IEICE TRANS. FUNDAMENTALS, VOL.E103–A, NO.9 SEPTEMBER 2020

PAPER *Special Section on Circuits and Systems*

Active Vibration Control of Nonlinear 2DOF Mechanical Systems via IDA-PBC

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SUMMARY This paper proposes an active vibration-suppression control method for the systems with multiple disturbances using only the relative displacements and velocities. The controller can suppress the vibration of the main body in the world coordinate, where a velocity disturbance and a force disturbance affect the system simultaneously. The added device plays a similar role as an accelerometer, but we avoid the algebraic loop. The main idea of the feedback law is to convert a nonlinear system into an aseismatic desired system by using the energy shaping technique. A parameter selection procedure is derived by combining the constraints of nonlinear IDA-PBC and the evaluation of the control performance of the linearly approximated system. The effectiveness of the proposed method is confirmed by simulations for an example.
key words: *nonlinear control, vibration measurement, sensors*

1. Introduction

Vibration suppression is a basic problem in the design of mechanical systems, and active vibration suppression methods have been used in actual mechanical systems for some decades. From the viewpoint of vibration suppression effect, active vibration control can give us much better vibration suppression performance than passive methods [1]. Unfortunately, most of active vibration control methods are based on the assumption that all the states are exactly known [2]–[6]. If the state is the relative information with reference plane, it will be easy to be observed by sensors. On the other hand, if the reference plane is vibrating, it is difficult to observe the absolute position and velocity directly by inexpensive sensors.

Although it is possible to observe the absolute information by the development of sensor technology, it should be pointed out that there are some problems such as expensive equipment and limited-frequency characteristics. We can use cheap MEMS accelerometers nowadays, and the methods using an accelerometer to obtain absolute information are developed [7], whereas they will result in the generation of an algebraic loop, and such methods are difficult to be applied in nonlinear cases. The methods using observer can estimate the absolute information [8], but those methods are also difficult to be applied in nonlinear case and it is essential to remember that they typically lead to a degradation in performance.

In automobile industry, many works on the suspension design using robust control method against the unmeasured external vibration have been developed during the last two decades. The robust controller can be designed by Linear-Quadratic-Gaussian (LQG) methodology [8], H_∞ technique [9], [10], saturated adaptive robust control (ARC) strategy [11] and so on. Especially, the works based on H_∞ technique show good performance on the robustness with respect to the uncertainties due to sensors. Even so, in order to apply these methods to nonlinear systems, we often need to solve Hamilton-Jacobi equations.

Motivated by aforementioned issues, Aoki et al. [12] proposed a method that sets a device similar to tuned mass damper (TMD) [13] on the controlled object and utilizes interconnection and damping assignment passivity-based control (IDA-PBC) method [14], which is a very general energy shaping control method, to gain the ideal vibration suppression performance. In that research, Aoki et.al use the device like an accelerometer, but they considered its dynamics so that the algebraic loop is avoided. They use IDA-PBC to convert the controlled system to the system with skyhook damper [5]. Although Aoki et al. [12] were successful in suppressing the vibration only with relative displacement and velocity and without any accelerometer signal, the method only handles the linear case and simple nonlinear-spring case.

In this paper, we obtain an IDA-PBC vibration suppression controller for more general port Hamiltonian systems. We first identify a class of port Hamiltonian systems with force and velocity disturbances, which is a general system expression for the cases with a floating nonlinear mechanical structure with additional spring and damper. We show a control law including some free parameters with some constraints. The controller uses only relative information, which can be easily measured. We propose a new parameter design method, which is more accomplished than that of [12]. The parameter selection can be made constructively. Finally, we show an example with simulation results, which verify the good vibration effect of the proposed controller. The stability of the nonlinear closed-loop system is guaranteed theoretically by the IDA-PBC method.

Compared to the preliminary study [15] of this research, the parameter design method in this paper is more sophisticated than that in [15]. With this new method, the parameter design become flexible. Moreover, we expect that the parameter design that is robust against the parameter uncertainties becomes possible by the new scheme, and it is our future work.

Manuscript received November 27, 2019.
Manuscript revised March 23, 2020.
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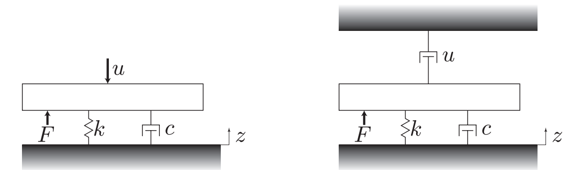
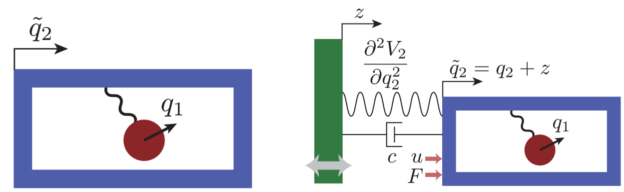
DOI: 10.1587/transfun.2019KEP0007



(a) Basic structure (b) Skyhook system

Fig. 1 Conventional vibration suppression method.



(a) Floating system (b) Controlled object

Fig. 2 Structure of controlled object.

2. Motivation of the Proposed Method

Let us consider a simple linear structure like Fig. 1(a), where u, F, k, c and z denote feedback input, force disturbance, elastic coefficient of the spring, damping coefficient, and displacement of the floor, respectively. If we know the absolute displacement of the main body, it is easy to suppress the vibration through active vibration control methods. One of the most commonly applied method is skyhook damper method, where the input force works as a virtual damper that is set between main body and rigid ceiling as showed in Fig. 1(b). With this method, we can realize ideal control performance. However, the reference point of the absolute position is lost due to the floor vibration, making it difficult to obtain the absolute information of the main body which is required for the skyhook damper method.

We can obtain the absolute information indirectly via an accelerometer. A direct feedback of the acceleration signal causes a static loop, and integration of the acceleration signal to obtain the velocity causes a drift problem. Filtering techniques may solve the static-loop problem, but a frequency-domain design is often required due to undesired phase-lag of the filter, which cannot be applied to nonlinear systems.

Therefore, in this paper we propose a new vibration-suppression control that utilizes only relative informations. To obtain more rich information from the measurements of the relative movements, we assume that there exists an additional mechanical degree-of-freedom (DOF) in the controlled object. This approach is almost equivalent to considering the internal dynamics of an accelerometer precisely. In addition to the cases with accelerometers, our method can be applied to the systems in which additional (nonlinear) dynamic structure is naturally contained. In these cases, the additional structure may have a mass that cannot be ignored, and the motion the additional structure has a phase lag even for the low frequncy range.

3. Controlled Object and Problem Setting

In this section, we specify a class of port Hamiltonian systems with force and velocity disturbances, which commonly appear in vibration suppression problem. In our research, we set a nonlinear additional mass on the main body as shown in Fig. 2(a), where q_1 denote the relative displacement of the additional mass and $\tilde{q}_2$ denote the displacement of the main body in world coordinate. The system is a part of a controlled object, but is not the target system itself.

We assume that there exists a symmetry on the change of $\tilde{q}_2$, which derives a law of conservation of momentum with respect to the movement of the whole mass to $\tilde{q}_2$ direction by the Noether's theorem [16]. According to the symmetry, the Hamiltonian of this system is not the function of $\tilde{q}_2$, i.e. the inertia matrix and the potential energy only depend on q_1. Consequently, the Hamiltonian of the system of Fig. 2(a) can be written as

$$\tilde{H}(q_1, p) = \frac{1}{2}p^\top M(q_1)^{-1}p + V_1(q_1)$$
$$M(q_1) = \begin{bmatrix} m_1(q_1) & m_2(q_1) \\ m_2(q_1) & m_3(q_1) \end{bmatrix} \tag{1}$$

$$p = M(q_1)\begin{pmatrix} \dot{q}_1 \\ \dot{\tilde{q}}_2 \end{pmatrix}, \tag{2}$$

and the friction coefficient matrix becomes $\tilde{C} = \mathrm{diag}(\mu, 0)$, where $\mu > 0$ is the friction coefficient of the additional movement. The positive-definite matrix $M(q_1)$ is the inertia matrix, $V_1(q_1)$ is the potential energy of the internal structure, and p is the generalized momentum. We assume that $V_1(q_1)$ is positive definite with respect to q_1. The law of conservation of momentum of the basic structure is $p_2 = m_2(q_1)\dot{q}_1 + m_3(q_1)\dot{\tilde{q}}_2 = \text{const}$. We assume that there exists an interconnection between the motion of q_1 and $\tilde{q}_2$, and thus $m_2(q_1) \neq 0$.

By adding a potential force $V_2(q_2)$ and a damping term $c\dot{q}_2$ with respect to the relative movement between the main body and a vibrating object, a force disturbance F, and a control input u, we obatin the controlled object like Fig. 2(b). We assume that the control force and the force disturbance act on the main body. The Hamiltonian of the controlled system is

$$H(p, q) = \tilde{H}(q_1, p) + V_2(q_2)$$
$$q = (q_1, q_2)^\top, \quad q_2 = \tilde{q}_2 - z(t), \tag{3}$$

where z and q_2 denote the displacement of the vibrating object and the relative displacement of main body from the object, respectively. We let the additional potential $V_2(q_2)$ be positive definite with respect to q_2 as well. The definition (2) of p can be rewritten as

$$p = M(q_1)\left(\dot{q} - a\omega\right), \tag{4}$$

where $\omega = \dot{z}$ is a velocity disturbance, and $a = (0, -1)^\top$. Notice that p is defined in the world coordinate, while q is a relative displacement vector.

Thus, the controlled object can be expressed by a port-Hamiltonian system (PH system)

$$\dot{x} = (J - R)\frac{\partial H}{\partial x}^{\top} + D\omega + B(u + F), \tag{5}$$

where $x = (q^{\top}, p^{\top})^{\top}$ is the state, and

$$J = \begin{bmatrix} O & I \\ -I & O \end{bmatrix},\ R = \begin{bmatrix} O & O \\ O & C \end{bmatrix},\ C = \tilde{C} + \tilde{C}_a = \begin{bmatrix} \mu & 0 \\ 0 & c \end{bmatrix},$$
$$B = (0\,0\,0\,1)^{\top},$$
$$D = (a^{\top}\ -(Ca)^{\top})^{\top} = (0\ -1\ 0\ c)^{\top}.$$

Our main purpose is the vibration suppression of $\tilde{q}_2$ against the velocity disturbance $\omega(t)$ and the force disturbance $F(t)$. Note that $\tilde{q}_2 \approx 0$ means $q_2 \approx -z(t)$. Since the second element of D is -1, a feedforward term of ω exists in the dynamics of q_2, and therefore suppression of p will achieve the control objective. In this study, we construct an IDA passivity-based controller using only the relative displacements q and velocities $\dot{q}$, which can be easily measured by sensors. Note that our control law is not a function of q and p but q and $\dot{q}$, because p is defined in the world coordinate and (4) includes ω.

4. Application of IDA-PBC

4.1 Overview

In this section, we define the dynamics of desired system at first, and then obain a matching condition between the controlled system and the desired system. The matching condition clearfy the degree of freedom in the controller design and the expression of feedback law with free parameters as well as equality and inequality constraints.

4.2 Desired System

We construct the desired system with artificial strucutre matrix as follows:

$$\dot{x} = (J_d(q_1) - R_d(q_1))\frac{\partial H_d}{\partial x}^{\top} + D_d(q_1)\omega + D_{dp}(q_1)p\cdot\omega + D_{d\omega}(q_1)\omega^2 + BF, \tag{6}$$

where

$$H_d(x) = \frac{1}{2}p^{\top}M_d(q_1)^{-1}p + V_d(q_1, q_2) \tag{7}$$

denotes the Hamiltonian of desired system, and

$$M_d(q_1) = \begin{bmatrix} m_{d1}(q_1) & m_{d2}(q_1) \\ m_{d2}(q_1) & m_{d3}(q_1) \end{bmatrix},$$
$$J_d(q_1) = \begin{bmatrix} O & M(q_1)^{-1}M_d(q_1) \\ -M_d(q_1)M(q_1)^{-1} & J_2(q_1) \end{bmatrix},$$
$$J_2(q_1) = \begin{bmatrix} 0 & j_e(q_1) \\ -j_e(q_1) & 0 \end{bmatrix},$$
$$R_d(q_1) = \begin{bmatrix} O & O \\ O & C_d(q_1) \end{bmatrix},\ C_d(q_1) = \begin{bmatrix} c_{d1}(q_1) & c_{d2}(q_1) \\ c_{d2}(q_1) & c_{d3}(q_1) \end{bmatrix},$$
$$D_d(q_1) = (0\ -1\ 0\ d_1(q_1))^{\top},$$
$$D_{dp}(q_1) = \begin{bmatrix} 0 & 0 & 0 & d_2(q_1) \\ 0 & 0 & 0 & d_3(q_1) \end{bmatrix}^{\top},$$
$$D_{d\omega}(q_1) = (0\,0\,0\,d_4(q_1))^{\top}.$$

$J_d(q_1), R_d(q_1), V_d(q)$, and $M_d(q_1)$ denote an artificial skew-symmetric structure matrix, a positive semidefinite damping matrix, a potential energy, and the inertia matrix in the desired Hamiltonian, respectively.

4.3 Application of IDA-PBC Method

We can derive the expression of feedback law with equality and inequality constraints on the parameters of the desired system by matching the dynamics of desired system with that of controlled system as follows:

$$(J_d - R_d)\frac{\partial H_d}{\partial x}^{\top} = (J - R)\frac{\partial H}{\partial x}^{\top} + Bu + (D - D_d)\omega - D_{dp}(q_1)p\cdot\omega - D_{d\omega}(q_1)\omega^2. \tag{8}$$

For convenience of calculations, we set

$$S(q_1) = M^{-1}(q_1) = \begin{bmatrix} s_1(q_1) & s_2(q_1) \\ s_2(q_1) & s_3(q_1) \end{bmatrix}$$
$$S_d(q_1) = M_d^{-1}(q_1) = \begin{bmatrix} s_{d1}(q_1) & s_{d2}(q_1) \\ s_{d2}(q_1) & s_{d3}(q_1) \end{bmatrix}. \tag{9}$$

Hereafter, by omitting '(q_1)', we simply express them as S, S_d, s_i and s_{di}. Each side of (8) is four dimensional vector. The first two components of (8) are already satisfied for all x and ω. We can easily derive the equality constraints of parameters by extracting the coefficients of p, q and thier higher-order terms. By focusing on the coefficients of p_1^2, p_1p_2 and p_2^2 in the third component of (8), we obtain

$$s_{d1}' = \frac{|S_d|s_1'}{s_1s_{d3} - s_2s_{d2}}$$
$$s_{d2}' = \frac{|S_d|s_2'}{s_1s_{d3} - s_2s_{d2}} \tag{10}$$
$$s_{d3}' = \frac{|S_d|s_3'}{s_1s_{d3} - s_2s_{d2}},$$

where $*'$ means the derivative with respect to q_1.

The coefficients of p_1 and p_2 in the third component of (8) derive the following relations:

$$c_{d1}(q_1) = \frac{\mu}{|S_d|}(s_1s_{d3} - s_2s_{d2}) \tag{11}$$

$$j_e(q_1) = c_{d2}(q_1) + \frac{\mu}{|S_d|}(s_1s_{d2} - s_2s_{d1}). \tag{12}$$

The rest of the third component of (8) leads an equation for the potential energy

$$\frac{s_2 s_{d2} - s_1 s_{d3}}{|S_d|} \cdot \frac{\partial V_d}{\partial q_1} + \frac{s_3 s_{d2} - s_2 s_{d3}}{|S_d|} \cdot \frac{\partial V_d}{\partial q_2} + V_1' = 0.$$

The general solution of the above equation is

$$\begin{aligned} V_d(q) = P &\left[q_2 + \int_0^{q_1} \left. \frac{s_3 s_{d2} - s_2 s_{d3}}{s_1 s_{d3} - s_2 s_{d2}} \right|_{q_1=\tau} d\tau \right] \\ &+ \int_0^{q_1} \left. \frac{V_1' |S_d|}{s_1 s_{d3} - s_2 s_{d2}} \right|_{q_1=\tau} d\tau, \end{aligned} \tag{13}$$

where P will be an arbitrary positive-definite function.

By solving the forth equation of (8) with respect to u, we can obtain a feedback law $u = \alpha_{\mathrm{raw}}(q, p, \omega)$. Notice that the feedback should be a function of q and $\dot{q}$ only. Hence, we decompose α_{raw} as

$$\alpha_{\mathrm{raw}}\,(q, M(q_1)(\dot{q} - a\omega), \omega) = \alpha(q, \dot{q}) + \alpha_{\mathrm{rest}}(q, \dot{q}, \omega)\omega.$$

The coefficient $\alpha_{\mathrm{rest}}(\cdot)$ should be identically zero, and thus we decompose it again as

$$\begin{aligned} &\alpha_{\mathrm{rest}}(q, S(q_1)p + a\omega, \omega) = \\ &\quad \alpha_1(q_1) + \alpha_2(q_1)p_1 + \alpha_3(q_1)p_2 + \alpha_4(q_1)\omega. \end{aligned}$$

By solving $\alpha_i(q_1) = 0$ ($i = 1, \ldots, 4$)with respect to $d_1(q_1), \ldots, d_4(q_1)$ and applying (10), we obtain additional equality constraints

$$\begin{aligned} d_1(q_1) = \frac{1}{|S|}\{&(s_1 s_{d3} - s_2 s_{d2})c_{d3}(q_1) \\ &+ (s_1 s_{d2} - s_2 s_{d1})(j_e(q_1) + c_{d2}(q_1)\} \end{aligned} \tag{14}$$

$$(d_2(q_1)\ d_3(q_1)) = g(q_1) \cdot (0\ 1)M'S \tag{15}$$

$$d_4(q_1) = \frac{g(q_1)}{2} \cdot (0\ 1)M'(0\ 1)^\top, \tag{16}$$

where $M' = \partial M / \partial q_1$ and

$$g(q_1) = \frac{s_2 s_{d1} - s_1 s_{d2}}{s_1 s_{d3} - s_2 s_{d2}}.$$

The control input can be written as

$$\begin{aligned} u &= \alpha(q, \dot{q}) \\ &= \frac{(s_2 s_{d3} - s_3 s_{d2})c_{d3} - (s_3 s_{d1} - s_2 s_{d2})(c_{d2} + j_e)}{|S|}\dot{q}_1 \\ &+ (c - d_1(q_1))\dot{q}_2 + \frac{g(q_1)}{2} \cdot \dot{q}^\top M' \dot{q} + \frac{\partial V_2(q_2)}{\partial q_2} \\ &+ \frac{s_1 s_{d2} - s_2 s_{d1}}{|S_d|} \cdot \frac{\partial V_d}{\partial q_1} - \frac{s_3 s_{d1} - s_2 s_{d2}}{|S_d|} \cdot \frac{\partial V_d}{\partial q_2}. \end{aligned} \tag{17}$$

Because of the feature of IDA-PBC, the closed-loop system is identical to the desired system. Therefore, the asymptotic stability of zero-disturbance case can be guaranteed by the nature of port-Hamiltonian system. Thus we need to ensure the positive definiteness of M_d, V_d and C_d, and the following inequality constraints can be derived:

$$s_{d3}(q_1) > 0, \quad |S_d(q_1)| > 0, \tag{18}$$

$$s_1 s_{d3} - s_2 s_{d2} > 0, \quad {}^\forall q_1, \tag{19}$$

$$|C_d(q_1)| > 0, \tag{20}$$

$$P[\sigma] > 0, \quad \sigma \neq 0. \tag{21}$$

Inequalities (18) show the positive definiteness of the inertia matrix of the desired system. We can show $c_{d1}(q_1) > 0$ from (19) and (11), and therefore (19) and (20) means that the damping matrix of the desired system is positive definite. Because of (19), the positivity of the second term of (13) will be automatically satisfied if $q_1 V_1' \geq 0$. Hence, under the constraint (21), the potential energy function $V_d(q)$ is positive definite.

We can gain $s_{di}(q_1)$ by solving (10), while the initial value $S_d(0) = S_{d0}$ is a degree of freedom. The inequality constraints of parameters are (18), (19), (20), and (21). The equality constraints of parameters are (11), (12), (13), (14), (15), (16), and (17).

Note that the asymtotic satbility is guaranteed by the positive definiteness of M_d, V_d and C_d. Therefore, stability of the numerical solution process of differential equation (10) is not required when desgining the control law.

In next section, we derive the guideline of parameter selection in order that we can obtain an aseismatic desired system.

5. Guideline for Parameter Selection

5.1 Linear Approximation

To design the parameters of desired system, we need to know what role the each parameters play in vibration dynamics. However, because of the nonlinear term, the pratical meaning of parameters in inertia matrix is unclear. Hence, we will derive the guideline of the parameter selection based on the linearly approximated systems of (5) and (6) at first, which determines the low-order terms of the free parameters. Then we will apply it into nonlinear case. By the quadratic approximation of H, the Hamiltonian of the linearized plant is

$$H_L(p, q) = \frac{1}{2}p^\top S_0 p + \frac{1}{2}(K_1 q_1^2 + K_2 q_2^2),$$

where

$$S_0 = \begin{bmatrix} s_{10} & s_{20} \\ s_{20} & s_{30} \end{bmatrix} = S(0)$$

$$K_1 = \frac{\partial^2 V_1}{\partial q_1^2}(0), \quad K_2 = \frac{\partial^2 V_2}{\partial q_2^2}(0).$$

The linearized controlled object can be described as

$$\dot{x} = \begin{bmatrix} 0 & I \\ -I & -C \end{bmatrix} \begin{pmatrix} \mathrm{diag}(K_1, K_2)q \\ S_0 p \end{pmatrix} + \begin{pmatrix} a \\ -Ca \end{pmatrix} \omega. \tag{22}$$

The quadratic approximation of H_d can be also obtained as

$$H_{dL}(p, q) = \frac{1}{2}\{p^\top S_{d0} p + K_{d1} q_1^2 + K_{d2}(q_2 + h q_1)^2\},$$

where

$$S_{d0} = \begin{bmatrix} s_{d10} & s_{d20} \\ s_{d20} & s_{d30} \end{bmatrix} = S_d(0),$$

$$h = \frac{s_{30}s_{d20} - s_{20}s_{d30}}{s_{10}s_{d30} - s_{20}s_{d20}} \tag{23}$$

$$K_{d1} = \frac{K_1|S_{d0}|}{s_{10}s_{d30} - s_{20}s_{d20}}, \quad K_{d2} = \left.\frac{\partial^2 P(y')}{\partial y'^2}\right|_{y'=0}. \tag{24}$$

The linearized desired system is

$$\dot{x} = (J_{d0} - R_{d0}) \begin{pmatrix} K_{d0} q \\ S_{d0} p \end{pmatrix} + D_{w0}\omega + BF, \tag{25}$$

where

$$J_{d0} = \begin{bmatrix} O & M_0^{-1} M_{d0} \\ -M_{d0} M_0^{-1} & J_{20} \end{bmatrix}, \ J_{20} = \begin{bmatrix} 0 & j_{e0} \\ -j_{e0} & 0 \end{bmatrix}$$

$$M_{d0} = S_{d0}^{-1} = \begin{bmatrix} m_{d1} & m_{d2} \\ m_{d2} & m_{d3} \end{bmatrix}, K_{d0} = \mathrm{diag}(K_{d1}, K_{d2})$$

$$R_{d0} = \begin{bmatrix} O & O \\ O & C_{d0} \end{bmatrix}, \ C_{d0} = \begin{bmatrix} c_{d10} & c_{d20} \\ c_{d20} & c_{d30} \end{bmatrix}$$

$$D_{w0} = (0 \ -1 \ 0 \ d_{10})^\top$$

5.2 Coordinate Transformation

The diagnolized inertia matrix can help us clarify the structure of linearized system, thus we consider new transformed variables

$$\hat{q} = L^{-1} q, \quad \hat{p} = L^\top p, \tag{26}$$

where

$$L = \begin{bmatrix} r_0 & -r_0 \\ 0 & 1 \end{bmatrix}, \tag{27}$$

$$r_0 = \frac{m_{20}}{m_{10}} = -\frac{s_{20}}{s_{30}}. \tag{28}$$

To simplify the problem, we choose S_{d0} such that h defined by (23) becomes zero, i.e.

$$\frac{s_{20}}{s_{30}} = \frac{s_{d20}}{s_{d30}}. \tag{29}$$

Under the new constraint (29),

$$r_0 = \frac{m_{d20}}{m_{d10}} = -\frac{s_{d20}}{s_{d30}}$$

is also satisfied as well as (28).

The coordinate of main mass q_2 is maintained with this coordinate transformation, i.e. $\hat{q}_2 = q_2$. Please recall that the control objective is the vibration suppression of the main body. Then, the linear approximation of (4) can be transformed to

$$\hat{p} = L^\top M_0 L(\dot{\hat{q}} - L^{-1} L^\top a\omega) = \hat{M}(\dot{\hat{q}} - \hat{a}\omega),$$

where

$$\hat{M} = L^\top M_0 L = \mathrm{diag.}(\hat{m}_1, \hat{m}_2)$$

$$\hat{m}_1 = m_{10} r_0^2, \quad \hat{m}_2 = m_{30} - m_{10} r_0^2$$

$$\hat{a} = L^{-1} L^\top a = (-1 \ -1)^\top.$$

Consequently, the linearized controlled object (22) can be transformed to

$$\begin{pmatrix} \dot{\hat{q}} \\ \dot{\hat{p}} \end{pmatrix} = \begin{bmatrix} 0 & I \\ -I & -\hat{C} \end{bmatrix} \begin{pmatrix} \hat{K}\hat{q} \\ \hat{S}\hat{p} \end{pmatrix} + \begin{pmatrix} \hat{a} \\ -\hat{C}\hat{a} \end{pmatrix} \omega, \tag{30}$$

where

$$\hat{S} = \hat{M}^{-1} = \mathrm{diag}(\hat{m}_1^{-1}, \hat{m}_2^{-1})$$

$$\hat{K} = L^T \mathrm{diag}(K_1, K_2) L = \begin{bmatrix} \hat{K}_1 & -\hat{K}_1 \\ -\hat{K}_1 & \hat{K}_1 + \hat{K}_2 \end{bmatrix}$$

$$\hat{C} = L^T C L = \begin{bmatrix} \hat{C}_1 & -\hat{C}_1 \\ -\hat{C}_1 & \hat{C}_1 + \hat{C}_2 \end{bmatrix}$$

$$\hat{K}_1 = K_1 r_0^2, \quad \hat{K}_2 = K_2, \quad \hat{C}_1 = \mu r_0^2, \quad \hat{C}_2 = c.$$

Under the assumption (29), the inertia matrix of the linearized desired system (25) in the new coordinate is

$$\hat{M}_d = L^\top M_{d0} L = \mathrm{diag}(\hat{m}_{d1}, \hat{m}_{d2}) = \mathrm{diag}(m_{d10} r_0^2, m_{d30} - m_{d10} r_0^2),$$

which is also a diagonal matrix under the constraint of (29). The linearized desired system is also converted into

$$\begin{pmatrix} \dot{\hat{q}} \\ \dot{\hat{p}} \end{pmatrix} = (\hat{J}_d - \hat{R}_d) \begin{pmatrix} \hat{K}_d \hat{q} \\ \hat{S}_d \hat{p} \end{pmatrix} + \hat{D}_d \omega + \hat{B} F, \tag{31}$$

where

$$\hat{J}_d = \begin{bmatrix} O & \hat{M}^{-1}\hat{M}_d \\ -\hat{M}_d \hat{M}^{-1} & \hat{J}_2 \end{bmatrix}, \quad \hat{J}_2 = L^T J_{20} L,$$

$$\hat{S}_d = \hat{M}_d^{-1} = \mathrm{diag}(\hat{m}_{d1}^{-1}, \hat{m}_{d2}^{-1}),$$

$$\hat{K}_d = L^T \begin{bmatrix} K_{d1} & 0 \\ 0 & K_{d2} \end{bmatrix} L = \begin{bmatrix} \hat{K}_{d1} & -\hat{K}_{d1} \\ -\hat{K}_{d1} & \hat{K}_{d1} + \hat{K}_{d2} \end{bmatrix},$$

$$\hat{K}_{d1} = K_{d1} r_0^2, \quad \hat{K}_{d2} = K_{d2},$$

$$\hat{R}_d = \mathrm{diag}\,(O, \hat{C}_d),$$

$$\hat{C}_d = L^T C_{d0} L = \begin{bmatrix} \hat{C}_{d1} + \hat{C}_{d2} & -\hat{C}_{d2} \\ -\hat{C}_{d2} & \hat{C}_{d2} + \hat{C}_{d3} + \hat{C}_{d4} \end{bmatrix},$$

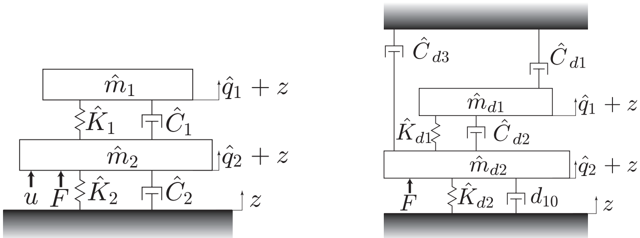
$$\hat{C}_{d1} = r_0 c_{d2}, \quad \hat{C}_{d2} = r_0^2 c_{d10} - r_0 c_{d2},$$

$$\hat{C}_{d3} = c_{d30} - r_0 c_{d20} - d_{10}, \quad \hat{C}_{d4} = d_{10},$$

$$\hat{D}_d = \mathrm{diag}(L^{-1}, L^T) D_{w0} = (-1 \ -1 \ 0 \ d_{10})^\top,$$

$$\hat{B} = \mathrm{diag}(L^{-1}, L^T) B = B.$$

The linearized controlled object (30) can be considered as a mass-spring-damper (MSD) system with a device similar to tuned mass damper (TMD) in Fig. 3(a). When we ignore the difference between J and $\hat{J}_d$, the linearized desired system (31) is regarded as an MSD system with a TMD-like



(a) Controlled object (b) Desired system

Fig. 3 Linearized system.

device and multiple skyhook dampers in Fig. 3(b). Thus, in linearized case, the feedback law in this research realizes the virtual skyhook dampers by only relative displacements and velocities.

5.3 Parameter Design

We define mass ratios

$$r_1 = \frac{\hat{m}_{d1}}{\hat{m}_1} = \frac{m_{d10}}{m_{10}}, \quad r_2 = \frac{\hat{m}_{d2}}{\hat{m}_2}. \tag{32}$$

The values r_1 and r_2 are positive, if and only if $S_{d0} > 0$.

From the definition (32), the inequality constraint (20) can be rewritten as

$$|C_{d0}| = \mu d_{10} r_1 r_2 - \left[c_{d20} - \mu r_0 (r_1 - r_2)\right]^2 > 0. \tag{33}$$

From the view point of energy, the small dissipation matrix is unsuitable for the control objective. Thus we set c_{d20} as

$$c_{d20} = \mu r_0 (r_1 - r_2), \tag{34}$$

which maximizes $|C_{d0}|$ for fixed r_1 and r_2. Hence, positive r_1, r_2, and d_{10} make C_{d0} and M_{d0} positive definite, and the asymptotical stability of the linearized desired system is guaranteed. We will design d_{10}, r_1, and r_2 such that $|C_{d0}|$ is sufficiently large, under the new constraint (34).

Under the assumptions (29) and (34), we obtain

$$c_{d30} = r_2 d_{10} + \frac{\mu r_0^2 (r_2 - r_1)^2}{r_1}. \tag{35}$$

The value of skyhook-damper coefficient of the mainbody becomes

$$\hat{C}_{d3} = d_{10}(r_2 - 1) + \frac{\mu r_0^2 r_2 (r_2 - r_1)}{r_1}. \tag{36}$$

Obviously, large d_{10}, r_2 and small r_1 can make skyhook damper coefficient (36) be large. However, large d_{10} will lead to the increasement of the high frequency gain from z to q_2, bacause d_{10} indicates the damping coefficient between the vibrating object and the main body, as seen in Fig. 3(b).

Hence, we choose small d_{10} first, and design small r_1 and large r_2 so that skyhook damper term coefficient $\hat{C}_{d3}$ is sufficiently large, because of (36). From empirical knowledges, small d_{10} and large $\hat{C}_{d3}$ in Fig. 3(b) make a good vibration suppression effect.

The selection (34) makes $\hat{C}_{d1}$, which is the coefficient of the skyhook damper of the additional mass in Fig. 3(b), negative, but $|C_{d0}| > 0$ is guaranteed by a large c_{d30}. A large r_2 also decreases the low-frequency gain from F as

$$G_{F\tilde{q}_2}(0) = \frac{K_{d2}}{r_2}. \tag{37}$$

The above parameter selection guideline is more sophisticated than that in Aoki, et al. [12]. The parameter selection procedure is summarized as follows.

1. Choose sufficiently small $d_{10} > 0$, sufficiently small $r_1 > 0$, and sufficiently large $r_2 > 0$. Select a small low-frequency gain with r_2 and K_{d2} in (37). Then design a positive-definite function $P[\cdot]$ by (24). From (29) and (32), S_{d0} (> 0) is determined.
2. Calculate $S_d(q_1)$ by solving the differential equations (10) with the initial condition $S_d(0) = S_{d0}$.
3. Check the conditions (18) and (19). If these inequalities are not satisfied for all q_1, return to the first step and choose the parameters again.
4. Set $c_{d1}(q_1)$ as (11). The values of $c_{d2}(0) = c_{d20}$ and $c_{d3}(0) = c_{d30}$ are determined by (34) and (35), respectively, and then $C_{d0} = C_d(0) > 0$ is guaranteed. Choose the high-order terms of $c_{d2}(q_1)$ and $c_{d3}(q_1)$ adequately so that $C_d(q_1) > 0$.
5. Calculate $j_e(q_1)$, $V_d(q)$, and $d_1(q_1)$ by (12), (13), and (14), respectively.
6. Obtain the control law (17).

Although the design procedure is constructive, the high-order terms of c_{d2} and c_{d3} should be chosen to satisfy $C_d(q_1) > 0$. Along with the increasing of q_1, the practical meaning of inertia matrix in desired system will be far different from that of linear approximated case. Therefore, $C_d(q_1)$ far from the origin must be varied along with changes of the inertia matrix. On the other way, large $|q_1|$ often means that the current vibration is violent, hence it is natural to make the feedback gain high when $|q_1|$ reaches a threshold. The idea of control barrier function may be utilized for this purpose, but this topic is one of our future work. In this paper, the parameter selection guideline is focusing on making the skyhook damper term large, while the free parameter selection in this proposed method can control not only the skyhook damper term but also the mass and spring term. The problem of how to adjust those terms to suppress the vibration in a wider frequency domain will depend on the sensitivity from the free-parameter selection to the control performance, and that is also our future work.

6. An Example and Simulation

In this section, in order to verify the vibration suppression effect of the proposed feedback law (17), we will construct a control system for an example and make simulations for the system.

We consider a main body (cart) with a pendulum like

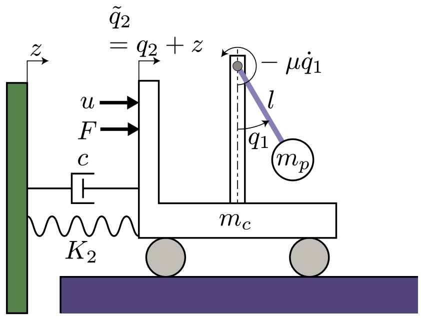


Fig. 4 Cart and pendulum system.

Fig. 4, where m_p, m_c, l, c, μ, and K_2 denote mass of the pendulum, mass of the cart, length of massless bar, viscosity dumper coefficient between the cart and the vibrating wall, rotational friction coefficient at the axis of the pendulum, and elastic coefficient between the cart and the vibrating wall, respectively. We choose the variables q_1, $\tilde{q}_2$, z, F, and u as swing angle of the pendulum, displacement of the cart in world coordinate, displacement disturbance of the basement, force disturbance on main body, and input force, respectively. Here we set the parameters of controlled object as $m_p = 0.2$, $m_c = 10$, $l = 5$, $c = 2$, $\mu = 10$, and $K_2 = 3$. The disturbances are $z = \sin(bt)$ and $F = 100\cos(bt)$ for $b = 1$ and $b = 10$.

To verify the performance of the proposed method, we perform simulations for the open loop system, the closed-loop system using the proposed feedback law, and the closed-loop system using conventional method which is skyhook damper method. We consider the skyhook damper feedback law

$$u = -\hat{C}_{d3}\dot{\tilde{q}}_2,$$

where $\hat{C}_{d3}$ is the same value as the desired skyhook damper used in our proposed method. However, we apply the skyhook damper method with the assumption that absolute displacement and velocity of the cart are measurable, while our proposed method only uses relative displacement and velocity information.

According to the proposed guideline of parameter selection in section 5, we firstly setting small d_{10}, samll r_1 and large r_2 as $d_{10} = 2$, $r_1 = 15$ and $r_2 = 1000$. Since a small low-frequency gain $G_{F\tilde{q}_2}(0)$ is preferred, we design the part of the desired potential energy fucntion $P[\cdot]$ as $P[\gamma] = 5\gamma^2$, so that $G_{F\tilde{q}_2}(0) = 0.005$. By setting $c_{d2}(q_1) = c_{d20}$ and $c_{d3}(q_1) = c_{d30}$, we can ensure that the inequality constraints (18), (19) and (20) are satisfied.

Through the simulations, we evaluate the displacements of the main body $\tilde{q}_2$ whose vibration should be attenuated. Figures 5 and 6 show the time responses of the cart displacement in the open loop system and closed-loop systems, when $b = 1$ and 10, respectively. We can see that the vibration of main body is suppressed effectively in the closed-loop system with the proposed controller, while the open-loop system has only a small vibration suppression effect. On the other hand, there is no obvious difference between the performance of skyhook damper method and the one of proposed method.

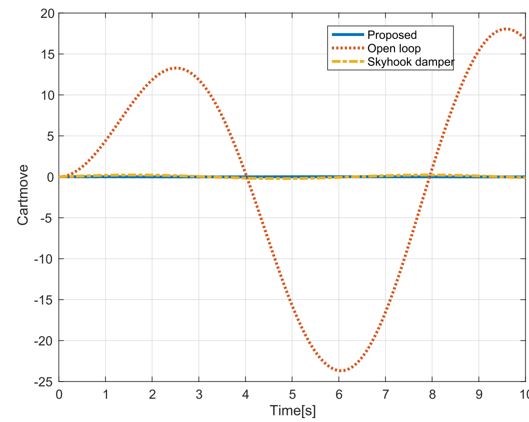


Fig. 5 Time responses of the cart displacement ($b = 1$).

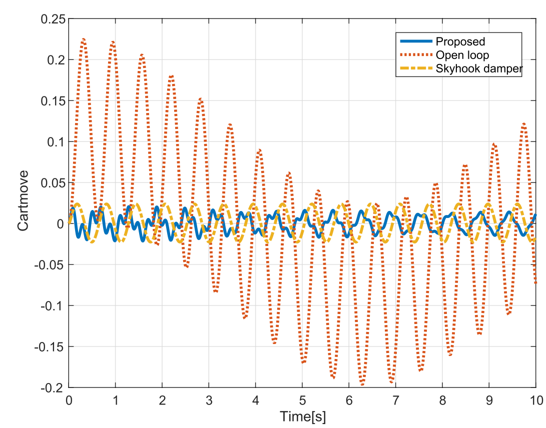


Fig. 6 Time responses of the cart displacement ($b = 10$).

Thus, we confirmed that the proposed method can achieve the same good vibration suppression effect as the skyhook damper method without world-coordinate measurements.

7. Conclusion

In this paper, we have solved vibration suppression problem of the general port-Hamiltonian system via designing IDA-PBC controller. The considered system can be expressed as any floating nonlinear mechanical structure with spring and damper. We have shown the matching condition between the controlled system and the desired system. We show a control law including some free parameters with some constraints. The controller uses only relative information, which can be easily measured. We propose a new parameter design method for more generalized nonlinear controlled objects than the previous work [12]. We show differential equations that determine the inertia matrix of the desired closed-loop system. The stability of the nonlinear closed-loop system is guaranteed theoretically by the IDA-PBC method. We have proposed an efficient parameter selection scheme achieving a good vibration suppression effect. Under the proposed parameter selection, the proposed control law realized a virtual skyhook damper using only relative informations. Simulation results for an example verify the good vibration effect of the proposed controller.

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