nu}^{(l)}\left[1 + h_{n+N+1}^{(l+1)}/h_\nu^{(l)}\right]e^{ih_{n+N+1}^{(l)}d_\Delta}/2,$$
$$\mathbf{V} \triangleq \left[v_{n,\nu}^{(l)}\right] = \left[\mathbf{U}^{(l)}\right]^{-1}\left[\mathbf{U}^{(l+1)}\right],$$
$$n = (-N, \cdots, 0, \cdots, N),\ \nu = 1 \sim (2N+1).$$

Next we obtain the matrix form for the relations between the metallic region C and the dielectric region $\bar{C}$ shown in Fig. 2 using Point Matching Method [9] at the matching points

$$Z_j = (j-1)p/[(2N+1)];\ j = 1 \sim (2N+1) \tag{8}$$

The boundary condition for one period $(0 \leq x < p)$ on $x = 0$ and $x = D$ are as follows:

$$Z_j \in C_1;\ \left[E_y^{(1)} = 0,\ E_y^{(2,1)} = 0\right]_{x=0} \tag{9}$$
$$Z_j \in \overline{C_1};\ \left[E_y^{(1)} = E_y^{(2,1)}\right]_{x=0},\ \left[H_z^{(1)} = H_z^{(2,1)}\right]_{x=0} \tag{10}$$
$$Z_j \in C_2;\ \left[E_y^{(2,M)} = 0,\ E_y^{(3)} = 0\right]_{x=D} \tag{11}$$
$$Z_j \in \overline{C_2};\ \left[E_y^{(2,M)} = E_y^{(3)}\right]_{x=D},\ \left[H_z^{(2,M)} = H_z^{(3)}\right]_{x=D} \tag{12}$$

Substitution of Eqs. (1)~(4) into Eqs. (9)~(12) yields the following equations.

$$1 + \sum_{n=-N}^{N} a_n e^{inZ_j} = 0 \tag{13}$$

$$\sum_{\nu=1}^{2N+1}\left[A_\nu^{(1)}+B_\nu^{(1)}e^{ih_\nu^{(1)}d_\Delta}\right]\sum_{n=-N}^{N}u_n^{(\nu,1)}e^{inZ_j}=0 \tag{14}$$

for $Z_j \in C_1$.

$$1+\sum_{n=-N}^{N}a_n e^{inZ_j}=\sum_{\nu=1}^{2N+1}\left[A_\nu^{(1)}+B_\nu^{(1)}e^{ih_\nu^{(1)}d_\Delta}\right]\sum_{n=-N}^{N}u_n^{(\nu,1)}e^{inZ_j} \tag{15}$$

$$k_0^{(1)}-\sum_{n=-N}^{N}k_n^{(1)}a_n e^{inZ_j} = \sum_{\nu=1}^{2N+1}h_\nu^{(M)}\left[A_\nu^{(1)}-B_\nu^{(1)}e^{ih_\nu^{(1)}d_\Delta}\right]\sum_{n=-N}^{N}u_n^{(\nu,1)}e^{inZ_j} \tag{16}$$

for $Z_j \in \overline{C_1}$.

$$\sum_{n=-N}^{N}b_n e^{inZ_j}=0 \tag{17}$$

$$\sum_{\nu=1}^{2N+1}\left[A_\nu^{(M)}e^{ih_\nu^{(M)}d_\Delta}+B_\nu^{(M)}\right]\sum_{m=-N}^{N}{}^{(M)}u_m^{(\nu)}e^{imZ_j}=0 \tag{18}$$

for $Z_j \in C_2$.

$$\sum_{n=-N}^{N}b_n e^{inZ_j}=\sum_{\nu=1}^{2N+1}\left[A_\nu^{(M)}e^{ih_\nu^{(M)}d_\Delta}+B_\nu^{(M)}\right]\sum_{m=-N}^{N}{}^{(M)}u_m^{(\nu)}e^{imZ_j} \tag{19}$$

$$\sum_{m=-N}^{N}k_n^{(3)}b_n e^{inZ_j} = \sum_{\nu=1}^{2N+1}h^{(M)}{}_\nu\left[A_\nu^{(M)}e^{ih_\nu^{(M)}d_\Delta}-B_\nu^{(M)}\right]\sum_{m=-N}^{N}{}^{(M)}u_m^{(\nu)}e^{imZ_j} \tag{20}$$

for $Z_j \in \overline{C_2}$.

In the Eq. (15) and Eq. (19) in an electric field satisfied in all matching points around one period. Therefore using the orthogonality properties of $\{e^{i\,2\pi nz/p}\}$ we obtained a_n in Eq. (2) and b_n in Eq. (3) are obtained as follows:

$$a_n=\sum_{\nu=1}^{2N+1}\left[A_\nu^{(1)}+B_\nu^{(1)}e^{ih_\nu^{(1)}d_\Delta}\right]u_n^{(\nu,1)}-\delta_{0,n}, \tag{21}$$

$$b_n=\sum_{\nu=1}^{2N+1}\left[A_\nu^{(M)}e^{ih_\nu^{(1)}d_\Delta}+B_\nu^{(M)}\right]u_n^{(\nu,1)} \tag{22}$$

Substitution of Eq. (21) and Eq. (22) into Eq. (15) and Eq. (19) yields, including metallic region C in Eq. (14) and Eq. (18), following equations with $\mathbf{A}_\nu^{(l)}$ and $\mathbf{B}_\nu^{(l)}$ ($l=1,\ M$).

$$Z_j\in C_1:\ \sum_{\nu=1}^{2N+1}A_\nu^{(1)}\left[\sum_{m=-N}^{N}u_m^{(1,\nu)}e^{imZ_j}\right] + \sum_{\nu=1}^{2N+1}B_\nu^{(1)}\left[e^{ih_\nu^{(1)}d_\Delta}\sum_{m=-N}^{N}u_m^{(\nu,l)}e^{imZ_j}\right]=0$$

$$Z_j\in \overline{C}_1:\ \sum_{\nu=1}^{2N+1}A_\nu^{(1)}\left[\sum_{m=-N}^{N}(k_m^{(1)}+h_\nu^{(1)})u_m^{(\nu,l)}e^{imZ_j}\right] + \sum_{\nu=1}^{2N+1}B_\nu^{(M)}\left[e^{ih_\nu^{(1)}d_\Delta}\sum_{m=-N}^{N}(k_m^{(1)}-h_\nu^{(1)})u_m^{(\nu,l)}e^{imZ_j}\right]=2k_0^{(1)} \tag{23}$$

$$Z_j\in C_2:\ \sum_{\nu=1}^{2N+1}A_\nu^{(M)}\left[e^{ih_\nu^{(M)}d_\Delta}\sum_{m=-N}^{N}u_m^{(\nu,M)}e^{imzZ_j}\right] + \sum_{\nu=1}^{2N+1}B_\nu^{(1)}\left[\sum_{m=-N}^{N}u_m^{(\nu,M)}e^{imZ_j}\right]=0$$

$$Z_j\in \overline{C}_2:\ \sum_{\nu=1}^{2N+1}A_\nu^{(M)}\left[e^{ih_\nu^{(M)}d_\Delta}\sum_{m=-N}^{N}(k_m^{(3)}-h_\nu^{(M)})u_m^{(\nu,M)}e^{imZ_j}\right] + \sum_{\nu=1}^{2N+1}B_\nu^{(M)}\left[\sum_{m=-N}^{N}(k_m^{(3)}+h_\nu^{(M)})u_m^{(\nu,M)}e^{imZ_j}\right]=0 \tag{24}$$

To combine with the matrix by the boundary region C and $\bar{C}$, we defined following matrixes:

$$\mathbf{X}^C \triangleq \begin{bmatrix} \vdots & \vdots & \vdots & & \vdots & \vdots \\ e^{-iNZ_j} & \cdots & e^{in(=0)Z_j} & \cdots & & e^{iNZ_j} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & \cdots & 0 & & \cdots & 0 \end{bmatrix} \begin{matrix} \left.\right\} Z_j\in C_k \\ \left.\right\} Z_j\in \overline{C}_k, \end{matrix} \tag{25}$$

$$\mathbf{X}^{\overline{C}} \triangleq \begin{bmatrix} \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & & 0 & & & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ e^{-iNZ_j} & & e^{i0Z_j} & & & e^{iNZ_j} \end{bmatrix} \begin{matrix} \left.\right\} Z_j\in C_k \\ \left.\right\} Z_j\in \overline{C}_k, \end{matrix} \tag{26}$$

By using new matrix relationship between Eq. (25) and Eq. (26), we get following equation in the boundary condition at Eq. (14), Eq. (16), Eq. (18) and Eq. (20) in regard to $A_\nu{}^{(1)}$, $B_\nu{}^{(1)}$, $A_\nu{}^{(M)}$, and $B_\nu{}^{(M)}$.

$$\mathbf{Q}_1\mathbf{A}^{(1)}+\mathbf{Q}_2\mathbf{B}^{(1)}=\mathbf{F}, \tag{27}$$

$$\mathbf{Q}_3\mathbf{A}^{(M)}+\mathbf{Q}_4\mathbf{B}^{(M)}=\mathbf{0}, \tag{28}$$

where $\mathbf{F}\triangleq\left[0\,(Z_k\in C_1),\ 2k_0^{(1)}\,(Z_k\in\overline{C_1})\right]^T$,

$$\mathbf{A}^{(k)}\triangleq\left[A_1^{(k)},A_2^{(k)},\cdots,A_{2N+1}^{(k)}\right]^T,\ k=1,\ M,$$
$$\mathbf{B}^{(k)}\triangleq\left[B_1^{(k)},B_2^{(k)},\cdots,B_{2N+1}^{(k)}\right]^T,\ k=1,\ M,$$
$$\mathbf{Q}_1\triangleq\mathbf{X}^C\mathbf{U}^{(1)}+\mathbf{K}^{(1)}\mathbf{X}^{\overline{C}}\mathbf{U}^{(1)}+\mathbf{X}^{\overline{C}}\mathbf{U}^{(l)}\mathbf{H}^{(1)},$$
$$\mathbf{Q}_2\triangleq(\mathbf{X}^C\mathbf{U}^{(1)}+\mathbf{K}^{(1)}\mathbf{X}^{\overline{C}}\mathbf{U}^{(1)}-\mathbf{X}^{\overline{C}}\mathbf{U}^{(l)}\mathbf{H}^{(1)})\mathbf{D},$$
$$\mathbf{Q}_3\triangleq\mathbf{X}^C\mathbf{U}^{(M)}\mathbf{H}^{(M)}+\mathbf{K}^{(3)}\mathbf{X}^{\overline{C}}\mathbf{U}^{(M)}-\mathbf{X}^{\overline{C}}\mathbf{U}^{(M)}\mathbf{H}^{(M)}),$$
$$\mathbf{Q}_4\triangleq-\mathbf{X}^C\mathbf{U}^{(M)}\mathbf{H}^{(M)}\mathbf{D}+\mathbf{K}^{(3)}\mathbf{X}^{\overline{C}}\mathbf{U}^{(M)}+\mathbf{X}^{\overline{C}}\mathbf{U}^{(M)}\mathbf{H}^{(M)},$$
$$\mathbf{K}^{(l)}\triangleq\left[k_n^{(l)}\cdot\delta_{n,m}\right],\ \mathbf{H}^{(l)}\triangleq\left[h_\gamma^{(l)}\cdot\delta_{\gamma,\nu}\right],\ \mathbf{D}^{(l)}\triangleq\left[e^{ih_\gamma^{(l)}d_\Delta}\bullet\delta_{\gamma,\nu}\right]$$
$$l=1,\ M,\ \gamma=1\sim(2N+1),\ n=-N\sim N.$$

By using matrix relationship Eq. (7),

$$\begin{pmatrix}\mathbf{A}^{(1)}\\ \mathbf{B}^{(1)}\end{pmatrix}=\begin{pmatrix}\mathbf{S}_1 & \mathbf{S}_2\\ \mathbf{S}_3 & \mathbf{S}_4\end{pmatrix}\begin{pmatrix}\mathbf{A}^{(M)}\\ \mathbf{B}^{(M)}\end{pmatrix} \tag{7}$$

we get the following homogeneous matrix equation in regard to $A_\nu^{(M)}$ ($\nu = 1 \sim 2N+1$).

$$\mathbf{W} \bullet \mathbf{A}^{(M)} = \mathbf{F}, \tag{29}$$

where $\mathbf{W} \triangleq \mathbf{Q}_1\mathbf{S}_1 + \mathbf{Q}_2\mathbf{S}_3 - (\mathbf{Q}_1\mathbf{S}_2 + \mathbf{Q}_2\mathbf{S}_4).\mathbf{Q}_4^{-1}\mathbf{Q}_3$.

An advantage of our method, the order of the matrix size for the simultaneous equation depends on the modal truncation number, but it does not depend on number of strip layers using matrix formulations.

The mode power transmission coefficients $\rho_t^{(TE)}$ and reflection coefficients $\rho_r^{(TE)}$ are given by

$$\rho_t^{(TE)} \triangleq \sum_{n=-N}^{N} \mathrm{Re}\left[k_n^{(3)}\right]|b_n|^2/k_0^{(1)}, \tag{30}$$

$$\rho_r^{(TE)} \triangleq \sum_{n=-N}^{N} \mathrm{Re}\left[k_n^{(1)}\right]|a_n|^2/k_0^{(1)}. \tag{31}$$

The energy error $\varepsilon_e^{(TE)}$ for TE case is

$$\varepsilon_e^{(TE)} \triangleq 1 - (\rho_t^{(TE)} + \rho_r^{(TE)}). \tag{32}$$

2.2 TM Wave

When the TM wave (the magnetic field has only the y-component)

$$H_y^{(i)} = e^{ik_0(z\sin\theta_0 - x\cos\theta_0)}, \quad k_0 \triangleq \omega\sqrt{\varepsilon_{10}\mu_0}, \tag{33}$$

is assumed to be incident from $x < 0$ at the angle θ_0, the electric fields in the regions S_1 ($x \le 0$), and S_3 ($x \ge D$) are expressed [10] as

$\underline{S_1\ (x \le 0)}$:

$$H_y^{(1)} = H_y^{(i)} + e^{ik_0 z\sin\theta_0}\sum_{n=-N}^{N} a_n e^{i(-k_n^{(1)}x + 2\pi nz/p)} \tag{34}$$

$\underline{S_3\ (x \ge D)}$:

$$H_y^{(3)} = e^{ik_0 z\sin\theta_0}\sum_{n=-N}^{N} b_n e^{i\{k_n^{(3)}(x-D) + 2\pi nz/p\}} \tag{35}$$

$$E_z^{(j)} = \{i\omega\varepsilon_j\}^{-1}\partial H_y^{(j)}\big/\partial x,\ (j = 1, 3),$$

where $k_n{}^{(j)} \triangleq \sqrt{k_0\varepsilon_j/\varepsilon_0 - (2\pi n/p + k_0\sin\theta_0)^2}$.

The electromagnetic fields are expanded appropriately by a finite Fourier series as follows:

$\underline{S_2\ (0 < x < D)}$:

$$H_y^{(2,l)} = \sum_{\nu=1}^{2N+1}\left[A_\nu^{(l)} e^{ih_\nu^{(l)}x} + B_\nu^{(1)} e^{-ih_\nu^{(l)}(x-d_\Delta)}\right] f_\nu^{(l)}(z)$$

$$E_z^{(2,j)} = -\{i\omega\varepsilon_2^{(l)}(z)\}^{-1}\partial H_y^{(2,j)}\big/\partial x,\ (l = 1 \sim M) \tag{36}$$

$$f_\nu^{(l)}(z) \triangleq e^{ik_0\sin\theta_0 z}\sum_{m=-N}^{N} u_m^{(\nu,l)} e^{i2\pi mz/p},\ l = 1 \sim M$$

where $A_\nu^{(1\sim M)}$, $B_\nu^{(1\sim M)}$. a_n and b_n are unknown coefficients to be determined from boundary conditions. $h_\nu^{(l)}$ and $u_n^{(\nu,l)}$ are the propagation constant and eigenvectors, respectively.

In the same way for TE case, we have obtained the following eigenvalue equation in regard to $h_\nu^{(l)}$ [8].

$$\mathbf{\Lambda}^{(l)}\mathbf{U}^{(l)} = \left\{h_\nu^{(l)}\right\}^2\mathbf{U}^{(l)} \tag{37}$$

where,

$$\begin{aligned}
&\mathbf{U}^{(l)} \triangleq \left[u_n^{(\nu,l)}\right] = \left[u_{-\mathrm{N}}^{(\nu,l)}, \cdots\ u_0^{(\nu,l)}, \cdots\ u_N^{(\nu,l)}\right]^T, T\text{: transpose}\\
&\mathbf{\Lambda}^{(l)} \triangleq \left[\alpha_{n,m}^{(l)}\right]^{-1}\left[\beta_{n,m}^{(l)}\right], \quad g_n \triangleq (2\pi n/p + k_0\sin\theta_0)\\
&\alpha_{n,m}^{(l)} \triangleq \frac{1}{p}\int_0^p \left\{\frac{\varepsilon_2^{(l)}(z)}{\varepsilon_0}\right\} e^{i2\pi(n-m)z/p}dz,\\
&m,\ n = (-N, \cdots, 0, \cdots, N),\\
&\beta_{n,m}^{(l)} \triangleq k_0^2\xi_{n,m}^{(l)} - g_n[g_n + 2\pi(n-m)/p]\alpha_{n,m}^{(l)} \qquad (38)\\
&\xi_{n,m}^{(l)} \triangleq \frac{1}{p}\int_0^p \left\{\frac{\varepsilon_2^{(l)}(z)}{\varepsilon_0}\right\} e^{i2\pi(n-m)z/p}dz,\\
&m,\ n = (-N, \cdots, 0, \cdots, N),
\end{aligned}$$

For the TM case, if $\varepsilon_2^{(l)}(z)$ contains discontinuity, such as the step function, matrix elements $\alpha_{n,m}^{(l)}[2\pi(n-m)/p]$ do not converge. Therefore this method cannot be directly applied to the step distribution.

To solve this difficulty, the function containing the discontinuity is approximated by finite Fourier series of N_f terms as follows:

$$\varepsilon_2^{(l)}(z) \triangleq \sum_{n=-N_f}^{N_f} \tau_n e^{i2\pi nz/p} \tag{39}$$

The Gibb's phenomenon occurs at the discontinuities of permittivity, but it does not affect the solution as long as a sufficiently large number of modes are used for the electromagnetic fields [15]. So we have experienced that the relation $\mathrm{N} = 1.5\mathrm{N_f}$ is sufficient to get the proper solution when N and $\mathrm{N_f}$ are increased [7].

In the boundary conditions at $x = -ld_\Delta$ ($l = 1 \sim M-1$),

$$\left[H_y^{(2,l)} = H_y^{(2,l+1)}\right]_{x=-ld_\Delta},\ \left[E_z^{(2,l)} = E_z^{(2,l+1)}\right]_{x=-ld_\Delta}, \tag{40}$$

we can obtain the relationship between $\mathbf{A}^{(1)}$, $\mathbf{B}^{(1)}$ in the first layer and, $\mathbf{A}^{(M)}$, $\mathbf{B}^{(M)}$ in the end of unit layer using the orthogonality properties of $\{e^{i2\pi nz/p}\}$, and for the electric components of Eq. (40) by multiplying by $\varepsilon_2^{(l)}(z)\{e^{-i2\pi mz/p}\}/\varepsilon_0$ and integrating with respect to over the interval $0 \le z < p$.

$$\begin{aligned}
&\begin{pmatrix}\mathbf{A}^{(1)}\\ \mathbf{B}^{(1)}\end{pmatrix}=\begin{pmatrix}\mathbf{S}_1^{(1)} & \mathbf{S}_2^{(1)}\\ \mathbf{S}_3^{(1)} & \mathbf{S}_4^{(1)}\end{pmatrix}\begin{pmatrix}\mathbf{S}_1^{(2)} & \mathbf{S}_2^{(2)}\\ \mathbf{S}_3^{(2)} & \mathbf{S}_4^{(2)}\end{pmatrix}\cdots\\
&\cdots\begin{pmatrix}\mathbf{S}_1^{(M)} & \mathbf{S}_2^{(M)}\\ \mathbf{S}_3^{(M)} & \mathbf{S}_4^{(M)}\end{pmatrix}\begin{pmatrix}\mathbf{A}^{(M)}\\ \mathbf{B}^{(M)}\end{pmatrix}=\begin{pmatrix}\mathbf{S}_1 & \mathbf{S}_2\\ \mathbf{S}_3 & \mathbf{S}_4\end{pmatrix}\begin{pmatrix}\mathbf{A}^{(M)}\\ \mathbf{B}^{(M)}\end{pmatrix}, \qquad (41)
\end{aligned}$$

where $\mathbf{S}_k^{(l)} \triangleq \left[{}^{(l)}s_{n,\nu}^{(k)}\right],\ k = 1 \sim 4,\ l = 1 \sim M$

$${}^{(l)}s_{n,\nu}^{(1)} \triangleq \left[v_{n,\nu}^{(l)} + \eta_{\nu,n}^{(l)} h_{n+N+1}^{(l+1)}/h_\nu^{(l)}\right] e^{-ih_\nu^{(l)} d_\Delta}/2,$$

$${}^{(l)}s_{n,\nu}^{(2)} \triangleq {}^{(l)}s_{n,\nu}^{(3)} \cdot e^{i\{h_{n+N+1}^{(l)} - h_\nu^{(l)}\} d_\Delta},$$

$${}^{(l)}s_{n,\nu}^{(3)} \triangleq \left[v_{n,\nu}^{(l)} - \eta_{\nu,n}^{(l)} h_{n+N+1}^{(l+1)}/h_\nu^{(l)}\right]/2,$$

$${}^{(l)}s_{n,\nu}^{(4)} \triangleq \left[v_{n,\nu}^{(l)} + \eta_{\nu,n}^{(l)} h_{n+N+1}^{(l+1)}/h_\nu^{(l)}\right] e^{ih_{n+N+1}^{(l)} d_\Delta}/2,$$

$$\mathbf{V} \triangleq \left[v_{n,\nu}^{(l)}\right] = \left[\mathbf{U}^{(l)}\right]^{-1}\left[\mathbf{U}^{(l+1)}\right],$$

$$\boldsymbol{\eta} \triangleq \left[\eta_{\nu,n}^{(l)}\right] = \left[\boldsymbol{\Psi}^{(l)}\right]^{-1}\left[\boldsymbol{\Phi}^{(l+1)}\right] = \left[\psi_{\nu,n}^{(l)}\right]^{-1}\left[\phi_{\nu,n}^{(l)}\right],$$

$$\psi_{\nu,n}^{(l)} \triangleq \sum_{m=-N}^{N} u_{\nu,m}^{(l)} \alpha_{m,n}^{(l+1)},\ \phi_{\nu,n}^{(l)} \triangleq \sum_{m=-N}^{N} u_{\nu,m}^{(l+1)} \alpha_{m,n}^{(l)},$$

$$n = (-N, \cdots, 0, \cdots, N),\ \nu = 1 \sim (2N+1).$$

We also obtain the matrix form combination of metallic region C and the dielectric region $\bar{C}$ using boundary condition $Z_j = (j-1)p/[(2N+1)];\ j = 1 \sim (2N+1)$ at the matching points on $x = 0$, and D for TM case.

Boundary condition using Point Matching are as follows:

$$Z_j \in C_1;\ \left[E_z^{(1)} = 0,\ E_z^{(2,1)} = 0\right]_{x=0},$$

$$k_0{}^{(1)} - \sum_{n=-N}^{N} k_n{}^{(1)} a_n e^{inZ_j} = 0, \tag{42}$$

$$\sum_{\nu=1}^{2N+1} h_\nu^{(1)}\left[A_\nu^{(1)} - B_\nu^{(1)} e^{ih_\nu^{(1)} d_\Delta}\right] \sum_{n=-N}^{N} u_n^{(1,\nu)} e^{inZ_j} = 0, \tag{43}$$

$$Z_j \in \overline{C_1};\ \left[H_y^{(1)} = H_y^{(2,1)}\right]_{x=0},\ \left[E_z^{(1)} = E_z^{(2,1)}\right]_{x=0}$$

$$1 + \sum_{n=-N}^{N} a_n e^{inZ_j} = \sum_{\nu=1}^{2N+1}\left[A_\nu^{(1)} + B_\nu^{(2)} e^{ih_\nu^{(1)} d_\Delta}\right] \sum_{n=-N}^{N} u_n^{(1,\nu)} e^{inZ_j}, \tag{44}$$

$$\frac{1}{\varepsilon_0}\left(k_0{}^{(1)} - \sum_{n=-N}^{N} k_n{}^{(1)} a_n e^{inZ_j}\right) =$$
$$\frac{1}{\varepsilon_2^{(1)}(z)} \sum_{\nu=1}^{2N+1} h_\nu^{(1)}\left[A_\nu^{(1)} - B_\nu^{(1)} e^{ih_\nu^{(1)} d_\Delta}\right] \sum_{n=-N}^{N} u_n^{(1,\nu)} e^{inZ_j}, \tag{45}$$

$$Z_j \in C_2;\ \left[E_z^{(2,M)} = 0,\ E_z^{(3)} = 0\right]_{x=D},$$

$$\sum_{n=-N}^{N} k_n{}^{(3)} b_n e^{inz_j} = 0, \tag{46}$$

$$\sum_{\nu=1}^{2N+1} h_\nu^{(M)}\left[A_\nu^{(M)} e^{ih_\nu{}^{(M)} d_\Delta} - B_\nu^{(M)}\right] \sum_{n=-N}^{N} u_m^{(M,\nu)} e^{inZ_j} = 0 \tag{47}$$

$$Z_j \in \overline{C_2};\ \left[H_y^{(2,M)} = H_y^{(3)}\right]_{x=D},\ \left[E_z^{(2,M)} = E_z^{(3)}\right]_{x=D}$$

$$\sum_{n=-N}^{N} b_n e^{inZ_j} = \sum_{\nu=1}^{2N+1}\left[A_\nu^{(M)} e^{ih_\nu{}^{(M)} d_\Delta} + B_\nu^{(M)}\right] \sum_{n=-N}^{N} u_n^{(M,\nu)} e^{inZ_j} \tag{48}$$

$$\frac{1}{\varepsilon_0} \sum_{n=-N}^{N} k_n{}^{(3)} b_n e^{inZ_j}$$
$$= \frac{1}{\varepsilon_2^{(M)}(z)} \sum_{\nu=1}^{2N+1} h_\nu^{(M)}\left[A_\nu^{(M)} e^{ih_\nu{}^{(M)} d_\Delta} - B_\nu^{(M)}\right] \sum_{n=-N}^{N} u_n^{(M,\nu)} e^{inZ_j} \tag{49}$$

Using the orthogonality properties of $\{e^{i\,2\pi nz/p}\}$ by a boundary conditional around one period in an electric field of the Eq. (45) and Eq. (49) by multiplying by $\varepsilon_2^{(l)}(z)\{e^{-i\,2\pi mz/p}\}/\varepsilon_0$ are as follows

$$k_0^{(1)} \alpha_{0,m}^{(1)} - \sum_{n=-N}^{N} k_n^{(1)} \alpha_{n,m}^{(1)} a_n = \sum_{\nu=1}^{2N+1}\left[A_\nu^{(1)} - B_\nu^{(1)} e^{ih_\nu{}^{(1)} d_\Delta}\right] u_m^{(\nu,1)} \tag{50}$$

$$\sum_{n=-N}^{N} k_n^{(3)} \alpha_{n,m}^{(M)} b_n = \sum_{\nu=1}^{2N+1} h_\nu^{(M)}\left[A_\nu^{(M)} e^{ih_\nu{}^{(M)} d_\Delta} - B_\nu^{(M)}\right] u_m^{(\nu,M)} \tag{51}$$

$$n,\ m = -N \sim N.$$

In the same way for TE wave, substitution of Eq. (50) and Eq. (51) into Eq. (44) and Eq. (48) yields, including metallic region in Eq. (43) and Eq. (47), following equations with.

$$\mathbf{K}^{(1)}\boldsymbol{\alpha}^{(1)}\mathbf{a} = \mathbf{K}_0^{(1)}\boldsymbol{\alpha}_0^{(1)} - \mathbf{U}^{(1)}\mathbf{A}^{(1)} + \mathbf{U}^{(1)}\mathbf{D}^{(1)}\mathbf{B}^{(1)}, \tag{52}$$
$$\mathbf{K}^{(M)}\boldsymbol{\alpha}^{(M)}\mathbf{b} = \mathbf{U}^{(M)}\mathbf{H}^{(M)}\mathbf{D}^{(M)}\mathbf{A}^{(M)} - \mathbf{U}^{(M)}\mathbf{H}^{(M)}\mathbf{B}^{(\mathrm{M})}, \tag{53}$$

where,

$$\mathbf{a} \triangleq [a_n] \triangleq [a_{-N}, \cdots, a_0, \cdots, a_N]^T,\ T\text{: transpose}$$
$$\mathbf{b} \triangleq [b_n] \triangleq [b_{-N}, \cdots, b_0, \cdots, b_N]^T,$$
$$\mathbf{A}^{(k)} \triangleq \left[A_1^{(k)}, A_2^{(k)}, \cdots, A_{2N+1}^{(k)}\right]^T,\ k = 1,\ M,$$
$$\mathbf{B}^{(k)} \triangleq \left[B_1^{(k)}, B_2^{(k)}, \cdots, B_{2N+1}^{(k)}\right]^T,\ k = 1,\ M,$$
$$\mathbf{K}^{(k)} \triangleq \left[k_n^{(k)} \cdot \delta_{n,m}\right],\ \mathbf{K}_0^{(1)} \triangleq \left[k_0^{(1)} \cdot \delta_{0,,m}\right],\ \mathbf{H}^{(l)} \triangleq \left[h_\gamma^{(l)} \cdot \delta_{\gamma,\nu}\right],$$
$$\boldsymbol{\alpha}^{(l)} \triangleq \left[\alpha_{n,m}^{(l)}\right],\ \boldsymbol{\alpha}_0^{(l)} \triangleq \left[\alpha_{0,m}^{(l)} \cdot \delta_{0,m}\right],\ \mathbf{D}^{(l)} \triangleq h_\gamma^{(l)}\left[e^{ih_\nu^{(1)} d_\Delta} \cdot \delta_{\gamma,\nu}\right]$$
$$k = 1,\ 3,\ l = 1,\ M,\ \nu = 1 \sim (2N+1),\ n,\ m = -N \sim N.$$

We get also the following matrix form in regard to **a** and **b**.

$$\mathbf{a} = \left[\mathbf{K}^{(1)}\boldsymbol{\alpha}^{(1)}\right]^{-1}\left[\mathbf{K}_0^{(1)}\boldsymbol{\alpha}_0^{(1)} - \mathbf{U}^{(1)}\mathbf{A}^{(1)} + \mathbf{U}^{(1)}\mathbf{D}^{(1)}\mathbf{B}^{(1)})\right]$$
$$= \left[\mathbf{K}^{(1)}\boldsymbol{\alpha}^{(1)}\right]^{-1}\left[\mathbf{K}_0^{(1)}\boldsymbol{\alpha}_0^{(1)}\right] - \left[\mathbf{K}^{(1)}\boldsymbol{\alpha}^{(1)}\right]^{-1}\mathbf{U}^{(1)}\mathbf{A}^{(1)}$$
$$+ \left[\mathbf{K}^{(1)}\boldsymbol{\alpha}^{(1)}\right]^{-1}\mathbf{U}^{(1)}\mathbf{D}^{(1)}\mathbf{B}^{(1)})]$$
$$= \mathbf{P}^{(1)}\mathbf{A}^{(1)} + \mathbf{P}^{(2)}\mathbf{B}^{(1)} + \mathbf{R}, \tag{54}$$
$$\mathbf{b} = \left[\mathbf{K}^{(M)}\boldsymbol{\alpha}^{(M)}\right]^{-1}\left[\mathbf{U}^{(M)}\mathbf{H}^{(M)}\mathbf{D}^{(M)}\mathbf{A}^{(M)} - \mathbf{U}^{(M)}\mathbf{H}^{(M)}\mathbf{B}^{(M)}\right]$$
$$= \mathbf{P}^{(3)}\mathbf{A}^{(M)} + \mathbf{P}^{(4)}\mathbf{B}^{(M)}, \tag{55}$$

where $\mathbf{P}^{(1)} \triangleq -\left[\mathbf{K}^{(1)}\boldsymbol{\alpha}^{(1)}\right]^{-1}\mathbf{U}^{(1)}$,

$$\mathbf{P}^{(2)} \triangleq \left[\mathbf{K}^{(1)}\boldsymbol{\alpha}^{(1)}\right]^{-1}\left[\mathbf{U}^{(1)}\mathbf{D}^{(1)}\right],$$

$$\mathbf{P}^{(3)} \triangleq \left[\mathbf{K}^{(3)}\boldsymbol{\alpha}^{(M)}\right]^{-1}\left[\mathbf{U}^{(M)}\mathbf{H}^{(M)}\mathbf{D}^{(M)}\right],$$

$$\mathbf{P}^{(4)} \triangleq -\left[\mathbf{K}^{(3)}\boldsymbol{\alpha}^{(M)}\right]^{-1}\mathbf{U}^{(M)}\mathbf{H}^{(M)},$$
$$\mathbf{R} \triangleq \left[\mathbf{K}^{(1)}\boldsymbol{\alpha}^{(1)}\right]^{-1}\left[\mathbf{K}_0^{(1)}\boldsymbol{\alpha}_0^{(1)}\right]$$

By using new matrix relationship between Eq. (54) and Eq. (55) using the same equations $\mathbf{X}^C$ in Eq. (25) and $\mathbf{X}^{\overline{C}}$ in Eq. (26) for the TE case, we get following equation in the boundary condition at Eq. (43), Eq. (44), Eq. (47) and Eq. (48) in regard to $A_\nu{}^{(1)}$, $B_\nu{}^{(1)}$, $A_\nu{}^{(M)}$, and $B_\nu{}^{(M)}$.

$$\mathbf{Q}_1\mathbf{A}^{(1)} + \mathbf{Q}_2\mathbf{B}^{(1)} = \mathbf{F}, \tag{56}$$
$$\mathbf{Q}_3\mathbf{A}^{(M)} + \mathbf{Q}_4\mathbf{B}^{(M)} = \mathbf{0}, \tag{57}$$

where, $\mathbf{F} \triangleq \left[0\ (Z_k \in C_1),\ 1 + \mathbf{X}^C\mathbf{R}\ (Z_k \in \overline{C_1})\right]^T$,

$$\mathbf{Q}_1 \triangleq \mathbf{X}^C\mathbf{U}^{(1)}\mathbf{H}^{(1)} + \mathbf{X}^{\overline{C}}(\mathbf{U}^{(1)} - \mathbf{P}^{(1)}),$$
$$\mathbf{Q}_2 \triangleq -\mathbf{X}^C\mathbf{U}^{(1)}\mathbf{H}^{(1)}\mathbf{D}^{(1)} + \mathbf{X}^{\overline{C}}(\mathbf{U}^{(1)}\mathbf{D}^{(1)} - \mathbf{P}^{(2)}),$$
$$\mathbf{Q}_3 \triangleq \mathbf{X}^C\mathbf{U}^{(M)}\mathbf{H}^{(M)}\mathbf{D}^{(M)} + \mathbf{X}^{\overline{C}}(\mathbf{U}^{(M)}\mathbf{D}^{(M)} - \mathbf{P}^{(3)}),$$
$$\mathbf{Q}_4 \triangleq -\mathbf{X}^C\mathbf{U}^{(M)} + \mathbf{X}^{\overline{C}}(\mathbf{U}^{(M)} - \mathbf{P}^{(4)}),$$
$$\mathbf{H}^{(l)} \triangleq \left[h_\gamma^{(l)} \cdot \delta_{\gamma,\nu}\right],\ \mathbf{D}^{(l)} \triangleq \left[e^{ih_\gamma^{(l)}d_\Delta} \bullet \delta_{\gamma,\nu}\right]$$
$$l = 1,\ M,\ \gamma = 1 \sim (2N+1),\ n = -N \sim N.$$

By using matrix relationship between Eq. (56), and Eq. (57), we get the following homogeneous matrix equation in regard to $A_\nu^{(M)}$ ($\nu = 1 \sim 2N+1$).

$$\mathbf{W} \bullet \mathbf{A}^{(M)} = \mathbf{F}, \tag{58}$$

where $\mathbf{W} \triangleq \mathbf{Q}_1\mathbf{S}_1 + \mathbf{Q}_2\mathbf{S}_3 - (\mathbf{Q}_1\mathbf{S}_2 + \mathbf{Q}_2\mathbf{S}_4).\mathbf{Q}_4^{-1}\mathbf{Q}_3$.

For the TM wave, an advantage of our method, the order of the matrix size for the simultaneous equation depends on the modal truncation number, but it does not depend on number of strip layers using matrix formulations.

The mode power transmission coefficients $\rho_t^{(TE)}$ and reflection coefficients $\rho_r^{(TE)}$ are given by

$$\rho_t^{(TM)} \triangleq \sum_{n=-N}^{N} \mathrm{Re}\left[k_n^{(3)}\right]|b_n|^2/k_0^{(1)}, \tag{59}$$

$$\rho_r^{(TM)} \triangleq \sum_{n=-N}^{N} \mathrm{Re}\left[k_n^{(1)}\right]|a_n|^2/k_0^{(1)}. \tag{60}$$

The energy error $\varepsilon_e^{(TM)}$ for TM case is

$$\varepsilon_e^{(TM)} \triangleq 1 - (\rho_t^{(TM)} + \rho_r^{(TM)}). \tag{61}$$

3. Numerical Analysis

We consider the three cases of strip profiles with inhomogeneous dielectric gratings as follows in Fig. 1:

$$\varepsilon_2(x,z)/\varepsilon_0 \triangleq \begin{cases} 1.0 : (0 \le \phi \le \alpha \text{ and } \beta) \\ 1.5 : (\alpha \text{ and } \beta < \phi \le \pi/2) \end{cases} \tag{62}$$
$$\varepsilon_1 = \varepsilon_3 = \varepsilon_o,\ b/p = 0.5 \text{ and } D/p = 0.3.$$

The cases of parameters chosen are as follows in Fig. 3:

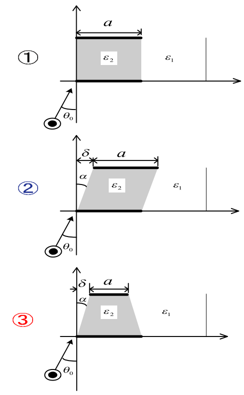


Fig. 3 Three cases of strip gratings.

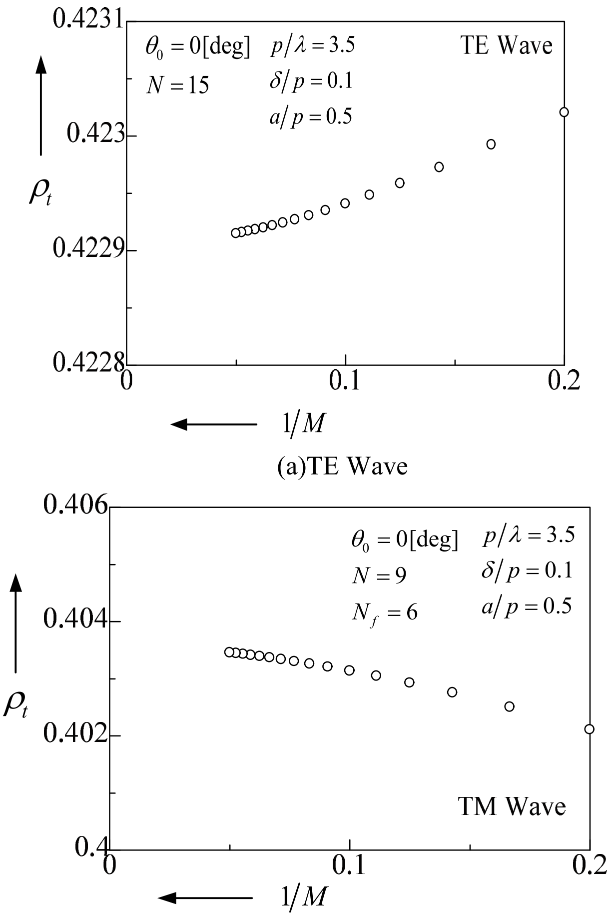


Fig. 4 Convergence of ρ_t vs. $1/M$ [(a) TE Wave and (b) TM Wave].

Case circle ①:

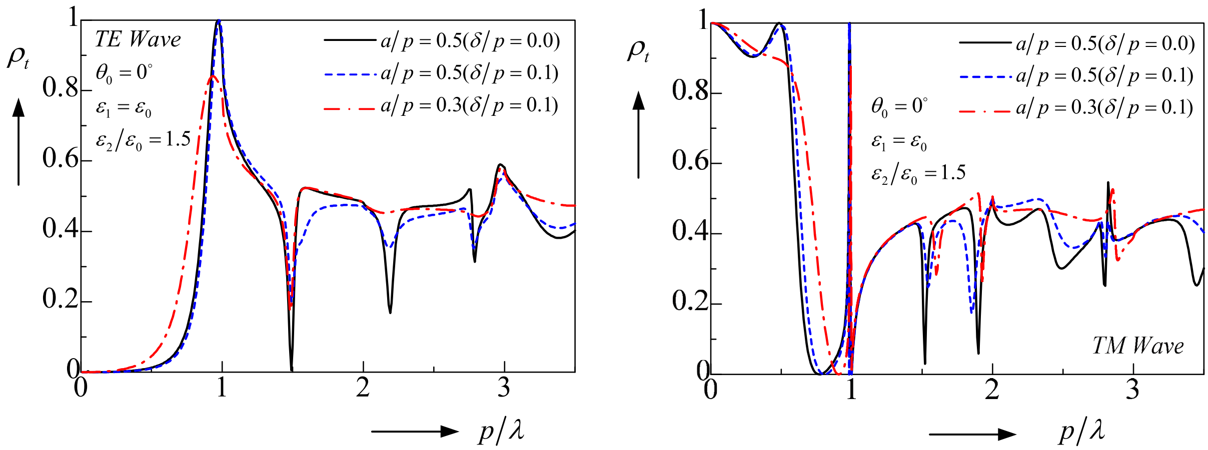
$a = b$ and $\alpha = \beta = 0$ ($\delta/p = 0$),

Case circle ②:

$a = b$ and $\beta = -\alpha$ ($\delta/p \neq 0$),

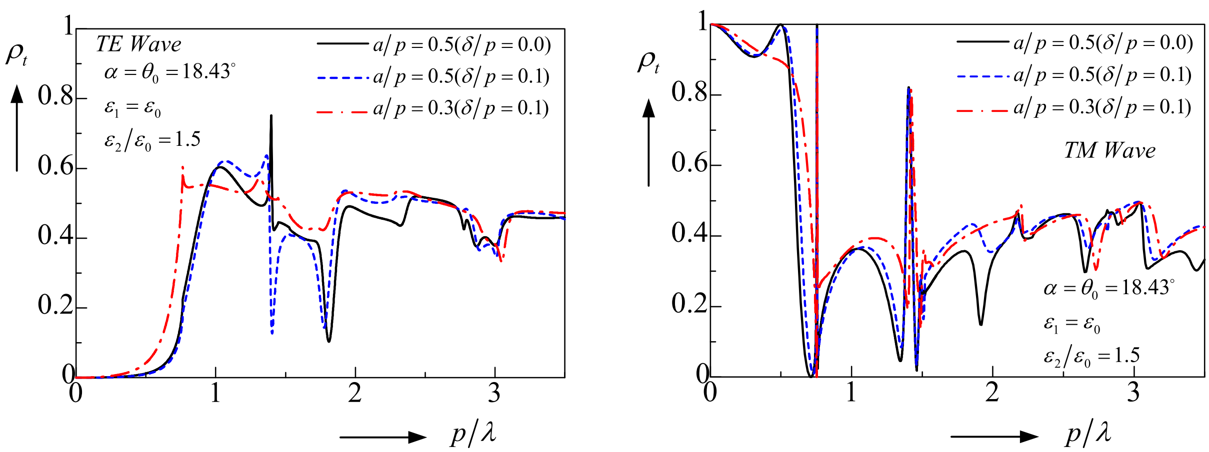
Case circle ③:

$a < b$ and $\alpha = \beta$ ($\delta/p \neq 0$).



(a) TE Wave (b) TM Wave

Fig. 5 Mode power transmission coefficients ρ_t vs. normalized frequency p/λ.



(a) TE Wave (b) TM Wave

Fig. 6 Mode power transmission coefficients ρ_t vs. normalized frequency p/λ.

The accuracy of our method in inhomogeneous already given by reference [16] compared with the exact solution [17] in the circular cylinder.

Figures 4 (a) and 4 (b) show the convergence of ρ_t versus $1/M$ for the case of circle ② with the fixed N and N_f: (a) TE Wave and (b) TM Wave, respectively.

From Fig. 3, for the TE Wave we take N = 15 and M = 20 and for the TM Wave we take N = 9 (N_f = 6) and M = 20.

The relative errors (≙ [extrapolated true value − computed value]/[extrapolated true value] [9] of ρ_t both TE and TM Wave is less than 0.1 % for M > 20 in Fig. 4. The energy errors ($\varepsilon_e^{(TE)}$ and $\varepsilon_e^{(TM)}$) are less than about 10^{-7} for TE Wave and 10^{-3} for TM Wave, respectively. For the case of circle ③, the order of relative error and energy error are about same, but for the case of circle ①, the energy error is about 10^{-7} for both Waves, because of the homogeneous layer.

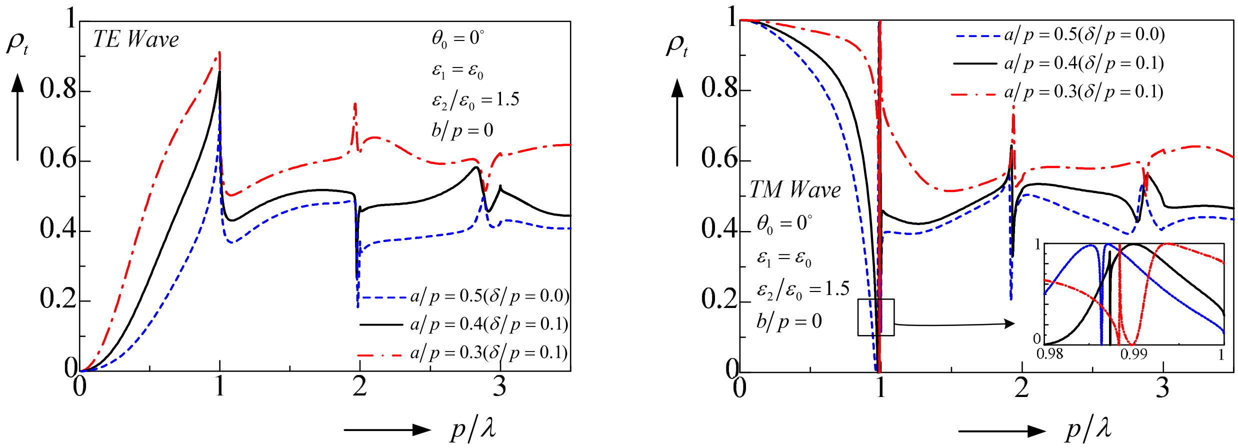
Figures 5 shows ρ_t for various values of normalized frequency λ/p at $\theta_0 = 0^\circ$. Figure 5 (a) is TE wave and Fig. 5 (b) is TM wave. From Fig. 5 we can see following features:

(1) Characteristic tendencies of coupling resonance cases of circle ①, circle ② and circle ③ are approximately same for the TE and TM wave at 1.4 < p/λ < 1.5.
(2) For the TE wave, the peak of $\rho_t^{(\mathrm{TE})}$ in $p/\lambda < 1.0$ is suppressed and the resonant peaks of frequency moves toward smaller in the case of circle ③.
(3) For the TM wave in $p/\lambda < 1.0$ the resonant peaks of frequency moves toward larger circle ③.

Figures 6 shows for the various values of normalized frequency with the case of for the same parameters Fig. 5.

We can see following features:

(1) The characteristic tendencies for the effect of equivalent permittivity are approximately same.
(2) For the TE wave, as the incidence angle θ_0 is equal to α = 18.43°, the p/λ is around 1.4 and resonant peaks appears, and p/λ is 0.7 < λ/p < 1.4 in case of the trapezoid type of circle ③, and it is interesting that $\rho_t^{(\mathrm{TE})}$ becomes nearly constant.



(a) TE Wave

(b) TM Wave

Fig. 7 Mode power transmission coefficients ρ_t vs. normalized frequency p/λ.

(3) For the TM wave, it doesn't change into resonant peaks around $p/\lambda = 0.75$ and $p/\lambda = 1.4$ in case of circle ③. But it peaks moves toward smaller p/λ.

Figures 7 shows ρ_t when only the strip is upper part (the lower part of strip is removed) for the same parameters Fig. 5 at $\theta_0 = 0^\circ$. Figure 7 (a) are TE wave and Fig. 7 (b) are TM wave.

We can see following features:

(4) For the TE wave, characteristic tendency of $\rho_t^{(TE)}$ is similar in case of a dielectric strip type and influences of p/λ around Wood's anomaly by a frequency response of a TE wave becomes larger in case of the trapezoid type circle ③ because a/p is small.

(5) For the TM wave, characteristic tendency of $\rho_t^{(TM)}$ is similar in case of a dielectric strip type, but the resonant peaks $\rho_t^{(\mathrm{TM})}$ of p/λ around 1.0 indicates the double switching effects [18] (two resonance peaks appear for both low and high frequency region) in case of the trapezoid type circle ③ as it also indicates in an enlarged drawing in the Fig. 7 (b) for $0.985 < \lambda/\mathrm{p} < 0.995$.

4. Conclusion

In this paper, we have proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings loaded with perfectly conducting strips using the combination of improved Fourier series expansion method and point matching method. An advantage of our method, the order of the matrix size for the simultaneous equation depends on the modal truncation number, but it does not depend on number of strip layers using matrix formulations.

Numerical results are given for the transmitted scattered characteristics for the case of frequency loaded with the parallel perfectly conducting strips for TE and TM cases. The effects of the inhomogeneous dielectric gratings compared with that of the slant angle on the transmitted power are discussed.

This method also can be applied to the inhomogeneous dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials [19].

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