INVITED PAPER Special Section on Recent Progress in Electromagnetic Theory and Its Application

Kobayashi Potential in Electromagnetism

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SUMMARY The Kobayashi potential in electromagnetic theory is reviewed. As an illustration we consider two problems, diffraction of plane wave by disk and rectangular plate of perfect conductor. Some numerical results are compared with approximated and experimental results when they are available to verify the validity of the present method. We think the present method can be used as reference solutions of the related problems.
key words: *Kobayashi potential, mixed boundary value problem, diffraction and scattering, edge condition*

1. Introduction

The name of the Kobayashi potential (KP) was given by I. Sneddon in his book "Mixed Boundary Value Problems in Potential Theory" [1] for the paper [2] published by Kobayashi in 1931. This paper treated the potential problems associated with single disk or two disks. He assumed a potential function so that the function becomes in the form of Weber-Schafheitlin's discontinuous integrals in the plane $z = 0$ where the circular plate is located. This makes the function satisfy a part of the required boundary conditions like the eigen function expansion methods in circular cylinder and sphere scattering problems. The potential functions in static problems were extended to dynamic wave problems by Yukiti (Yukichi) Nomura and his associates. Their noteworthy work is on the paper of the diffraction of electromagnetic wave by a ribbon and a disk of perfect conductor (with Shigetoshi Katsura) [3]. Electromagnetic scattering problems using the Kobayashi potential have been studied by K. Hongo and his students [4], [5]. In this paper we introduce an excellent technique developed in Japan since very few researchers may be familiar to this method. To deepen the understanding we show two problems as an illustration, the first is the scattering by a rectangular plate and the second one is by a disk (see Fig. 1).

The reference solution for scattering by rectangular plate is very few except the moment method solution. And this method serves its aims. Also this subject has some interesting problem, for example, singularity at the vertex, numerical solutions of the grazing incidence case, and so on. In this paper we present rigorous formulation of this problem by using the KP method. Two components of the vector potential are expressed in terms of two dimensional Fourier sine and cosine transform. We use the discontinuous properties of the Weber-Schafheitlin integrals so that the required boundary conditions at the exterior to the plate or hole in the plane where it is located. By using the concept of the projection for the remaining condition on the plate, the solution is reduced to matrix equations. The matrix elements are given by double infinite integrals with rather slow convergence. We have developed a method of computation that enables one to get precise results. The expressions thus derived have properties similar to those for eigen function solutions for a circular cylinder and a sphere.

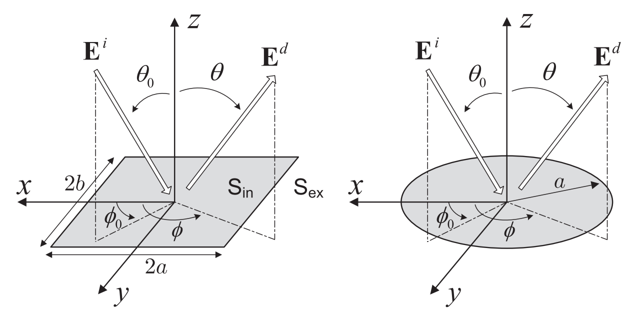

Fig. 1 Scattering of an electromagnetic plane wave by a rectangular plate and by a circular disk. (θ_0, ϕ_0) denote the angles of incidence.

Diffraction of EM wave by a circular hole is classical problem and exact solutions exist. Meixner formulated the field in terms of Spheroidal functions and Andrejewsky [6] gave some numerical results. Katsura and Nomura solved the same problem by using the Weber-Schafheitlin's integral [3]. The present formulation belongs to the similar category with the work of Katsura and Nomura, but there are also some differences. Theoretically, diffracted field can be expressed by two scalar wave functions [7]. The works [3] and [6] used three components which are related by imposing edge conditions. In this paper we show that the field can be derived from two components of the vector potential functions and the functions themselves satisfy the edge condition [8]. Therefore the philosophy of the formulation is very simple. The procedure of the formulation is as follows. The two components of the vector potential are expanded in the form of Fourier Hankel transform [9]. By imposing the required boundary conditions dual integral equations are derived. These equations are transformed into matrix equations by applying the properties of Weber-Schafheitlin's integral and projection. Matrix elements may be evaluated in

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a closed form and computed rather easily. Computed results are compared with asymptotic solutions and experimental results. Agreement among them is fairly well.

2. Diffraction of Plane EM Waves by a Rectangular Plate [4], [5]

In this chapter we present how to formulate the electromagnetic plane wave diffracted by a rectangular plate or hole (hereafter we treat only plate problem). The incident wave is given by

$$\mathbf{E}^i = (E_2\mathbf{i}_\theta + E_1\mathbf{i}_\phi)\exp[jk\Phi^i(\mathbf{r})] \tag{1a}$$

$$\mathbf{H}^i = Y_0(-E_2\mathbf{i}_\phi + E_1\mathbf{i}_\theta)\exp[jk\Phi^i(\mathbf{r})] \tag{1b}$$

and the magnetic vector potential $\mathbf{A}^s$ of the scattered wave, which is used instead of the electromagnetic field for the convenience of later analysis, is given by

$$\begin{aligned}\begin{pmatrix}A_x^s\\A_y^s\end{pmatrix} &= \mu_0 Y_0 a\int_0^\infty\int_0^\infty\left[\begin{pmatrix}f_{cc}^\pm(\alpha,\beta)\\g_{cc}^\pm(\alpha,\beta)\end{pmatrix}\cos\alpha\xi\cos\beta\eta\right.\\&\quad+\begin{pmatrix}f_{cs}^\pm(\alpha,\beta)\\g_{cs}^\pm(\alpha,\beta)\end{pmatrix}\cos\alpha\xi\sin\beta\eta+\begin{pmatrix}f_{sc}^\pm(\alpha,\beta)\\g_{sc}^\pm(\alpha,\beta)\end{pmatrix}\sin\alpha\xi\\&\quad\left.\times\cos\beta\eta+\begin{pmatrix}f_{ss}^\pm(\alpha,\beta)\\g_{ss}^\pm(\alpha,\beta)\end{pmatrix}\sin\alpha\xi\sin\beta\eta\right]\\&\quad\times\exp\left[\mp\zeta(\alpha,\beta)z_a\right]d\alpha d\beta\quad (z\gtrless 0)\end{aligned}\tag{2}$$

where $f(\alpha,\beta)$ and $g(\alpha,\beta)$ are unknown functions determined later. The symbols used in (1) and (2) are defined by

$$\begin{aligned}&\mathbf{i}_\theta = \cos\theta_0\cos\phi_0\mathbf{i}_x + \cos\theta_0\sin\phi_0\mathbf{i}_y - \sin\theta_0\mathbf{i}_z,\\&\mathbf{i}_\phi = -\sin\phi_0\mathbf{i}_x + \cos\phi_0\mathbf{i}_y\end{aligned}\tag{3a}$$

$$\begin{pmatrix}\Phi^i(\mathbf{r})\\\Phi^r(\mathbf{r})\end{pmatrix} = x\sin\theta_0\cos\phi_0 + y\sin\theta_0\sin\phi_0 \pm z\cos\theta_0 \tag{3b}$$

$$\begin{aligned}&\zeta(\alpha,\beta) = \sqrt{\alpha^2+p^2\beta^2-\kappa^2},\quad \xi=\frac{x}{a},\quad \eta=\frac{y}{b},\\&z_a=\frac{z}{a},\quad p=\frac{a}{b}\left(=\frac{1}{q}\right),\quad \kappa=ka,\quad Y_0=\sqrt{\frac{\epsilon_0}{\mu_0}}.\end{aligned}\tag{3c}$$

In this analysis, harmonic time dependence $\exp(j\omega t)$ is assumed and omitted in equations. Imposing the required boundary conditions: H_x^t and H_y^t are continuous for $(x,y)\in S_{ex}$ and $E_x^t=0$ and $E_y^t=0$ on $(x,y)\in S_{in}$, we derive the dual integral equations. Equations for the continuities of magnetic field components can be solved by using the discontinuous properties of the Weber-Schafheitlin's integrals [10, p.99] and the results are given by

$$f_{cc}(\alpha,\beta) = \sum_{m=0}^\infty\sum_{n=0}^\infty\frac{1}{\alpha\zeta(\alpha,\beta)}A_{mn}^{(x)}J_{2m+1}(\alpha)J_{2n}(\beta) \tag{4a}$$

$$g_{cc}(\alpha,\beta) = \sum_{m=0}^\infty\sum_{n=0}^\infty\frac{1}{\beta\zeta(\alpha,\beta)}A_{mn}^{(y)}J_{2m}(\alpha)J_{2n+1}(\beta). \tag{4b}$$

Similar relations can be derived for other components. The solutions for the electric field components are obtained by applying the projection and we have matrix equations for the expansion coefficients

$$\begin{aligned}&\begin{bmatrix}K_A(2m{+}1,2n,2s{+}1,2t) & pG(2m{+}1,2n{+}2,2s{+}1,2t)\\qG(2m{+}1,2n,2s{+}1,2t{+}2) & K_B(2m{+}1,2n{+}2,2s{+}1,2t{+}2)\end{bmatrix}\\&\times\begin{bmatrix}A_{mn}^{(x)}\\D_{mn}^{(y)}\end{bmatrix}=\begin{bmatrix}-j\Lambda_{2s+1}(\kappa\sin\theta_0\cos\phi_0)J_{2t}(\kappa\sin\theta_0\sin\phi_0)\Pi_x\\jq^2J_{2s+1}(\kappa\sin\theta_0\cos\phi_0)\Lambda_{2t+2}(\kappa\sin\theta_0\sin\phi_0)\Pi_y\end{bmatrix}\end{aligned}\tag{5a}$$

$$\begin{aligned}&\begin{bmatrix}K_A(2m{+}1,2n{+}1,2s{+}1,2t{+}1) & -pG(2m{+}1,2n{+}1,2s{+}1,2t{+}1)\\-qG(2m{+}1,2n{+}1,2s{+}1,2t{+}1) & K_B(2m{+}1,2n{+}1,2s{+}1,2t{+}1)\end{bmatrix}\\&\times\begin{bmatrix}B_{mn}^{(x)}\\C_{mn}^{(y)}\end{bmatrix}=\begin{bmatrix}\Lambda_{2s+1}(\kappa\sin\theta_0\cos\phi_0)J_{2t+1}(\kappa\sin\theta_0\sin\phi_0)\Pi_x\\q^2J_{2s+1}(\kappa\sin\theta_0\cos\phi_0)\Lambda_{2t+1}(\kappa\sin\theta_0\sin\phi_0)\Pi_y\end{bmatrix}\end{aligned}\tag{5b}$$

$$\begin{aligned}&\begin{bmatrix}K_A(2m{+}2,2n,2s{+}2,2t) & -pG(2m,2n{+}2,2s{+}2,2t)\\-qG(2m{+}2,2n,2s,2t{+}2) & K_B(2m,2n{+}2,2s,2t{+}2)\end{bmatrix}\\&\times\begin{bmatrix}C_{mn}^{(x)}\\B_{mn}^{(y)}\end{bmatrix}=\begin{bmatrix}\Lambda_{2s+2}(\kappa\sin\theta_0\cos\phi_0)J_{2t}(\kappa\sin\theta_0\sin\phi_0)\Pi_x\\q^2J_{2s}(\kappa\sin\theta_0\cos\phi_0)\Lambda_{2t+2}(\kappa\sin\theta_0\sin\phi_0)\Pi_y\end{bmatrix}\end{aligned}\tag{5c}$$

$$\begin{aligned}&\begin{bmatrix}K_A(2m{+}2,2n{+}1,2s{+}2,2t{+}1) & pG(2m,2n{+}1,2s{+}2,2t{+}1)\\qG(2m{+}2,2n{+}1,2s,2t{+}1) & K_B(2m,2n{+}1,2s,2t{+}1)\end{bmatrix}\\&\times\begin{bmatrix}D_{mn}^{(x)}\\A_{mn}^{(y)}\end{bmatrix}=\begin{bmatrix}j\Lambda_{2s+2}(\kappa\sin\theta_0\cos\phi_0)J_{2t+1}(\kappa\sin\theta_0\sin\phi_0)\Pi_x\\-jq^2J_{2s}(\kappa\sin\theta_0\cos\phi_0)\Lambda_{2t+2}(\kappa\sin\theta_0\sin\phi_0)\Pi_y\end{bmatrix}\end{aligned}\tag{5d}$$

where Π_x and Π_y are the amplitude of the incident wave. In the above equations the matrix elements are defined by

$$\begin{aligned}K_A(m,n,\mu,\nu) &= \int_0^\infty\int_0^\infty\frac{\kappa^2-\alpha^2}{\sqrt{\alpha^2+p^2\beta^2-\kappa^2}}\frac{J_m(\alpha)J_\mu(\alpha)}{\alpha^2}\\&\quad\times J_n(\beta)J_\nu(\beta)d\alpha d\beta\end{aligned}\tag{6a}$$

$$\begin{aligned}K_B(m,n,\mu,\nu) &= \int_0^\infty\int_0^\infty\frac{q^2\kappa^2-\beta^2}{\sqrt{\alpha^2+p^2\beta^2-\kappa^2}}J_m(\alpha)J_\mu(\alpha)\\&\quad\times\frac{J_n(\beta)J_\nu(\beta)}{\beta^2}d\alpha d\beta\end{aligned}\tag{6b}$$

$$\begin{aligned}G(m,n,\mu,\nu) &= \int_0^\infty\int_0^\infty\left(\alpha^2+p^2\beta^2-\kappa^2\right)^{-\frac{1}{2}}J_m(\alpha)J_\mu(\alpha)\\&\quad\times J_n(\beta)J_\nu(\beta)d\alpha d\beta\end{aligned}\tag{6c}$$

$$\Lambda_\nu(x) = \frac{J_\nu(x)}{x}. \tag{6d}$$

Thus the problem is reduced to matrix equations, and they can be solved in a standard manner. Once the expansion coefficients are obtained, the far field, current density, and other physical quantities can be obtained. The far field expression is derived by applying the stationary phase method of integration. It is found that the expression of the components of the current density is represented in terms of the Chebyshev polynomials of the first and second kind (see [4], [5]). And it is found that J_x is proportional to $(1-\xi^2)^{\frac{1}{2}}(1-\eta^2)^{-\frac{1}{2}}$ and, J_y is proportional to $(1-\xi^2)^{-\frac{1}{2}}(1-\eta^2)^{\frac{1}{2}}$. These are consistent with the required edge conditions for the field components. It is considered that the singularity at the vertex is higher than the straight

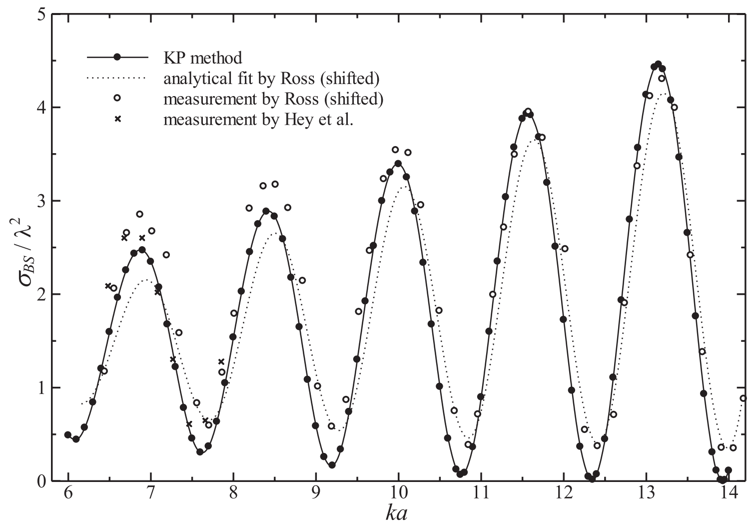

Fig. 2 Radar cross section as function of plate's one side size ka ($kb = 2\pi$, $\phi_0 = 0°$). Ross's data are shifted to right direction to consider plate thickness.

edge, but it is not actually correct and the singularity becomes indefinite. It depends on how to approach to the vertex. As a numerical result we present here RCS of the rectangular plate of width 2λ ($kb = 2\pi$) as a function of the normalized half-length ka at glancing incidence since this problem has only approximate and experiment results. It is shown in Fig. 2. The experiment was done by Hey and Senior [11] and Ross [12]. In Fig. 2, the Ross's data (dotted line and open circles) are shifted for the thickness.

3. Diffraction of Electromagnetic Wave by a Circular Disk [8]

This problem was studied by Meixner and Andrejewski (Spheroidal function), and Nomura and Katsura (Weber Schafheitlin's integral) [3] as reproduced in the handbook by Bowman et al. [13] Scattered field can be expressed in terms of two scalar wave functions, but the authors cited above use three scalar wave functions since two functions lead to electromagnetic field with higher singularities. In this paper we show that the field can be derived from two scalar wave functions and these functions satisfy the required boundary conditions and edge conditions [8]. The incident wave is same as that given in (1) and the magnetic and electric vector potentials of the scattered field are given by

$$\begin{pmatrix} A_z^s(\rho,\phi,z) \\ F_z^s(\rho,\phi,z)\end{pmatrix} = \begin{pmatrix} \pm\mu_0 a\kappa Y_0 \\ \epsilon_0 a\end{pmatrix} \sum_{m=0}^{\infty} \int_0^{\infty} \left[\begin{pmatrix}\widetilde{f}_{cm}(\xi)\\ \widetilde{g}_{cm}(\xi)\end{pmatrix}\cos m\phi + \begin{pmatrix}\widetilde{f}_{sm}(\xi)\\ \widetilde{g}_{sm}(\xi)\end{pmatrix}\sin m\phi \right] J_m(\rho_a\xi) \times \exp[\mp\sqrt{\xi^2-\kappa^2}\, z_a]\xi^{-1}d\xi \qquad (7)$$

where the upper and lower signs refer to the region $z > 0$ and $z < 0$, respectively, and $\rho_a = \rho/a$ and $z_a = z/a$ are the normalized variables with respect to the radius a of the disk ($\kappa = ka$ is the normalized radius). In the above equations $\widetilde{f}(\xi)$ and $\widetilde{g}(\xi)$ are the unknown spectrum functions and they are to be determined so that they satisfy all the required boundary conditions. First we consider the surface fields at the plane $z = 0$ to derive the dual integral equations associated with them. By using the relation between the vector potentials and the electromagnetic field, the tangential components of the electric field and the current density on the disk become

$$\begin{pmatrix} E_\rho^d(\rho,\phi,0) \\ E_\phi^d(\rho,\phi,0)\end{pmatrix} = \sum_{m=0}^{\infty}\left[\begin{pmatrix}E_{\rho c,m}(\rho_a)\\ E_{\phi c,m}(\rho_a)\end{pmatrix}\cos m\phi + \begin{pmatrix}E_{\rho s,m}(\rho_a)\\ E_{\phi s,m}(\rho_a)\end{pmatrix}\sin m\phi\right] \qquad (8a)$$

$$\begin{pmatrix} K_\rho(\rho,\phi) \\ K_\phi(\rho,\phi)\end{pmatrix} = \begin{pmatrix} -2H_\phi^d(\rho,\phi,0) \\ 2H_\rho^d(\rho,\phi,0)\end{pmatrix} = \sum_{m=0}^{\infty}\left[\begin{pmatrix}K_{\rho c,m}(\rho_a)\\ K_{\phi c,m}(\rho_a)\end{pmatrix}\cos m\phi + \begin{pmatrix}K_{\rho s,m}(\rho_a)\\ K_{\phi s,m}(\rho_a)\end{pmatrix}\sin m\phi\right] \qquad (8b)$$

where the Fourier components are written in the form of the vector Hankel transform given below:

$$\begin{bmatrix}E_{\rho c,m}(\rho_a)\\ E_{\phi s,m}(\rho_a)\end{bmatrix} = \int_0^{\infty}\left[H^-(\xi\rho_a)\right]\begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\,\widetilde{f}_{cm}(\xi)\xi^{-1} \\ \widetilde{g}_{sm}(\xi)\xi^{-1}\end{bmatrix}\xi d\xi \qquad (9a)$$

$$\begin{bmatrix}E_{\rho s,m}(\rho_a)\\ E_{\phi c,m}(\rho_a)\end{bmatrix} = \int_0^{\infty}\left[H^+(\xi\rho_a)\right]\begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\,\widetilde{f}_{sm}(\xi)\xi^{-1} \\ \widetilde{g}_{cm}(\xi)\xi^{-1}\end{bmatrix}\xi d\xi \qquad (9b)$$

$$\begin{bmatrix}K_{\rho c,m}(\rho_a)\\ K_{\phi s,m}(\rho_a)\end{bmatrix} = 2Y_0\int_0^{\infty}\left[H^-(\xi\rho_a)\right]\begin{bmatrix} \kappa\widetilde{f}_{cm}(\xi)\xi^{-1} \\ j\sqrt{\xi^2-\kappa^2}\,\widetilde{g}_{sm}(\xi)(\kappa\xi)^{-1}\end{bmatrix}\xi d\xi = \int_0^{\infty}\left[H^-(\xi\rho_a)\right]\begin{bmatrix}\widetilde{K}_{\rho c,m}(\xi)\\ \widetilde{K}_{\phi s,m}(\xi)\end{bmatrix}\xi d\xi \qquad (9c)$$

$$\begin{bmatrix}K_{\rho s,m}(\rho_a)\\ K_{\phi c,m}(\rho_a)\end{bmatrix} = 2Y_0\int_0^{\infty}\left[H^+(\xi\rho_a)\right]\begin{bmatrix} \kappa\widetilde{f}_{sm}(\xi)\xi^{-1} \\ j\sqrt{\xi^2-\kappa^2}\,\widetilde{g}_{cm}(\xi)(\kappa\xi)^{-1}\end{bmatrix}\xi d\xi = \int_0^{\infty}\left[H^+(\xi\rho_a)\right]\begin{bmatrix}\widetilde{K}_{\rho s,m}(\xi)\\ \widetilde{K}_{\phi c,m}(\xi)\end{bmatrix}\xi d\xi. \qquad (9d)$$

In the above equations the kernel matrices $\left[H^{+}(\xi\rho_a)\right]$ and $\left[H^{-}(\xi\rho_a)\right]$ are given by

$$\left[H^{\pm}(\xi\rho_a)\right] = \begin{bmatrix} J'_m(\xi\rho_a) & \pm\dfrac{m}{\xi\rho_a}J_m(\xi\rho_a) \\ \pm\dfrac{m}{\xi\rho_a}J_m(\xi\rho_a) & J'_m(\xi\rho_a) \end{bmatrix}. \quad (10)$$

The spectrum functions given in the right hand sides of (9) are obtained by applying the vector Hankel transform pair defined by [9]. The required boundary conditions state that the current densities on the plane $z = 0$ are zero for $\rho_a \geq 1$ and the tangential components of the total electric field vanish on the disk. These are written as

$$\int_0^\infty \left[H^{-}(\xi\rho_a)\right]\begin{bmatrix}\widetilde{K}_{\rho c,m}(\xi)\\ \widetilde{K}_{\phi s,m}(\xi)\end{bmatrix}\xi d\xi = 0,$$

$$\int_0^\infty \left[H^{+}(\xi\rho_a)\right]\begin{bmatrix}\widetilde{K}_{\rho s,m}(\xi)\\ \widetilde{K}_{\phi c,m}(\xi)\end{bmatrix}\xi d\xi = 0, \qquad \rho_a \geq 1 \quad (11)$$

$$\begin{bmatrix}E^t_{\rho c,m}(\rho_a)\\ E^t_{\phi s,m}(\rho_a)\end{bmatrix} = \int_0^\infty \left[H^{-}(\xi\rho_a)\right]\begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\widetilde{f}_{cm}(\xi)\xi^{-1}\\ \widetilde{g}_{sm}(\xi)\xi^{-1}\end{bmatrix}\xi d\xi + \begin{bmatrix}E^i_{\rho c,m}(\rho_a)\\ E^i_{\phi s,m}(\rho_a)\end{bmatrix} = 0, \qquad \rho_a \leq 1 \quad (12a)$$

$$\begin{bmatrix}E^t_{\rho s,m}(\rho_a)\\ E^t_{\phi c,m}(\rho_a)\end{bmatrix} = \int_0^\infty \left[H^{+}(\xi\rho_a)\right]\begin{bmatrix} j\sqrt{\xi^2-\kappa^2}\widetilde{f}_{sm}(\xi)\xi^{-1}\\ \widetilde{g}_{cm}(\xi)\xi^{-1}\end{bmatrix}\xi d\xi + \begin{bmatrix}E^i_{\rho s,m}(\rho_a)\\ E^i_{\phi c,m}(\rho_a)\end{bmatrix} = 0, \qquad \rho_a \leq 1 \quad (12b)$$

where the superscript "t" refers to the total field. In the above equations $E^i_{\rho c,m}$ and $E^i_{\rho s,m}$ denote the $\cos m\phi$ and $\sin m\phi$ parts of the incident wave E^i_ρ, respectively, and same is true for $E^i_{\phi c,m}$ and $E^i_{\phi s,m}$. Equations (11) and (12) are the dual integral equations to determine the spectrum functions $\widetilde{f}_m(\xi)'s$ and $\widetilde{g}_m(\xi)'s$. To solve (11) we expand $\mathbf{K}(\rho_a)$ in terms of the functions which satisfy Maxwell's equations and the edge conditions. These functions can be found by taking into account the discontinuity property of the Weber-Schafheitlin integrals. Once the expressions for $\mathbf{K}(\rho_a)$ are established, the corresponding spectrum functions can be derived by applying the vector Hankel transform. It is noted that (K_ρ, K_ϕ) satisfy the vector Helmholtz equation $\nabla^2\mathbf{K} + k^2\mathbf{K} = 0$ in circular cylindrical coordinates on the plane $z = 0$ since $\mathbf{K}$ and $\mathbf{H}$ are related by $\mathbf{K} = \mathbf{n}\times\mathbf{H}$ on the plane. Furthermore (K_ρ, K_ϕ) have the properties $K_\rho \sim (1-\rho_a^2)^{\frac{1}{2}}$ and $K_\phi \sim (1-\rho_a^2)^{-\frac{1}{2}}$ near the edge of the disk. By taking into these facts, we set $K_{\rho c,m}(\rho_a) \sim K_{\phi s,m}(\rho_a)$ defined in (9c) and (9d)

$$K_{\rho c,m}(\rho_a) = \sum_{n=0}^\infty \left[A_{mn}F^-_{mn}(\rho_a) - B_{mn}G^+_{mn}(\rho_a)\right],$$

$$K_{\rho s,m}(\rho_a) = \sum_{n=0}^\infty \left[C_{mn}F^-_{mn}(\rho_a) + D_{mn}G^+_{mn}(\rho_a)\right],$$

$$K_{\phi s,m}(\rho_a) = \sum_{n=0}^\infty \left[-A_{mn}F^+_{mn}(\rho_a) + B_{mn}G^-_{mn}(\rho_a)\right],$$

$$K_{\phi c,m}(\rho_a) = \sum_{n=0}^\infty \left[C_{mn}F^+_{mn}(\rho_a) + D_{mn}G^-_{mn}(\rho_a)\right] \quad (13)$$

where

$$F^{\pm}_{mn}(\rho_a) = \int_0^\infty \left[J_{|m-1|}(\eta\rho_a)J_{|m-1|+2n+\frac{1}{2}}(\eta) \pm J_{m+1}(\eta\rho_a)J_{m+2n+\frac{3}{2}}(\eta)\right]\eta^{\frac{1}{2}}d\eta \quad (14a)$$

$$F^{+}_{0n}(\rho_a) = 2\int_0^\infty J_1(\eta\rho_a)J_{2n+\frac{3}{2}}(\eta)\eta^{\frac{1}{2}}d\eta \quad (14b)$$

$$G^{\pm}_{mn}(\rho_a) = \int_0^\infty \left[J_{|m-1|}(\eta\rho_a)J_{|m-1|+2n+\frac{3}{2}}(\eta) \pm J_{m+1}(\eta\rho_a)J_{m+2n+\frac{5}{2}}(\eta)\right]\eta^{-\frac{1}{2}}d\eta \quad (14c)$$

$$G^{+}_{0n}(\rho_a) = 2\int_0^\infty J_1(\eta\rho_a)J_{2n+\frac{5}{2}}(\eta)\eta^{-\frac{1}{2}}d\eta. \quad (14d)$$

These integrals are of the form of the discontinuous Weber-Schafheitlin's integral and they can be performed analytically and expressed in terms of the hypergeometric functions. It may readily be verified that $F^{\pm}_{mn}(\rho_a) = G^{\pm}_{mn}(\rho_a) = 0$ for $\rho_a \geq 1$, and $F^+_{mn}(\rho_a) \sim (1-\rho_a^2)^{-\frac{1}{2}}$, $F^-_{mn}(\rho_a) \sim (1-\rho_a^2)^{\frac{1}{2}}$, $G^+_{mn}(\rho_a) \sim (1-\rho_a^2)^{\frac{1}{2}}$, and $G^-_{mn}(\rho_a) \sim (1-\rho_a^2)^{\frac{3}{2}}$ near the edge $\rho_a \simeq 1$. Thus the expressions (14) satisfy one part of the dual integral equations (11) with the unknown expansion coefficients $A_{mn} \sim D_{mn}$. To derive the spectrum functions $\widetilde{f}(\xi)$ and $\widetilde{g}(\xi)$ of the vector potentials we first determine the spectrum functions of the current densities, since they are related each other. These are obtained by applying the vector Hankel transform. We see from (9c) and (9d) the spectral functions $\widetilde{f}_{cm}(\xi) \sim \widetilde{g}_{sm}(\xi)$ are represented in terms of $\widetilde{K}_{\rho c}(\xi) \sim \widetilde{K}_{\phi sc}(\xi)$. Thus the solutions of (11) are established. The next problem is to solve (12) and it is done by using the projection. As a set of functions we choose Jacobi's polynomials having similar form as (14). The result is given in the form of matrix equation. It is given below:

$$\sum_{n=0}^\infty \left[A_{mn}\begin{pmatrix}Z^{(1,1)}_{mp,n}\\ Z^{(2,1)}_{mp,n}\end{pmatrix} - B_{mn}\begin{pmatrix}Z^{(1,2)}_{mp,n}\\ Z^{(2,2)}_{mp,n}\end{pmatrix}\right] = \begin{pmatrix}H^{(1)}_{m,p}\\ H^{(2)}_{m,p}\end{pmatrix},$$
$$m = 1,2,3,\cdots, \qquad p = 0,1,2,3,\cdots \quad (15a)$$

$$\sum_{n=0}^\infty B_{0n}Z^{(1,2)}_{0p,n} = H^{(1)}_{0,p}, \qquad p = 0,1,2,3,\cdots. \quad (15b)$$

Equations for C_{mn} and D_{mn} are same except the right hand sides.

$$Z^{(1,1)}_{mp,n} = \frac{2(m+2n+\frac{1}{2})}{\kappa}\left[\alpha^m_p K^m_{pn}\left(\frac{1}{2},\frac{1}{2}\right) - (\alpha^m_p+3) \times K^m_{pn}\left(\frac{5}{2},\frac{1}{2}\right)\right] - \kappa m(2\alpha^m_p+3)\left[G^m_{2,pn}\left(\frac{3}{2},\frac{1}{2}\right) - G^m_{2,pn}\left(\frac{3}{2},\frac{3}{2}\right)\right] \quad (16a)$$

$$Z^{(1,2)}_{mp,n} = \frac{1}{\kappa}\left\{\alpha^m_p\left[K^m_{pn}\left(\frac{1}{2},\frac{1}{2}\right) - K^m_{pn}\left(\frac{1}{2},\frac{5}{2}\right)\right] - (\alpha^m_p+3)\right.$$

$$\times\left[K^m_{pn}\left(\frac{5}{2},\frac{1}{2}\right)-(\alpha^m_p+3)K^m_{pn}\left(\frac{5}{2},\frac{5}{2}\right)\right]\Bigg\}$$
$$-\kappa m\left[G^m_{2,pn}\left(\frac{1}{2},\frac{1}{2}\right)+G_{2,pn}\left(\frac{5}{2},\frac{1}{2}\right)+G^m_{2,pn}\left(\frac{1}{2},\frac{5}{2}\right)+G^m_{2,pn}\left(\frac{5}{2},\frac{5}{2}\right)\right] \quad (16b)$$

$$Z^{(2,1)}_{mp,n}=\frac{4m(m+2n+\frac{1}{2})(m+2p+\frac{1}{2})}{\kappa}K^m_{pn}\left(\frac{1}{2},\frac{1}{2}\right)-\kappa\left\{\alpha^m_p\left[G^m_{pn}\left(-\frac{1}{2},-\frac{1}{2}\right)-G^m_{pn}\left(-\frac{1}{2},\frac{3}{2}\right)\right]-(\alpha^m_p+1)\left[G^m_{pn}\left(\frac{3}{2},-\frac{1}{2}\right)-G^m_{pn}\left(\frac{3}{2},\frac{3}{2}\right)\right]\right\} \quad (16c)$$

$$Z^{(2,2)}_{mp,n}=\frac{2m(m+2p+\frac{1}{2})}{\kappa}\left[K^m_{pn}\left(\frac{1}{2},\frac{1}{2}\right)-K^m_{pn}\left(\frac{1}{2},\frac{5}{2}\right)\right]-2\kappa\left(m+2n+\frac{3}{2}\right)\left[\alpha^m_pG^m_{2,pn}\left(-\frac{1}{2},\frac{3}{2}\right)-(\alpha^m_p+1)G^m_{2,pn}\left(\frac{3}{2},\frac{3}{2}\right)\right] \quad (16d)$$

$$Z^{(1,2)}_{0p,n}=\frac{1}{\kappa}\left[-pK^0_{pn}\left(\frac{1}{2},\frac{5}{2}\right)+(p+1.5)K^0_{pn}\left(\frac{5}{2},\frac{5}{2}\right)\right] \quad (16e)$$

$$H^{(1)}_{m,p}=4Y_0E_2\cos\theta_0 j^m\Big[\alpha^m_pJ_{m+2p+\frac{1}{2}}(\kappa\sin\theta_0)-(\alpha^p_m+3)J_{m+2p+\frac{5}{2}}(\kappa\sin\theta_0)\Big](\kappa\sin\theta_0)^{-\frac{3}{2}} \quad (17a)$$
$$H^{(2)}_{m,p}=4Y_0E_2\cos\theta_0 j^m m\Big[J_{m+2p-\frac{1}{2}}(\kappa\sin\theta_0)+J_{m+2p+\frac{3}{2}}(\kappa\sin\theta_0)\Big](\kappa\sin\theta_0)^{-\frac{1}{2}} \quad (17b)$$

where

$$K^m_{pn}(\alpha,\beta)=\int_0^\infty\frac{\sqrt{\xi^2-\kappa^2}}{\xi^2}J_{m+2p+\alpha}(\xi)J_{m+2n+\beta}(\xi)d\xi,$$
$$G^m_{pn}(\alpha,\beta)=\int_0^\infty\frac{1}{\sqrt{\xi^2-\kappa^2}}J_{m+2p+\alpha}(\xi)J_{m+2n+\beta}(\xi)d\xi,$$
$$G^m_{2,pn}(\alpha,\beta)=\int_0^\infty\frac{1}{\xi^2\sqrt{\xi^2-\kappa^2}}J_{m+2p+\alpha}(\xi)J_{m+2n+\beta}(\xi)d\xi. \quad (18)$$

The infinite integrals (18) may be transformed into series expansion (see [8]). We computed the transmission coefficient "t" for the normal and oblique incidences for the range $0<\kappa(=ka)\leq 15$. The results are normalized by the area of the disk πa^2 and shown in Fig. 3. To our knowledge the numerical results for oblique incidence are not found. For normal incidence, results by Andrejewski [6] and results by Seshadri and Wu [14] are also shown for comparison. When the value of κ is very small, t is known to be proportional to

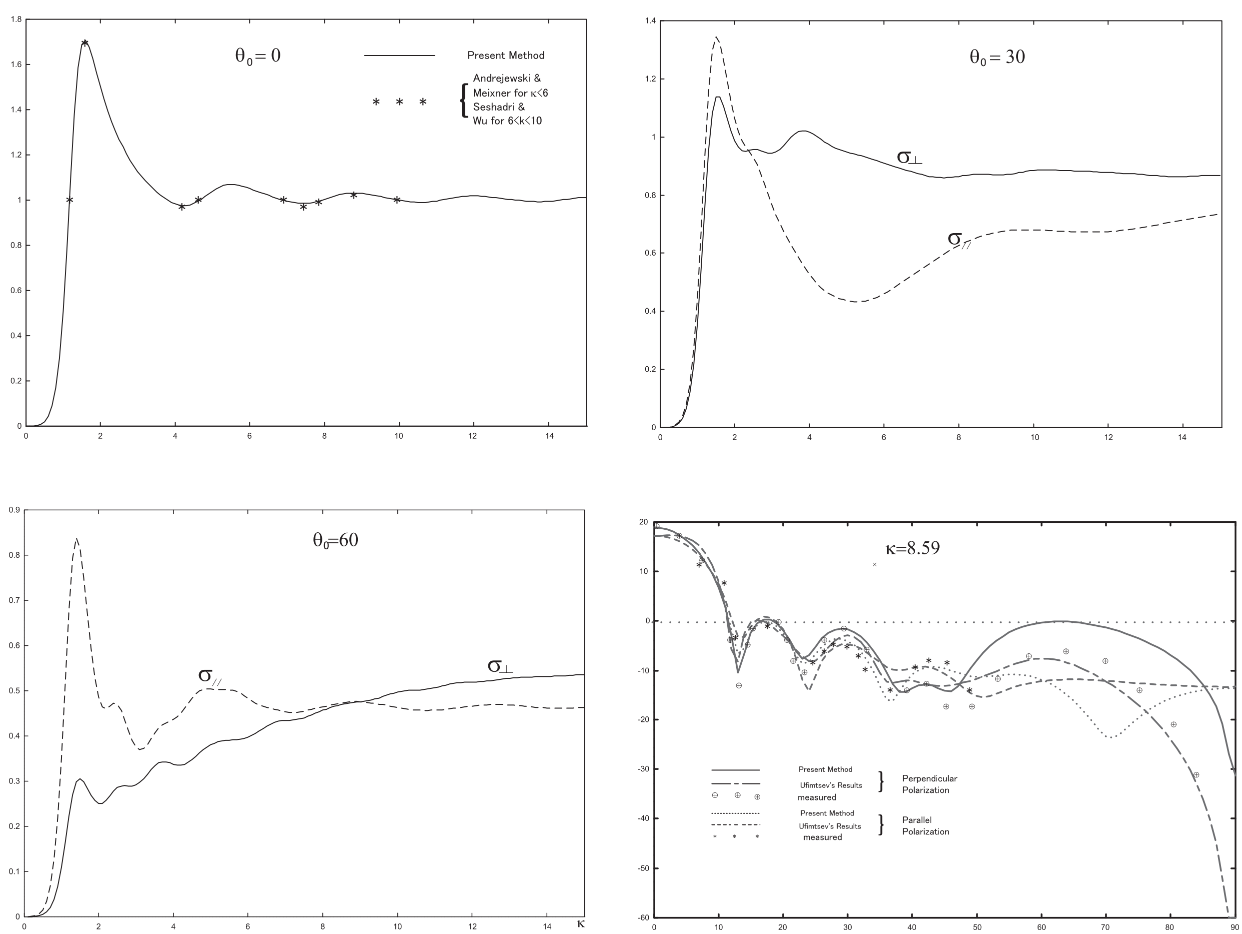

Fig. 3 Transmission coefficient of the circular aperture and RCS of the corresponding disk.

Table 1 Numerical comparison of transmission coefficients.

κ	Andre-jewski	Seshadri and Wu	Jones	Present Method
1	...	1.00257	1.01487	0.50462
2	...	1.38364	1.37742	1.50369
3	1.127	1.13291	1.13400	1.12731
4	0.992	0.98092	0.98158	0.98322
5	1.039	1.03367	1.03306	1.04012
6	1.047	1.05359	1.05368	1.05136
7	0.995	0.99365	0.99386	0.99469
8	0.999	1.00239	1.00222	1.00333
9	1.030	1.03044	1.03043	1.02953
10	1.001	0.99915	0.99925	0.99970
11	...	0.99572	0.99566	0.99581
12	...	1.01930	1.01928	1.01893
13	...	1.00197	1.00203	1.00227
14	...	0.99400	0.99438	0.99434
15	...	1.01254	1.01251	1.01241

$(ka)^4$, or more explicitly $t \simeq 64(ka)^4/27\pi$. When κ is very large, asymptotic expressions were derived by Seshadri and Wu, and Jones [15]. The last figure in Fig. 3 represents the RCS of the disk and compared with approximate and experimental resuls. The precise results of the transmission coefficients are shown in Table 1. It is found that our results cover wide range of ka. The results of the current densities are also obtained, but they are not shown here.

4. Conclusion

We have formulated the plane wave field scattered by a perfectly conducting rectangular plate and circular disk and their complementary hole in a perfectly conducting infinite plane. We derived dual integral equations for the induced current and the tangential components of the electric field on the disk. The equations for the current densities are solved by applying the discontinuous properties of the Weber-Schafheitlin's integrals and the vector Hankel transform. It is readily found that the solution satisfies Maxwell's equations and edge conditions. Therefore it may be considered as the eigen function expansion. The equations for the electric field are solved by applying the projection. We use the functional space of the Jacobi's polynomials. Thus the problem reduces to the matrix equations and their elements are given by infinite integrals of double variables for the rectangular plate and a single variable for the disk. These integrals are transformed into infinite series in terms of the normalized radius for single variable. Numerical computation is performed for the far field patterns, distribution of the current densities, and transmission coefficients for the circular hole in a perfectly conducting screen for $ka = 0.1$ to $ka = 15$. The results for the transmission coefficient for normal incident case are compared with some published results and we have a good agreement.

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