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Complex Orthogonal Variable Spreading Factor Codes Based on Polyphase Sequences*

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SUMMARY The direct sequence code division multiple access (DS-CDMA) technique is widely used in various communication systems. When adopting orthogonal variable spreading factor (OVSF) codes, DS-CDMA is particularly suitable for supporting multi-user/multi-rate data transmission services. A useful property of OVSF codes is that no two code sequences assigned to different users will ever interfere with each other, even if their spreading factors are different. Conventional OVSF codes are constructed based on binary orthogonal codes, called Walsh codes, and OVSF code sequences are binary sequences. In this paper, we propose new OVSF codes that are constructed based on polyphase orthogonal codes and consist of complex sequences in which each symbol is represented as a complex number. Construction of the proposed codes is based on a tree structure that is similar to conventional OVSF codes. Since the proposed codes are generalized versions of conventional OVSF codes, any conventional OVSF code can be presented as a special case of the proposed codes. Herein, we show the method used to construct the proposed OVSF codes, after which the orthogonality of the codes, including conventional OVSF codes, is investigated. Among the advantages of our proposed OVSF codes is that the spreading factor can be designed more flexibly in each layer than is possible with conventional OVSF codes. Furthermore, combination of the proposed code and a non-binary phase modulation is well suited to DS-CDMA systems where the level fluctuation of signal envelope is required to be suppressed.

key words: *DS-CDMA, OVSF codes, complex sequences, polyphase orthogonal codes*

1. Introduction

The direct sequence code division multiple access (DS-CDMA) technique is widely used in various communication systems [1]. One of the useful properties of this technique is that, when an orthogonal code is used as a signature code in DS-CDMA, multi-user interference is canceled because any two code sequences are orthogonal and will never interfere with each other. There are two kinds of orthogonal codes: binary and non-binary. Walsh-Hadamard codes, referred to as Walsh codes in this paper, are binary orthogonal codes whose code sequences are binary sequences. On the other hand, non-binary orthogonal codes include orthogonal codes over the complex number field and orthogonal codes over non-binary finite fields [2], [3]. Polyphase orthogonal codes are orthogonal codes over the complex number field whose code sequences are complex number sequences [4]–[6].

Recently, orthogonal variable spreading factor (OVSF) codes have been attracting attention for use in synchronous DS-CDMA systems that support multi-rate data transmission services [7]–[10]. For example, in some practical wireless communication systems, OVSF codes are used to realize multi-rate communications [11], [12]. These OVSF codes are represented by code trees in which the sequences at the mth layer are codewords of the Walsh code with a spreading factor of 2^m. When a code sequence is assigned to one user, its descendant and ancestor code sequences in the code tree cannot be assigned to another user because two sequences will not be orthogonal if one of them is an ancestor of the other. A useful property of OVSF codes is that any two code sequences assigned to different users will not interfere with each other, even if their spreading factors are different. Since the conventional OVSF codes are based on Walsh codes, their code sequences are binary sequences in which each symbol is presented as an element in $\{+1, -1\}$.

In this paper, we propose complex OVSF codes whose code sequences consist of complex number field elements. Since the proposed codes are generalized versions of conventional OVSF codes, any conventional OVSF code can be presented as a special case of the proposed codes. In the sections below, we show the construction method of the proposed complex OVSF codes, in which several polyphase orthogonal codes whose degrees are arbitrary positive integers are used in order to expand code trees, and then investigate the orthogonality of the proposed codes. An advantage of our proposed complex OVSF codes is that the spreading factor can be designed more flexibly in each layer than is possible with conventional OVSF codes. Furthermore, combination of the proposed code and a non-binary phase modulation is well suited to DS-CDMA systems where the level fluctuation of signal envelope is required to be suppressed.

The remainder of the paper is organized as follows. Section 2 reviews the Walsh codes, the polyphase orthogonal codes, and the conventional OVSF codes. Then, the construction of proposed complex OVSF codes is introduced in Sect. 3. In Sect. 4, the orthogonality of the proposed codes is discussed. Section 5 describes the features of the proposed codes in some application systems. Finally, Sect. 6 offers some concluding remarks.

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2. Preliminaries

2.1 Walsh Codes

A Hadamard matrix H_{2^m} contains all of the 2^m Walsh code sequences with a length of 2^m as its rows [1], [6]. H_{2^m} is generated by the following recursive algorithm:

$$H_1 = \begin{bmatrix} 1 \end{bmatrix}, \quad (1)$$

$$H_{2^{m+1}} = \begin{bmatrix} H_{2^m} & H_{2^m} \\ H_{2^m} & -H_{2^m} \end{bmatrix}, \quad (2)$$

where $m \geq 0$. Equation (2) can be also written with the Kronecker product as follows:

$$H_{2^{m+1}} = H_2 \otimes H_{2^m}, \quad (3)$$

where $A \otimes B$ is the Kronecker product of matrices A and B.

For example, H_2, H_4 and H_8 are calculated as follows:

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad (4)$$

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}, \quad (5)$$

$$H_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}. \quad (6)$$

Walsh codes have perfect orthogonality at zero time delay. Since any two code sequences with the same length are orthogonal, multi-user interference can be canceled when Walsh codes are used in synchronous DS-CDMA systems.

2.2 Polyphase Orthogonal Codes

A polyphase orthogonal code of degree q is given by the following matrix:

$$G_q = \begin{bmatrix} \omega^0 & \omega^0 & \omega^0 & \cdots & \omega^0 \\ \omega^0 & \omega^1 & \omega^2 & \cdots & \omega^{q-1} \\ \omega^0 & \omega^2 & \omega^4 & \cdots & \omega^{2(q-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega^0 & \omega^{q-1} & \omega^{2(q-1)} & \cdots & \omega^{(q-1)^2} \end{bmatrix}, \quad (7)$$

where q is a positive integer. A complex number ω is a primitive qth root of unity, whose order is q, and represented by

$$\omega = e^{2\pi j/q}, \quad (8)$$

where $j = \sqrt{-1}$. By using ω, the q different qth roots of unity are presented as ω^0, ω^1, ω^2, $\cdots$, ω^{q-1}. Polyphase orthogonal code sequences with a length of q are presented as the rows of G_q, and it is known that the following equality holds for any positive integer k:

$$\sum_{i=0}^{q-1} \omega^{ik} = \begin{cases} q, & \text{if } k \equiv 0 \pmod{q}, \\ 0, & \text{if } k \not\equiv 0 \pmod{q}. \end{cases} \quad (9)$$

For example, G_1, G_2, G_3, and G_4 are presented as follows:

$$G_1 = \begin{bmatrix} e^{0\pi j/1} \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}, \quad (10)$$

$$G_2 = \begin{bmatrix} e^{0\pi j/2} & e^{0\pi j/2} \\ e^{0\pi j/2} & e^{2\pi j/2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad (11)$$

$$G_3 = \begin{bmatrix} e^{0\pi j/3} & e^{0\pi j/3} & e^{0\pi j/3} \\ e^{0\pi j/3} & e^{2\pi j/3} & e^{4\pi j/3} \\ e^{0\pi j/3} & e^{4\pi j/3} & e^{8\pi j/3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & (-1+\sqrt{3}j)/2 & (-1-\sqrt{3}j)/2 \\ 1 & (-1-\sqrt{3}j)/2 & (-1+\sqrt{3}j)/2 \end{bmatrix}, \quad (12)$$

$$G_4 = \begin{bmatrix} e^{0\pi j/4} & e^{0\pi j/4} & e^{0\pi j/4} & e^{0\pi j/4} \\ e^{0\pi j/4} & e^{2\pi j/4} & e^{4\pi j/4} & e^{6\pi j/4} \\ e^{0\pi j/4} & e^{4\pi j/4} & e^{8\pi j/4} & e^{12\pi j/4} \\ e^{0\pi j/4} & e^{6\pi j/4} & e^{12\pi j/4} & e^{18\pi j/4} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}. \quad (13)$$

Although G_1 and G_2 are equal to H_1 and H_2, respectively, G_{2^m} is not equal to H_{2^m} for $m \geq 2$. When $q \geq 3$, the polyphase orthogonal codes sequences are complex sequences.

Let $\mathbf{a}_k$ be the kth sequence of a polyphase orthogonal code and be represented by

$$\begin{aligned} \mathbf{a}_k &= (a_{k,0}, a_{k,1}, a_{k,2}, \cdots, a_{k,q-1}) \\ &= (\omega^0, \omega^k, \omega^{2k}, \cdots, \omega^{(q-1)k}) \end{aligned} \quad (14)$$

for $k = 0, 1, \cdots, q-1$. The correlation between two sequences $\mathbf{a}_{k_1}$ and $\mathbf{a}_{k_2}$ at zero time delay is defined as follows:

$$R(\mathbf{a}_{k_1}, \mathbf{a}_{k_2}) = \sum_{i=0}^{q-1} a_{k_1,i} a^*_{k_2,i}, \quad (15)$$

where $a^*_{k,i}$ denotes the complex conjugate of $a_{k,i}$. From Eqs. (9) and (15), the auto-correlation and cross-correlation of a polyphase orthogonal code are given as follows:

$$R(\mathbf{a}_{k_1}, \mathbf{a}_{k_2}) = \begin{cases} q, & \text{if } k_1 = k_2, \\ 0, & \text{if } k_1 \neq k_2, \end{cases} \quad (16)$$

where q is a code length, $k_1 = 0, 1, \cdots, q-1$, and $k_2 = 0, 1, \cdots, q-1$. The above equation shows that any two different polyphase orthogonal code sequences with the same length are mutually orthogonal over the complex number field.

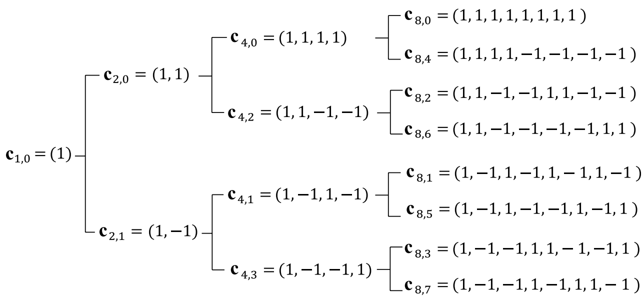

Fig. 1 Code tree for the conventional OVSF code with $L = 3$. This code is also represented as the complex OVSF (2, 2, 2) code.

2.3 Conventional OVSF Codes

OVSF codes have been proposed for synchronous DS-CDMA supporting multi-rate data transmission services [7]. An OVSF code is represented by a code tree in which sequences in the mth layer are codewords for a Walsh code with a spreading factor of 2^m. Each OVSF code sequence corresponds to a node of its code tree. When a code sequence is assigned to one user, its descendant and ancestor code sequences in the code tree cannot be assigned to any other user. Therefore, a particular property of OVSF codes is that no two code sequences assigned to different users will ever interfere with each other, even if the spreading factors of the sequences are different.

Suppose the code tree representing an OVSF code has $L + 1$ layers, where L is a positive integer. As for conventional OVSF codes, the number of code sequences at the mth layer is 2^m for $m = 0, 1, \cdots, L$. The spreading factor of code sequences at the mth layer is also 2^m. Shorter code sequences in the code tree are assigned to users transmitting data at higher rates, while longer code sequences are assigned to users transmitting data at lower rates.

Figure 1 shows a conventional OVSF code tree with $L = 3$. Here, we can see that the spreading factor of code sequences at the mth layer is twice of that of code sequences at the $(m-1)$th layer for $m = 1, 2, \cdots, L$. In Fig. 1, the code sequence $\mathbf{c}_{2^m,k}$ corresponds to the kth row in H_{2^m} ($m = 0, 1, 2, \cdots, L$, $k = 0, 1, 2, \cdots, 2^m - 1$).

3. Construction of Complex OVSF Codes

As mentioned above, this study proposes new OVSF codes constructed over the complex number field. The proposed codes, which are constructed based on polyphase orthogonal codes shown in 2.2, have a property similar to conventional OVSF codes in that no two code sequences assigned to different users will ever interfere with each other, even if their spreading factors are different.

3.1 Code Sequences

First, we describe the code sequences in each layer of the proposed complex OVSF codes. Suppose the code tree representing a proposed code has $L + 1$ layers and r_m is the spreading factor at the mth layer ($m = 0, 1, 2, \cdots, L$). r_m is given by

$$r_m = \prod_{i=0}^{m} q_i, \qquad m = 0, 1, 2, \cdots, L, \tag{17}$$

where $q_0 = 1$ and $q_1, q_2, \cdots, q_L$ are arbitrary integers greater than one. For $m \geq 1$, q_m shows the ratio between the spreading factors of the mth and $(m-1)$th layers. The proposed code with the above parameters is hereinafter referred to as a complex OVSF ($q_1, q_2, \cdots, q_L$) code.

Suppose $M_{q_1,q_2,\cdots,q_m}$ is a matrix containing all of the sequences at the mth layer in the code tree representing a complex OVSF ($q_1, q_2, \cdots, q_L$) code for $m \geq 1$. In that case,

$$M_{q_1} = G_{q_1}, \tag{18}$$

and $M_{q_1,q_2,\cdots,q_m}$ for $2 \leq m \leq L$ is represented with the Kronecker product as follows:

$$M_{q_1,q_2,\cdots,q_m} = G_{q_m} \otimes M_{q_1,q_2,\cdots,q_{m-1}}. \tag{19}$$

For example, the zeroth, first, second and third layer matrices of the complex OVSF (4, 2, 2) code are shown as follows:

$$H_1 = \begin{bmatrix} 1 \end{bmatrix}, \tag{20}$$

$$M_4 = G_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}, \tag{21}$$

$$M_{4,2} = \begin{bmatrix} M_4 & M_4 \\ M_4 & -M_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & j & -1 & -j & -1 & -j & 1 & j \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -j & -1 & j & -1 & j & 1 & -j \end{bmatrix}, \tag{22}$$

$$M_{4,2,2} = \begin{bmatrix} M_{4,2} & M_{4,2} \\ M_{4,2} & -M_{4,2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & j & -1 & -j & 1 & j & \cdots & -j \\ 1 & -1 & 1 & -1 & 1 & -1 & \cdots & -1 \\ 1 & -j & -1 & j & 1 & -j & \cdots & j \\ 1 & 1 & 1 & 1 & -1 & -1 & \cdots & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & -1 & 1 & -1 & -1 & 1 & \cdots & -1 \\ 1 & -j & -1 & j & -1 & j & \cdots & j \end{bmatrix}. \tag{23}$$

The following example shows the second layer matrix of the complex OVSF (4, 4) code whose zeroth and first layer matrices are equal to those of the complex OVSF (4, 2, 2) code:

$$M_{4,4} = \begin{bmatrix} M_4 & M_4 & M_4 & M_4 \\ M_4 & jM_4 & -M_4 & -jM_4 \\ M_4 & -M_4 & M_4 & -M_4 \\ M_4 & -jM_4 & -M_4 & jM_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & j & -1 & -j & 1 & j & \cdots & -j \\ 1 & -1 & 1 & -1 & 1 & -1 & \cdots & -1 \\ 1 & -j & -1 & j & 1 & -j & \cdots & j \\ 1 & 1 & 1 & 1 & j & j & \cdots & -j \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & -1 & 1 & -1 & -j & j & \cdots & -j \\ 1 & -j & -1 & j & -j & -1 & \cdots & -1 \end{bmatrix}. \quad (24)$$

Another example of the first and second layer matrices of the complex OVSF (3, 2) code whose zeroth layer matrix is equal to that of the complex OVSF (4, 2, 2) and (4, 4) codes is shown below:

$$M_3 = G_3 = \begin{bmatrix} e^{0\pi j/3} & e^{0\pi j/3} & e^{0\pi j/3} \\ e^{0\pi j/3} & e^{2\pi j/3} & e^{4\pi j/3} \\ e^{0\pi j/3} & e^{4\pi j/3} & e^{8\pi j/3} \end{bmatrix}, \quad (25)$$

$$M_{3,2} = \begin{bmatrix} M_3 & M_3 \\ M_3 & -M_3 \end{bmatrix} = \begin{bmatrix} e^{0\pi j/3} & e^{0\pi j/3} & e^{0\pi j/3} & e^{0\pi j/3} & e^{0\pi j/3} & e^{0\pi j/3} \\ e^{0\pi j/3} & e^{2\pi j/3} & e^{4\pi j/3} & e^{0\pi j/3} & e^{2\pi j/3} & e^{4\pi j/3} \\ e^{0\pi j/3} & e^{4\pi j/3} & e^{8\pi j/3} & e^{0\pi j/3} & e^{4\pi j/3} & e^{8\pi j/3} \\ e^{0\pi j/3} & e^{0\pi j/3} & e^{0\pi j/3} & e^{3\pi j/3} & e^{3\pi j/3} & e^{3\pi j/3} \\ e^{0\pi j/3} & e^{2\pi j/3} & e^{4\pi j/3} & e^{3\pi j/3} & e^{5\pi j/3} & e^{7\pi j/3} \\ e^{0\pi j/3} & e^{4\pi j/3} & e^{8\pi j/3} & e^{3\pi j/3} & e^{7\pi j/3} & e^{11\pi j/3} \end{bmatrix}, \quad (26)$$

All of the row sequences in the matrix $M_{q_1,q_2,\cdots,q_m}$ of any complex OVSF $(q_1, q_2, \cdots, q_L)$ code are mutually orthogonal over the complex number field for $m = 1, 2, \cdots, L$. This fact can be proven by mathematical induction as follows:

1. For $m = 1$ all of the row sequences in the matrix M_{q_1} are mutually orthogonal, since $M_{q_1} = G_{q_1}$, and all of the rows of G_{q_1} are mutually orthogonal as shown in Eq. (16).

2. Assuming that all of the row sequences in the matrix $M_{q_1,q_2,\cdots,q_{m-1}}$ are mutually orthogonal, all of the row sequences in the $M_{q_1,q_2,\cdots,q_m}$ are shown to be mutually orthogonal as follows:
   $M_{q_1,q_2,\cdots,q_m}$ is obtained by

$$M_{q_1,q_2,\cdots,q_m} = G_{q_m} \otimes M' = \begin{bmatrix} \omega_m^0 M' & \omega_m^0 M' & \cdots & \omega_m^0 M' \\ \omega_m^0 M' & \omega_m^1 M' & \cdots & \omega_m^{q_m-1} M' \\ \omega_m^0 M' & \omega_m^2 M' & \cdots & \omega_m^{2(q_m-1)} M' \\ \vdots & \vdots & \ddots & \vdots \\ \omega_m^0 M' & \omega_m^{q_m-1} M' & \cdots & \omega_m^{(q_m-1)^2} M' \end{bmatrix}, \quad (27)$$

   where ω_m is a primitive q_mth root of unity and is represented by

$$\omega_m = e^{2\pi j/q_m} \quad (28)$$

   and

$$M' = M_{q_1,q_2,\cdots,q_{m-1}}. \quad (29)$$

   In Eq. (27), each M' has r_{m-1} rows that are mutually orthogonal and $M_{q_1,q_2,\cdots,q_m}$ has $r_m = q_m r_{m-1}$ rows. Let $\mathbf{b}_0$, $\mathbf{b}_1$, $\cdots$, and $\mathbf{b}_{r_m-1}$ be the rows in $M_{q_1,q_2,\cdots,q_m}$ numbered beginning at the top. Two different sequences, $\mathbf{b}_{k_1}$ and $\mathbf{b}_{k_2}$ ($k_1 = 0, 1, \cdots, r_m - 1$, $k_2 = 0, 1, \cdots, r_m - 1$), are orthogonal for $k_1 \equiv k_2 \pmod{r_{m-1}}$, since all of the rows in G_{q_m} are orthogonal. Two sequences, $\mathbf{b}_{k_1}$ and $\mathbf{b}_{k_2}$, are orthogonal for $k_1 \not\equiv k_2 \pmod{r_{m-1}}$, since all of the rows in M' are orthogonal. Therefore, any two different rows in $M_{q_1,q_2,\cdots,q_m}$ are mutually orthogonal.

Due to the induction principle, all of the row sequences in the matrix $M_{q_1,q_2,\cdots,q_m}$ of any complex OVSF $(q_1, q_2, \cdots, q_L)$ code are mutually orthogonal for $m = 1, 2, \cdots, L$.

3.2 Code Trees

Secondly, we show the construction method of code trees for the proposed complex OVSF codes.

Figure 2 shows the code tree of the proposed complex OVSF (q_1, q_2, q_3) code. Here, it can be seen that in the code tree of the complex OVSF $(q_1, q_2, \cdots, q_L)$ code,

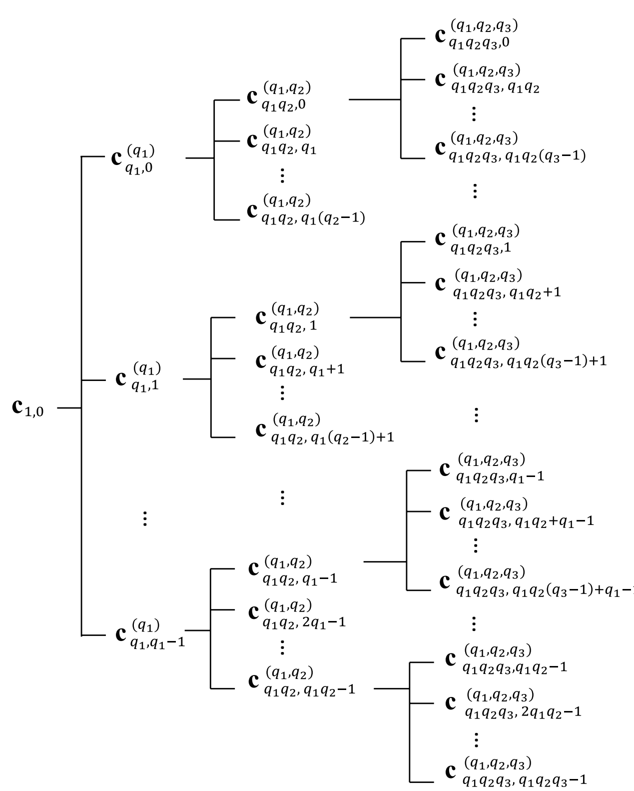

Fig. 2 Code tree for the proposed complex OVSF (q_1, q_2, q_3) code.

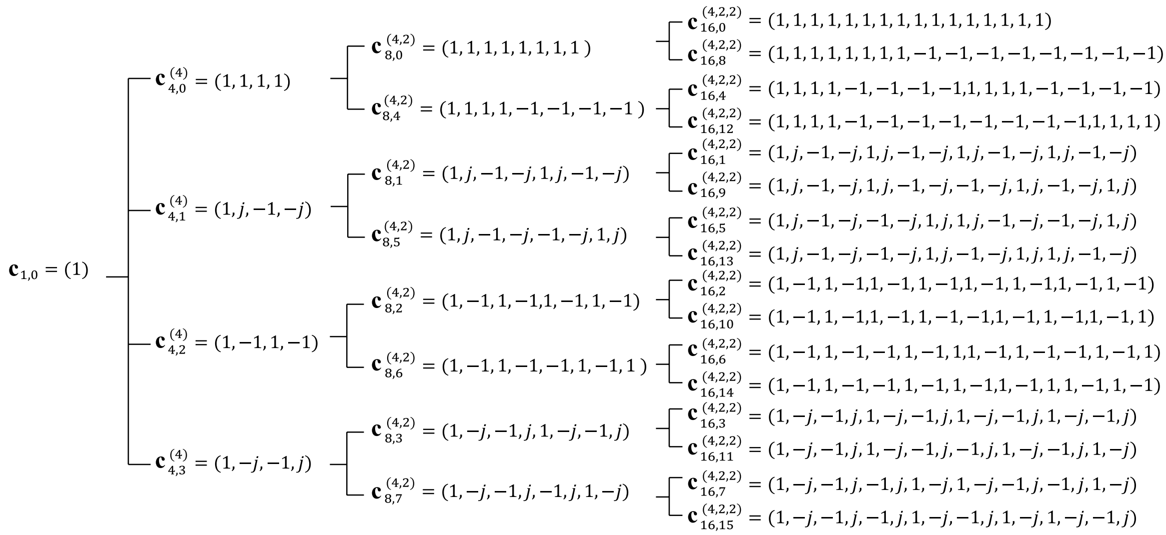

Fig. 3 Code tree for the proposed complex OVSF (4, 2, 2) code.

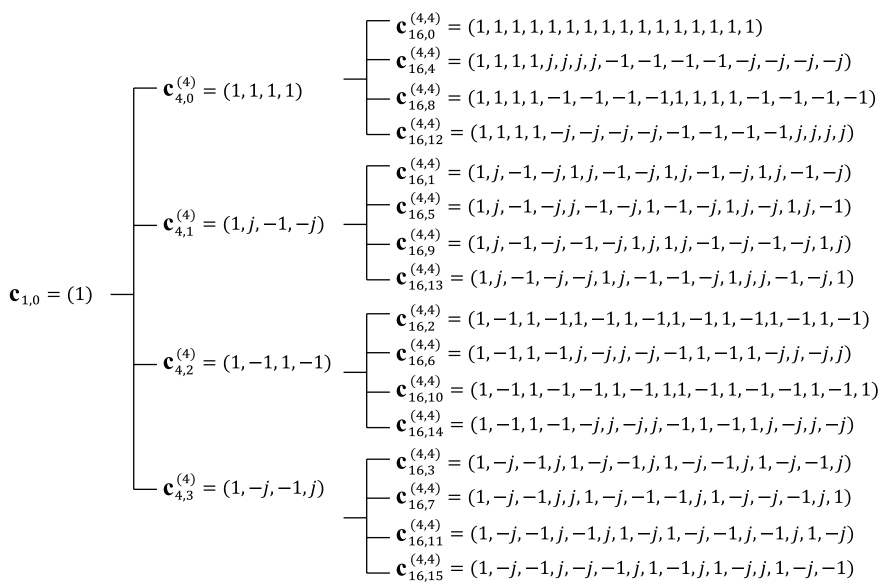

Fig. 4 Code tree for the proposed complex OVSF (4, 4) code.

the mth layer generally has r_m sequences with a spreading factor of r_m. A sequence $\mathbf{c}_{r_m,k}^{(q_1,q_2,\cdots,q_m)}$ in the mth layer in the code tree corresponds to the kth row of $M_{q_1,q_2,\cdots,q_m}$ ($k = 0, 1, \cdots, r_m - 1$, $m = 1, 2, \cdots, L$). Figures 3, 4, and 5 show specific examples of code trees for complex OVSF (4, 2, 2), (4, 4), and (3, 2) codes, respectively.

In general, the root node at the zeroth layer in the complex OVSF $(q_1, q_2, \cdots, q_L)$ code tree corresponds to the sequence

$$\mathbf{c}_{1,0} = (1).$$

The r_1 nodes at the first layer are the sequences

$$\mathbf{c}_{r_1,0}^{(q_1)}, \quad \mathbf{c}_{r_1,1}^{(q_1)}, \quad \cdots, \quad \mathbf{c}_{r_1,r_1-1}^{(q_1)},$$

where $r_1 = q_1$. These sequences are the r_1 rows of M_{q_1}, and M_{q_1} is equal to G_{q_1}. $\mathbf{c}_{r_1,k}^{(q_1)}$ corresponds to the kth row of M_{q_1}. The r_2 nodes at the second layer are the sequences

$$\mathbf{c}_{r_2,0}^{(q_1,q_2)}, \quad \mathbf{c}_{r_2,1}^{(q_1,q_2)}, \quad \cdots, \quad \mathbf{c}_{r_2,r_2-1}^{(q_1,q_2)},$$

where $r_2 = q_2 r_1$. These sequences are the r_2 rows of M_{q_1,q_2}, and M_{q_1,q_2} is equal to $G_{q_2} \otimes M_{q_1}$. $\mathbf{c}_{r_2,k}^{(q_1,q_2)}$ corresponds to the kth row of M_{q_1,q_2}.

The r_m nodes at the mth layer are the sequences

$$\mathbf{c}_{r_m,0}^{(q_1,q_2,\cdots,q_m)}, \quad \mathbf{c}_{r_m,1}^{(q_1,q_2,\cdots,q_m)}, \quad \cdots, \quad \mathbf{c}_{r_m,r_m-1}^{(q_1,q_2,\cdots,q_m)},$$

where $r_m = q_m r_{m-1}$ and $m = 1, 2, \cdots, L$. These sequences are the r_m rows of $M_{q_1,q_2,\cdots,q_m}$, and $M_{q_1,q_2,\cdots,q_m}$ is equal to $G_{q_m} \otimes M_{q_1,q_2,\cdots,q_{m-1}}$. $\mathbf{c}_{r_m,k}^{(q_1,q_2,\cdots,q_m)}$ corresponds to the kth row of $M_{q_1,q_2,\cdots,q_m}$.

The only important caution that must be made is that

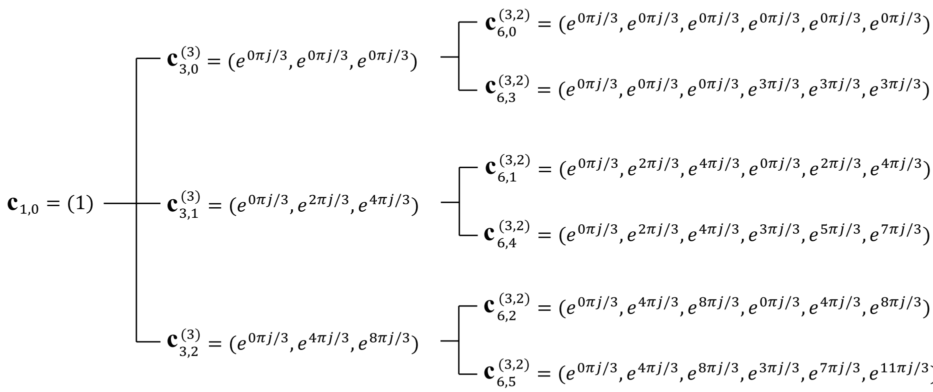

Fig. 5 Code tree for the proposed complex OVSF (3, 2) code.

the q_m descendants at the mth layer whose ancestor is $\mathbf{c}^{(q_1,q_2,\cdots,q_{m-1})}_{r_{m-1},i}$ must be

$$\mathbf{c}^{(q_1,q_2,\cdots,q_m)}_{r_m,i},\ \mathbf{c}^{(q_1,q_2,\cdots,q_m)}_{r_m,r_{m-1}+i},\ \mathbf{c}^{(q_1,q_2,\cdots,q_m)}_{r_m,2r_{m-1}+i},\ \cdots,\ \mathbf{c}^{(q_1,q_2,\cdots,q_m)}_{r_m,(q_m-1)r_{m-1}+i}$$

for $i = 0, 1, \cdots, r_{m-1} - 1$.

For example, in the case of the complex OVSF (3, 2) code whose code tree is described in Fig. 5, r_1 and r_2 are set to $r_1 = 3$ and $r_2 = 6$, respectively. For $m = 2$ and $i = 0$, the two descendants of $\mathbf{c}^{(3)}_{3,0}$ must be

$$\mathbf{c}^{(3,2)}_{6,0} \text{ and } \mathbf{c}^{(3,2)}_{6,3}.$$

For $m = 2$ and $i = 1$, the two descendants of $\mathbf{c}^{(3)}_{3,1}$ must be

$$\mathbf{c}^{(3,2)}_{6,1} \text{ and } \mathbf{c}^{(3,2)}_{6,4}.$$

And for $m = 2$ and $i = 2$, the two descendants of $\mathbf{c}^{(3)}_{3,2}$ must be

$$\mathbf{c}^{(3,2)}_{6,2} \text{ and } \mathbf{c}^{(3,2)}_{6,5}.$$

As with conventional OVSF codes, when a code sequence is assigned to one user, its descendant and ancestor code sequences cannot be assigned to any other user. As a result, the complex OVSF codes have a property that no two code sequences assigned to different users will ever interfere with each other, even if the spreading factors of the sequences are different.

For complex OVSF $(q_1, q_2, \cdots, q_L)$ codes, $q_1, q_2, \cdots, q_L$ are arbitrary integers greater than one. When $q_1 = q_2 = \cdots = q_L = 2$, the complex OVSF code is equivalent to the conventional OVSF code. Therefore, a particular property of the proposed OVSF codes is that the spreading factor can be designed more flexibly in each layer than is possible with conventional OVSF codes.

3.3 Code Symbols

Next, we will investigate the code symbols consisting the complex OVSF code sequences.

The mth layer code sequences consist of Q_m complex Q_mth roots of unity,

$$e^{0\pi j/Q_m},\ e^{2\pi j/Q_m},\ e^{4\pi j/Q_m},\ \ldots,\ e^{(Q_m-1)2\pi j/Q_m}, \tag{30}$$

where

$$Q_m = \mathrm{LCM}(q_1, q_2, \cdots, q_m) \tag{31}$$

and $m = 1, 2, \cdots, L$. In Eq. (31), $\mathrm{LCM}(q_1, q_2, \cdots, q_m)$ denotes the least common multiple of m integers $q_1, q_2, \cdots, q_m$.

For example, the symbols of the complex OVSF $(4, 2, 2)$ and $(4, 4)$ code sequences are in $\{1, j, -1, -j\}$, that is the set of the four complex fourth roots of unity, regardless of the layer. However, as for the complex OVSF $(3, 2)$ code, the symbols of the first layer sequences are in $\{e^{0\pi j/3}, e^{2\pi j/3}, e^{4\pi j/3}\}$, that is the set of the three complex third roots of unity, and the symbols of the second layer sequences are in $\{e^{0\pi j/3}, e^{1\pi j/3}, e^{2\pi j/3}, e^{3\pi j/3}, e^{4\pi j/3}, e^{5\pi j/3}\} = \{e^{0\pi j/6}, e^{2\pi j/6}, e^{4\pi j/6}, e^{6\pi j/6}, e^{8\pi j/6}, e^{10\pi j/6}\}$, that is the set of the six complex sixth roots of unity.

3.4 Code Sequence Assignment in Multi-Rate Transmission

Complex OVSF codes are useful in multi-rate DS-CDMA transmission systems in a way that is similar to conventional OVSF codes. Specifically, shorter code sequences are assigned to users transmitting data at higher rates, and longer code sequences are assigned to users transmitting data at lower rates. When a code sequence is assigned to one user, its descendant and ancestor code sequences are not assigned to any other user.

We will show an example of code sequence assignment in multi-rate transmission. Consider the DS-CDMA system using the complex OVSF (3, 2) code whose code tree is represented with Fig. 5. In Fig. 5, the spreading factors of the zeroth, first, and second layers are 1, 3, and 6, respectively. There are four cases of code sequence assignment depending on the number of users and their data rates.

Case 1: When six users transmit data at a low rate, each of the six code sequences at the second layer, specifically $\mathbf{c}^{(3,2)}_{6,0}$, $\mathbf{c}^{(3,2)}_{6,3}$, $\mathbf{c}^{(3,2)}_{6,1}$, $\mathbf{c}^{(3,2)}_{6,4}$, $\mathbf{c}^{(3,2)}_{6,2}$ and $\mathbf{c}^{(3,2)}_{6,5}$, is assigned to each user.

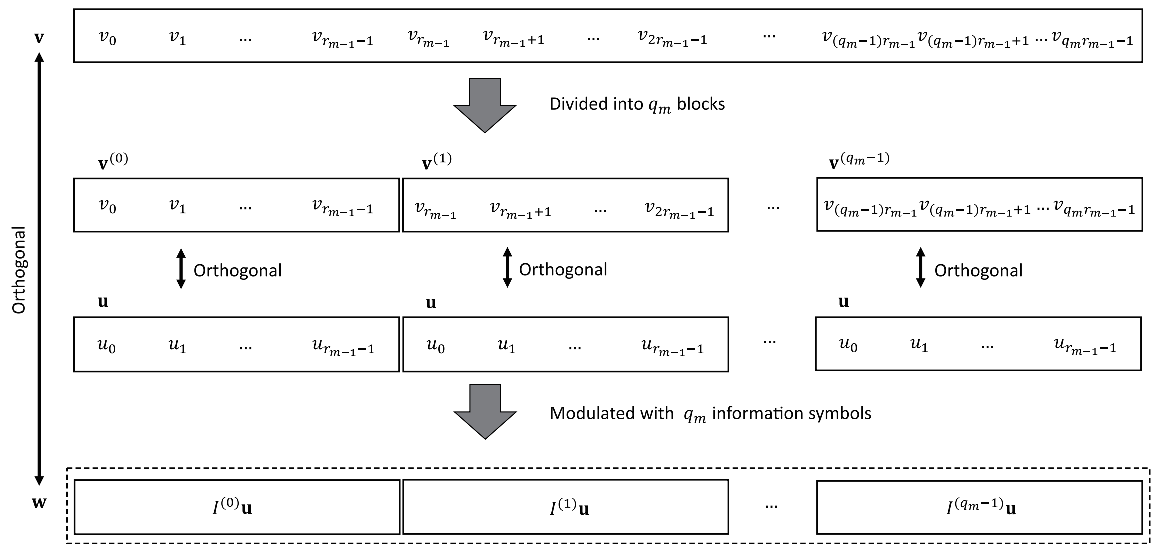

Fig. 6 Code orthogonality. Two sequences $\mathbf{v}^{(k)}$ and $\mathbf{u}$ are mutually orthogonal for $0 \le k \le q_m - 1$, and two sequences $\mathbf{v}$ and $\mathbf{w}$ are mutually orthogonal for any information symbols $I^{(0)}, I^{(1)}, \cdots, I^{(q_m-1)}$.

Case 2: When one user transmits data at a higher rate and remaining four users transmit data at a lower rate, one of the first layer sequences, e.g., $\mathbf{c}^{(3)}_{3,0}$, is assigned to the higher-rate user. The second layer sequences whose ancestors are not $\mathbf{c}^{(3)}_{3,0}$, specifically $\mathbf{c}^{(3,2)}_{6,1}$, $\mathbf{c}^{(3,2)}_{6,4}$, $\mathbf{c}^{(3,2)}_{6,2}$ and $\mathbf{c}^{(3,2)}_{6,5}$, are assigned to the four lower-rate users.

Case 3: When two users transmit data at a higher rate and remaining two users transmit data at a lower rate, two of the first layer sequences, e.g., $\mathbf{c}^{(3)}_{3,0}$ and $\mathbf{c}^{(3)}_{3,1}$, are assigned to the two higher-rate users. The second layer sequences whose ancestor is $\mathbf{c}^{(3)}_{3,2}$, specifically $\mathbf{c}^{(3,2)}_{6,2}$ and $\mathbf{c}^{(3,2)}_{6,5}$, are assigned to the two lower-rate users.

Case 4: When three users transmit data at a high rate, each of the three code sequences at the first layer, specifically $\mathbf{c}^{(3)}_{3,0}$, $\mathbf{c}^{(3)}_{3,1}$ and $\mathbf{c}^{(3)}_{3,2}$, is assigned to each user.

4. Orthogonality of Complex OVSF Codes

In this section, we will investigate the orthogonality of the proposed complex OVSF $(q_1, q_2, \cdots, q_L)$ codes, which include the conventional OVSF code as a case of $q_1 = q_2 = \cdots = q_L = 2$ as mentioned above.

Let $\mathbf{u} = (u_0, u_1, \cdots, u_{r_{m-1}-1})$ be a code sequence at the $(m-1)$th layer in a complex OVSF code, and $\mathbf{v} = (v_0, v_1, \cdots, v_{r_m-1})$ be a code sequence at the mth layer in the same code. Note that $\mathbf{u}$ is not an ancestor of $\mathbf{v}$, r_m and r_{m-1} are the spreading factors of the code sequences $\mathbf{v}$ and $\mathbf{u}$, respectively, and $r_m = q_m r_{m-1}$.

The sequence $\mathbf{v}$ can be divided into q_m blocks with a length of r_{m-1} symbols, as shown in Fig. 6. If we suppose $\mathbf{v}^{(0)}, \mathbf{v}^{(1)}, \cdots, \mathbf{v}^{(q_m-1)}$ are the blocks, each block $\mathbf{v}^{(k)}$ can be represented as $x\mathbf{u}'$, where x is one of the q_mth roots of unity and $\mathbf{u}'$ is the ancestor of $\mathbf{v}$ at the $(m-1)$th layer. Since $\mathbf{u}$ is not an ancestor of $\mathbf{v}$, the sequences $\mathbf{u}'$ and $\mathbf{u}$ are orthogonal over the complex number field. More specifically,

$$R(\mathbf{u}', \mathbf{u}) = \sum_{t=0}^{r_{m-1}-1} u'_t u^*_t = 0, \tag{32}$$

where $\mathbf{u}' = (u'_0, u'_1, \cdots, u'_{r_{m-1}-1})$. Thus, the two sequences $\mathbf{v}^{(k)}$ and $\mathbf{u}$ are orthogonal for $0 \le k \le q_m - 1$.

Furthermore, suppose $\mathbf{w}$ is a sequence

$$\mathbf{w} = (I^{(0)}\mathbf{u},\ I^{(1)}\mathbf{u},\ \cdots,\ I^{(q_m-1)}\mathbf{u}), \tag{33}$$

where $I^{(0)}, I^{(1)}, \cdots, I^{(q_m-1)}$ are q_m information symbols. The correlation of $\mathbf{v}$ and $\mathbf{w}$ is equal to zero, even if $\mathbf{u}$ is modulated by any information symbols. From these facts, it is shown that $\mathbf{u}$ in the $(m-1)$th layer and $\mathbf{v}$ in the mth layer do not interfere with each other if the ancestor code sequence of $\mathbf{v}$ is not $\mathbf{u}$. In a similar way, it is shown that two code sequences that are not ancestors or descendants of each other will not interfere with each other, regardless of whether their layers are adjacent in the code tree.

5. Features in Application Systems

5.1 Suppression of Signal Envelope Fluctuation

A system model of DS-CDMA transmitter can be illustrated with Fig. 7. Spreading a symbol by a polyphase code is the equivalent of rotating its signal phase in each chip duration. When data are modulated with p-ary phase shift keying (PSK) and spread with a q-phase code, a spread signal in each chip duration corresponds to one of the Q-phase signals, where $Q = \mathrm{LCM}(p, q)$. Spreading a symbol by a binary code increases the number of signal constellation points, when p is an odd number. For example, combination of ternary PSK modulation and a binary spreading code produces hexagonal phase signals. In contrast, combination of ternary PSK modulation and a ternary-phase spreading code does not increase the number of signal constellation points.

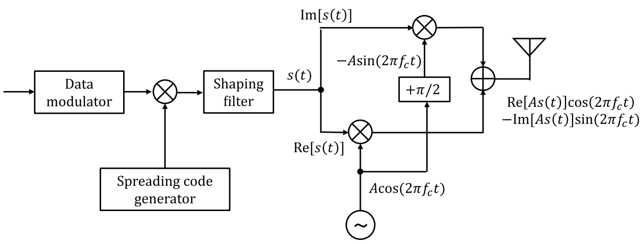

Fig. 7 System model of DS-CDMA transmitter.

Thus, properly designed polyphase codes are suited for DS-CDMA systems employing non-binary phase modulation, such as ternary, quadrature and hexagonal PSK [13], [14].

In addition, polyphase codes are inherently superior to binary codes in suppressing the level fluctuation of signal envelope. A binary code consists of two elements, 1 and -1. Each element rotates the signal phase 0 or 180 degrees. The phase shift transitions of spread symbols often pass nearby the origin of the complex plane in chip intervals. The probability of the phase shift transitions passing nearby the origin is about $1/2$, even if the data modulation scheme is not binary. Such transitions make the signal-envelope dynamic range wide [15]. On the other hand, when the system employs a complex OVSF code, the envelope fluctuation can be suppressed. For example, the probability of transitions passing nearby the origin for the complex OVSF (4, 4) code shown in Fig. 4 is about $1/4$. Thus, using the complex OVSF codes decreases the probability of transitions passing nearby the origin, and suppresses the level fluctuation of signal envelope. This property stems from polyphase orthogonal codes constructing the complex OVSF codes.

As mentioned above, polyphase orthogonal codes are inherently superior to binary orthogonal codes in suppressing the level fluctuation of signal envelope. This property of polyphase orthogonal codes is also attractive to orthogonal frequency division multiplexing (OFDM)-CDMA systems. A variable spreading factor code composed of polyphase symbols has been proposed for OFDM-CDMA systems [16], [17]. This code is constructed based on a binary tree, and its maximum spreading factor is restricted to $N = 2^n$. The code sequences of lengths 2^m $(m \leq n)$ have the property that the orthogonality in time domain is preserved in frequency domain as well. As a result, the code provides variable spreading factor property and low peak-to-average power ratio (PAPR) property in OFDM-CDMA systems, but its spreading factors are restricted to 2^m.

On the other hand, the OFDM-CDMA systems are out of the scope of this research. The complex OVSF codes in the present paper provide variable spreading factor property and smaller signal-envelope dynamic range. Furthermore, a particular property of the proposed OVSF codes is that the spreading factor can be designed more flexibly in each layer than is possible with conventional OVSF codes. This flexibility stems from the non-binary code trees which are extended by several polyphase codes with different or the same code lengths as shown in Sect. 3.

5.2 Effects of Multipath Fading and Synchronization Errors

The auto-correlations and cross-correlations between different shifts of code sequences are not always zero for the conventional OVSF codes. The proposed complex OVSF codes have similar auto-correlation and cross-correlation properties with the conventional OVSF codes. DS-CDMA systems using these OVSF codes normally require strict synchronization. In addition, in wireless communication environment, the orthogonality of the OVSF codes is often destroyed with time differences of multipath arrivals. Therefore, when the OVSF codes are applied to wireless communications, some techniques should be introduced to enforce the resistance against multipath channels and synchronization errors. Since the conventional OVSF codes have been practically used in the wideband DS-CDMA (W-CDMA) system [11], there exist a lot of studies to overcome these problems.

In the W-CDMA system, pilot symbols are inserted into every data slot for coherent RAKE combining at the receiver. In Refs. [8] and [12], performance of a DS-CDMA system employing a conventional OVSF code in combination with a long random code is evaluated under frequency selective Rayleigh fading environment. At the receiver of this system, a matched filter resolves the multipath and the pilot symbols are used for channel estimation to perform coherent RAKE combining. Other techniques have also been introduced to suppress the effect of multipath fading in CDMA systems. In Refs. [18] and [19], channel equalization techniques are presented in order to reduce the effect of multipath fading in CDMA systems. In addition, OVSF codes with Zero Correlation Zone (ZCZ) property are presented in Ref. [20], where ZCZ is a technique to mitigate interference by controlling synchronization. Though these techniques have been originally proposed for CDMA systems including W-CDMA, some of them would also be applied to the CDMA systems using proposed complex OVSF codes similarly.

6. Conclusion

In this paper, a class of complex OVSF codes, that is a new class of OVSF codes, is proposed. Since the complex OVSF codes are generalized versions of the conventional OVSF codes, any conventional OVSF code can be presented as a special case of the complex OVSF codes. Although the code trees of the proposed codes are similar to those of the conventional OVSF codes, the spreading factor of the mth layer in the proposed codes is not limited to 2^m. As a result, these complex OVSF codes can be applied to multi-rate data transmission services more flexibly than the conventional OVSF codes. In addition, the complex OVSF codes have the advantage of suppressing the level fluctuations of signal envelope in DS-CDMA systems employing non-binary PSK modulation.

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