PAPER Optical CDMA Scheme Using Generalized Modified Prime Sequence Codes and Extended Bi-Orthogonal Codes****

Kyohei ONO^{†*}, Nonmember, Shoichiro YAMASAKI^{†**}, Senior Member, Shinichiro MIYAZAKI[†], and Tomoko K. MATSUSHIMA^{†***}, Members

SUMMARY Optical code-division multiple-access (CDMA) techniques provide multi-user data transmission services in optical wireless and fiber communication systems. Several signature codes, such as modified prime sequence codes (MPSCs), generalized MPSCs (GMPSCs) and modified pseudo-orthogonal M-sequence sets, have been proposed for synchronous optical CDMA systems. In this paper, a new scheme is proposed for synchronous optical CDMA to increase the number of users and, consequently, to increase the total data rate without increasing the chip rate. The proposed scheme employs a GMPSC and an extended bi-orthogonal code which is a unipolar code generated from a bipolar Walsh code. Comprehensive comparisons between the proposed scheme and several conventional schemes are shown. Moreover, bit error rate performance and energy efficiency of the proposed scheme are evaluated comparing with those of the conventional optical CDMA schemes under atmospheric propagation environment.

key words: optical code division multiple access, generalized modified prime sequence code, extended bi-orthogonal code, multi-user interference, atmospheric propagation

1. Introduction

Recently, optical wireless communications have been gaining attention as a complementary technology to radiofrequency based wireless communications and physical links through cable or optical fiber. Especially, free-space optical (FSO) communications have been attracting growing attention as a solution to connect networks separating for a few miles economically [1], [2].

Code-division multiple-access (CDMA) technique provides multi-user data transmission services not only in radio-frequency based communications but in optical wireless and fiber communications [2], [3]. Several signature codes, such as modified prime sequence codes (MPSCs) [4], [5], generalized MPSCs (GMPSCs) [6], [7] and modified pseudo-orthogonal M-sequence sets (MPOMs) [8]–[11] have been proposed for synchronous optical CDMA systems. Some of these signature codes are also used in combination with code-shift-keying (CSK) techniques for optical communication systems [12]–[14]. Moreover, several

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schemes have been proposed to increase the data rate of optical CSK communication systems. The pseudo-orthogonal extended prime code set (POEPC) [15] is a combination of GMPSCs and bi-orthogonal codes [16], and the extended pseudo-orthogonal M-sequence (EPOM) [17] is a combination of MPOMs and bi-orthogonal codes. The data rates of POEPC and EPOM are higher than those of the conventional optical CSK employing a single signature code, while POEPC and EPOM pay the cost of doubling the chip rate.

In this paper, a new scheme is proposed for synchronous optical CDMA to increase the number of users and, consequently, to increase the total data rate without requiring to increase the chip rate. In the proposed scheme, data are spread with a GMPSC and an extended bi-orthogonal code which is a unipolar code generated from a bipolar Walsh code. It will be shown that the scheme cancels multi-user interference (MUI) completely and achieves error-free performance under an ideal link. The number of multiplexing users of the proposed scheme is larger than that of the conventional scheme using a single GMPSC. Another advantage of this scheme is that the decision distance of the proposed scheme does not decrease. We show comprehensive comparisons between the proposed scheme and several conventional schemes [8]–[18]. Moreover, the bit error rate (BER) performance and energy efficiency of the proposed scheme are evaluated comparing with those of the conventional optical CDMA schemes under atmospheric propagation environment, which is a typical model of FSO links [1], [19].

The remainder of the paper is organized as follows. Section 2 reviews GMPSCs and bi-orthogonal codes, and introduces extended bi-orthogonal codes which are used in the proposed scheme. Then, the coding procedure and system construction of the proposed scheme are presented in Sect. 3. Section 4 shows comprehensive comparisons between the proposed scheme and conventional schemes. In Sect. 5, the BER performance and energy efficiency of the proposed scheme are evaluated comparing with those of the conventional schemes under atmospheric propagation environment. Finally, Sect. 6 offers some concluding remarks.

2. Signature Codes for Synchronous CDMA

2.1 Generalized MPSC

In the optical CDMA system considered in this paper, in-

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[†]The authors are with Polytechnic University, Kodaira-shi, 187-0035 Japan.

^{*}Presently, with Japan Organization for Employment of the Elderly, Persons with Disabilities and Job Seekers.

^{**}Presently, with Hiroshima City University.

^{***}Presently, with Yokohama College of Commerce.

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tensity modulation and direct detection (IM/DD) are employed at the transmitter and the receiver, respectively. Signature codes are expected to be unipolar for optical CDMA employing IM/DD. MPSCs, sometimes called *synchronized prime codes* or *modified prime codes* in the literature, have been proposed for synchronous optical CDMA to increase the number of simultaneous users [4]. An MPSC is a binary code generated from a prime field GF(p) and has p^2 codewords with lengths of p^2 , where p is a prime number. All the codewords have a unique weight value of p. The set of p^2 codewords can be divided into p groups of p codewords. The cross correlation between any two codewords is zero when they belong to the same group. Otherwise, the cross correlation is one.

Generalized MPSCs (GMPSCs) generated from extension fields GF(q) have been proposed, where $q = p^m$, p is a prime number, and m is a positive integer [6], [7]. The code has q^2 codewords with lengths of q^2 and weights of q. These codewords are divided into q groups of q codewords, as with the case of the original MPSCs constructed from prime fields. The class of GMPSCs includes the class of original MPSCs as its subset.

Any GMPSC has the same following correlation property:

$$\Gamma(\mathbf{c}_{i_1,j_1},\mathbf{c}_{i_2,j_2}) = \begin{cases} q, & \text{if } i_1 = i_2 \text{ and } j_1 = j_2, \\ 0, & \text{if } i_1 = i_2 \text{ and } j_1 \neq j_2, \\ 1, & \text{if } i_1 \neq i_2, \end{cases}$$
(1)

where *q* is the size of the finite field, \mathbf{c}_{i_1,j_1} is the j_1 th codeword in the i_1 th group for i_1 , $j_1 = 0, 1, \dots, q-1$, and \mathbf{c}_{i_2,j_2} is the j_2 th codeword in the i_2 th group for i_2 , $j_2 = 0, 1, \dots, q-1$. In Eq. (1), $\Gamma(\mathbf{a}, \mathbf{b})$ is the correlation between two sequences $\mathbf{a} = (a_0, a_1, \dots, a_{n-1})$ and $\mathbf{b} = (b_0, b_1, \dots, b_{n-1})$ with lengths of *n*, and defined by

$$\Gamma(\mathbf{a}, \mathbf{b}) = \sum_{k=0}^{n-1} a_k b_k,$$
(2)

where $a_k \ge 0$ and $b_k \ge 0$. As for GMPSC, *n* is equal to q^2 .

Since the proposed scheme described later employs a GMPSC constructed from $GF(2^m)$, q represents 2^m , where m is an arbitrary positive integer, in the rest of this paper. As an example, a GMPSC with q = 4 is shown in Table 1. This code has sixteen codewords, which are divided into four groups of four codewords.

2.2 Extended Bi-Orthogonal Codes

The proposed scheme, which will be described in Sect. 3, uses an extended bi-orthogonal code as well as a GMPSC. Code-lengths of these two codes are assumed to be the same. An extended bi-orthogonal code with a code-length of q^2 is constructed from a bi-orthogonal code [16], which is generated from a $q \times q$ Hadamard matrix as follows.

A Hadamard matrix H_q is a $q \times q$ matrix, where $q = 2^m$. H_{2^m} is generated by the following recursive algorithm [20]:

Table 1An example of GMPSC and codeword assignment (q = 4).

Group	GMPSC codewords		Assignment
0	c _{0,0}	1000 1000 1000 1000	w _{0,0}
	c _{0,1}	0100 0100 0100 0100	$w_{0,1}$
	c _{0,2}	0010 0010 0010 0010	$w_{1,0}$
	c _{0,3}	0001 0001 0001 0001	$\mathbf{w}_{1,1}$
1	c _{1,0}	1000 0100 0010 0001	W _{2,0}
	c _{1,1}	0100 1000 0001 0010	w _{2,1}
	c _{1,2}	0010 0001 1000 0100	W 3,0
	c _{1,3}	0001 0010 0100 1000	w _{3,1}
2	c _{2,0}	1000 0010 0001 0100	W 4,0
	c _{2,1}	0100 0001 0010 1000	$w_{4,1}$
	c _{2,2}	0010 1000 0100 0001	W 5,0
	c _{2,3}	0001 0100 1000 0010	$w_{5,1}$
3	c _{3,0}	1000 0001 0100 0010	W 6,0
	c _{3,1}	0100 0010 1000 0001	w _{6,1}
	c _{3,2}	0010 0100 0001 1000	W 7,0
	C3 3	0001 1000 0010 0100	W7 1

$$H_2 = \begin{bmatrix} 1 & 1\\ 1 & -1 \end{bmatrix},\tag{3}$$

$$H_{2^{m+1}} = \begin{bmatrix} H_{2^m} & H_{2^m} \\ H_{2^m} & -H_{2^m} \end{bmatrix},$$
(4)

where $m \ge 1$. For example, H_4 is given by

Each row of a Hadamard matrix H_q is a codeword of Walsh code whose code-length is q. Walsh codes are bipolar codes which have perfect orthogonality at zero time delay.

A bi-orthogonal code with a length of q is given by a $2q \times q$ matrix B_q , whose upper half is H_q and lower half is $-H_q$ [16].

$$B_q = \begin{bmatrix} H_q \\ -H_q \end{bmatrix}.$$
 (6)

Each row of B_q specifies a codeword of a bi-orthogonal code whose code-length is q. The number of codewords of a bi-orthogonal code is 2q.

Next, we define an extended bi-orthogonal code with a length of q^2 . Let the *i*th row of B_q be a sequence $\mathbf{b}_i = (b_{i,0}, b_{i,1}, \dots, b_{i,q-1})$, where \mathbf{b}_i is a codeword of the bi-orthogonal code for $i = 0, 1, 2, \dots, 2q - 1$. The *i*th codeword of an extended bi-orthogonal code is $\mathbf{y}_i = (y_{i,0}, y_{i,1}, \dots, y_{i,q^2-1})$ and is specified by

$$y_{i,j} = \begin{cases} 1, & \text{if } b_{i,\lfloor j/q \rfloor} = 1, \\ 0, & \text{if } b_{i,\lfloor j/q \rfloor} = -1, \end{cases}$$
(7)

for $i = 0, 1, 2, \dots, 2q-1$, $j = 0, 1, \dots, q^2 - 1$. In Eq. (7), $\lfloor x \rfloor$ is the maximum integer that is not greater than the positive real number *x*.

The code-length of the extended bi-orthogonal code is $q^2 = 2^{2m}$ and the number of codewords is $2q = 2^{m+1}$. As an example, a bi-orthogonal code and an extended biorthogonal code with q = 4 are shown in Table 2. An example of bi-orthogonal code, extended bi-orthogonal code

Codewords of Codewords of Assignbi-orthogonal code extended bi-orthogonal code ment $\overline{\mathbf{b}}_0$ 1111 1111 1111 1111 1 1 1 \mathbf{y}_0 1111 0000 1111 0000 \mathbf{b}_1 1 -11 -1 **W**8,0 \mathbf{y}_1 \mathbf{b}_2 1 1 -1-1 1111 1111 0000 0000 **W**9.0 \mathbf{y}_2 b₂ 1 -1 -1 1 1111 0000 0000 1111 **w**_{10,0} V₂ \mathbf{b}_{4} 0000 0000 0000 0000 -1 -1 $^{-1}$ -1 \mathbf{y}_4 b₅ -11 $^{-1}$ 1 0000 1111 0000 1111 **y**₅ **W**_{8,1} 0000 0000 1111 1111 \mathbf{b}_6 -1-1 1 1 **W**9,1 **y**₆ h₇ $^{-1}$ 1 1 $^{-1}$ 0000 1111 1111 0000 V₇ **w**_{10,1}

Correlation Properties

and codeword assignment (q = 4).

Table 2

2.3

An extended bi-orthogonal code with a code-length of q^2 has 2q codewords including an all-one sequence \mathbf{y}_0 and an all-zero sequence \mathbf{y}_q . The 2q - 2 codewords excepting \mathbf{y}_0 and \mathbf{y}_q are equal-weight codewords. These 2q-2 codewords have the following correlation property:

$$\Gamma(\mathbf{y}_{k_1}, \mathbf{y}_{k_2}) = \begin{cases} q^2/2, & \text{if } k_1 = k_2, \\ 0, & \text{if } k_1 \le k_2 \text{ and } k_2 - k_1 = q, \\ q^2/4, & \text{if } k_1 \le k_2 \text{ and } k_2 - k_1 \ne q, \end{cases}$$
(8)

for $k_1, k_2 \in \{1, 2, \dots, q-1, q+1, q+2, \dots, 2q-1\}$. Equation (8) shows that the cross correlation between \mathbf{y}_{k_1} and \mathbf{y}_{k_2} is zero when $k_1 + q = k_2$. Otherwise, the cross correlation is $q^2/4$.

In addition, any cross correlation between a GMPSC codeword $\mathbf{c}_{i,j}$ and an extended bi-orthogonal codeword \mathbf{y}_k is always given by

$$\Gamma(\mathbf{c}_{i,j},\mathbf{y}_k) = q/2,\tag{9}$$

for $i, j \in \{0, 1, \dots, q-1\}$ and $k \in \{1, 2, \dots, q-1, q+1, q+2, \dots, 2q-1\}$.

3. Proposed Scheme

We assume that the system is synchronous, transmission data are binary, and equal-weight orthogonal (EWO) signaling is adopted [21], [22]. One advantage of the EWO signaling is that the threshold of the decision circuit at the receiver is fixed to zero, while the threshold for the on-off keying signaling depends on the signal strength and varies with channel conditions. Another advantage of the EWO signaling is that MUI is canceled completely when an MPSC or a GMPSC is used as a signature code [12], [18], [23], [24].

3.1 Coding and Multiplexing

It has been reported that the optical CDMA system using an MPSC or a GMPSC removes background light and MUI simultaneously at the decoder, when background light intensity is constant during a slot and an adequate MUI canceling scheme is adopted [25]. The EWO signaling is one of such schemes which cancel background light and MUI simultaneously.



(a) Sequences transmitted simultaneously



(b) Multiplexed sequence

Fig. 1 Examples of sequences transmitted simultaneously and an example of multiplexed sequence $(q = 4, N_1 = 8, N_2 = 3)$.

In this paper, a *slot* represents a time period during which one codeword consisting of q^2 chips is transmitted, and a *block* represents each part of q chips when one slot is divided into q parts of equal lengths as shown in Fig. 1. Through the discussion in Ref. [25], we have obtained a further result that the background light can be removed at the decoder when its intensity is constant during each block. In Appendix A, we will explain how the decoder removes background light varying by block in the system using a GMPSC. The decoder also cancels optical signals whose intensity varies by block. This result means that any GMPSC decoding is free from interference, even if optical signals encoded by an extended bi-orthogonal code are multiplexed to GMPSC sequences.

On the other hand, the number of mark chips in each block is always one for any GMPSC [6], [7]. This fact means that the decoder of an extended bi-orthogonal code cancels interference caused by the optical signals encoded by a GMPSC. As a result, an extended bi-orthogonal code does not interfere to a GMPSC with the same length, and vice versa.

The above result is utilized in our proposed scheme in order to increase the number of users in optical CDMA. In the proposed scheme, a GMPSC and an extended biorthogonal code with the same code-length of q^2 ($q = 2^m$) are used as signature codes. Although an extended biorthogonal code has $2q = 2^{m+1}$ codewords, the proposed scheme uses the $2^{m+1} - 2$ codewords excepting an all-one and all-zero sequences in the EWO signaling. In general, the number of total users is $N_0 = N_1 + N_2$, where $N_1 = 2^{2m-1}$ for a GMPSC and $N_2 = 2^m - 1$ for an extended bi-orthogonal code. As an example, in the case of q = 4, the code-length is 16, $N_1 = 8$, $N_2 = 3$, and $N_0 = 11$. As another example, in the case of q = 8, the code-length is 64, $N_1 = 32$, $N_2 = 7$, and $N_0 = 39$.



Fig. 2 Model of proposed optical CDMA system $(N_0 = N_1 + N_2)$.

(13)

Table 1 shows the codeword assignment of the EWO signaling for the GMPSC with q = 4. Two codewords assigned to a single user have to be in the same group of the GMPSC. Table 2 shows the codeword assignment of the EWO signaling for the extended bi-orthogonal code with q = 4. Though the number of codewords is eight for this extended bi-orthogonal code, the all-one and all-zero codewords are not used in the EWO signaling, and hence extended bi-orthogonal codewords are assigned to three users in our proposed scheme. In these tables, the two codewords assigned to the *k*th user are represented as \mathbf{w}_{k,l_k} , where I_k ($I_k \in \{0, 1\}$) is a datum to be transmitted by the user, and $0 \le k \le 2^{2m-1} + 2^m - 2$. In general, codeword assignment for the GMPSC in the proposed scheme is specified by

$$\mathbf{w}_{k,0} = \mathbf{c}_{i,2j},\tag{10}$$

$$\mathbf{w}_{k,1} = \mathbf{c}_{i,2\,j+1},\tag{11}$$

for $0 \le k \le 2^{2m-1}-1$, where k = (q/2)i+j, $i = 0, 1, \dots, q-1$, $j = 0, 1, \dots, q/2 - 1$. In addition, codeword assignment for the extended bi-orthogonal code is specified by

$$\mathbf{w}_{k,0} = \mathbf{y}_{k-2^{2m-1}+1},\tag{12}$$

$$\mathbf{w}_{k,1} = \mathbf{y}_{k-2^{2m-1}+q+1},$$

for $2^{2m-1} \le k \le 2^{2m-1} + 2^m - 2$.

Figure 1 illustrates examples of sequences transmitted simultaneously and an example of multiplexed sequence for q = 4, $N_1 = 8$ and $N_2 = 3$. In Fig. 1, (a) shows the sequences transmitted simultaneously by eleven users, and (b) shows the multiplexed sequence of the eleven sequences shown in (a). The GMPSC codewords are assigned to the 0th through 7th users and the extended bi-orthogonal codewords are assigned to the 8th through 10th users.



Fig. 3 Block diagram of EWO decoder.

3.2 System Model and EWO Decoder

In optical wireless communication systems, asynchronous CDMA can only allow a limited number of simultaneous users [5]. On the other hand, synchronous CDMA generally accommodates a larger number of users and provides higher transmission quality at the expense of complexity for synchronization. We suppose such systems as down-links from a base station to subscribers [3] and point-to-point links with multi-channel transmission using CDMA techniques [2] as applications of synchronous optical wireless CDMA systems. In such applications, all the encoders can be implemented in a single transmitter, and the slot timing and signal-intensity control between the encoders can be established easily.

Figure 2 illustrates a model of the optical CDMA system employing the proposed scheme. The N_0 users are divided into two groups of N_1 and N_2 users. Transmission data from each of the N_1 users are encoded at an EWO encoder with GMPSC codewords, and those from each of the N_2 users are encoded at an EWO encoder with extended bi-orthogonal codewords. The sequence generated by each encoder drives each light source, and optical signals from all the light sources are transmitted simultaneously and multiplexed spatially. The multiplexed optical signals are received and converted into electrical signals by a photo-detector (PD) at each receiver. The converted sig-

Scheme	Signature code	CDMA or CSK	Code-length	Number of users N_0	Average light intensity for N_0 users
Proposed	GMPSC and EBOC	CSK-CDMA ^{†1}	2^{2m}	$2^{2m-1} + 2^m - 1$	$(2^{m+1}-1)/2$
Conventional [12], [18]	GMPSC	CSK-CDMA ^{†1}	2^{2m}	2^{2m-1}	2^{m-1}
Conventional	BOC [16]	CSK-CDMA ^{†2}	2 ^m	$2^m - 1$	$(2^m - 1)/2$
Conventional	MPOMs [8]–[11]	CSK-CDMA ^{†2}	2 ^m	$2^m - 1$	$(2^m - 1)/2$
Scheme	Signature code	CDMA or CSK	Code-length	Number of bits transmitted per slot	Average light intensity for N sequences
Conventional [15], [17]	GMPSC and EBOC (POEPC)	CSK, N-CSK ^{†3}	2^{2m+1}	$\log_2 {\binom{2^m}{N}} + \log_2 {\binom{2^{m+1}}{N}}$	$N/2^{m+1}$
Conventional [17]	MPOMs and BOC (EPOM)	CSK, N-CSK ^{†3}	2 ^{<i>m</i>+1}	$\log_2 {\binom{2^m-1}{N}} + \log_2 {\binom{2^m}{N}}$	N/4
Conventional [13]	MPOMs	N-CSK	2 ^m	$\log_2 {\binom{2^m-1}{N}} + N$	N/2
Conventional [14]	GMPSC	VN-CSK	2^{2m}	$\log_2 {\binom{2^m}{N} \times 2^m}$ (channels)	$N/2^m \times 2^m$ (channels)

 Table 3
 Comparisons between the proposed and conventional schemes.

nal sequence is fed to each receiver's EWO decoder, and the decoded data are delivered to each destination.

Figure 3 illustrates a block diagram of the EWO decoder, which has been originally introduced in Ref. [21]. For the EWO scheme, the *k*th user's decoder computes Γ_1 and Γ_0 , which are correlation values between the received sequence **r** and the codewords **w**_{*k*,1} and **w**_{*k*,0}, respectively. Γ_1 and Γ_0 are represented as follows:

$$\Gamma_1 = \Gamma(\mathbf{r}, \mathbf{w}_{k,1}),\tag{14}$$

$$\Gamma_0 = \Gamma(\mathbf{r}, \mathbf{w}_{k,0}). \tag{15}$$

Then the decoder calculates the difference $\Gamma_1 - \Gamma_0$. If the difference is greater than or equal to the threshold *zero*, the decoder outputs $\hat{I}_k = 1$ as the decoded datum. Otherwise, the decoder outputs $\hat{I}_k = 0$. Because the threshold in the EWO decoder is always zero, this scheme cancels MUI without needing to estimate the light intensity of mark chips.

Suppose that the channel is an ideal link where the noise can be considered negligible, and that the received light intensity of each user's mark chip is 1. Let $\Delta(I_k)$ be the value $\Gamma_1 - \Gamma_0$ when the user transmits a datum I_k ($I_k \in \{0, 1\}$). From Eqs. (1), (8) and (9), it is shown that each EWO decoder cancels MUI completely even if two kinds of signature codes are used simultaneously. In Appendix B, we will explain how each decoder cancels MUI in the proposed scheme. Since the EWO decoders eliminate the effect of MUI completely, $\Delta(1)$ and $\Delta(0)$ at each decoder take certain specific values. At the EWO decoder for a GMPSC, $\Delta(1)$ and $\Delta(0)$ are certain to be q and -q, respectively. In contrast, at the EWO decoder for an extended bi-orthogonal code, $\Delta(1)$ and $\Delta(0)$ are certain to be $q^2/2$ and $-q^2/2$, respectively. $|\Delta(1) - \Delta(0)|$ is defined as the *decision distance* [24]. The decision distance of the EWO decoder for the GMPSC is 2q and that for an extended bi-orthogonal code is q^2 .

The proposed scheme can remove not only MUI but also background light varying by slot. In Appendix B, we explain how the decoder removes background light together with MUI in the proposed scheme.

4. Comparisons between the Proposed and Conventional Schemes

In this section, comprehensive comparisons between the proposed scheme and several conventional schemes [8]–[18] are investigated. We compare three conventional CDMA schemes and four conventional CSK schemes with the proposed CDMA scheme. Table 3 demonstrates codelengths, numbers of users, and average light intensities for the CDMA schemes. In addition, it demonstrates codelengths, numbers of bits transmitted per slot, and average light intensities for the conventional CSK (N-CSK) schemes. N-CSK is a scheme in which N codewords are selected from all the codewords depending on a transmission datum and are transmitted simultaneously. CSK is a special case of N-CSK when N = 1.

In Table 3, a bi-orthogonal code is referred to as a BOC, an extended bi-orthogonal code as an EBOC, and a variable N-parallel CSK as a VN-CSK. In addition, the class of GMPSCs includes the MPSC class as a subset. The notations \dagger 1, \dagger 2 and \dagger 3 in Table 3 are described below.

- ^{†1} EWO is a special case of CSK when assigned codewords to each user are equal-weight and orthogonal.
- ^{†2} CSK-CDMA using MPOMs or a BOC is sometimes referred to as *sequence inversion keying* (SIK)-CDMA [11]. Because code-weights of MPOMs and a BOC are half of their code-lengths, SIK has an equivalent meaning to EWO when CSK-CDMA uses these codes.
- ^{†3} N-CSK schemes using POEPC and EPOM have a tradeoff between the data rate and BER, and they bring about significant deterioration of their performances when N increases [17].

It should be noted that all the schemes in Table 3 except for N-CSK using POEPC or EPOM achieve error-free performance under an ideal link where the noise can be considered negligible.

Among the three conventional CSK-CDMA schemes, the scheme with GMPSC has the best performance in energy efficiency because its code-weight is much smaller than that of the other two schemes. However, the number of users of the scheme with GMPSC is limited to 2^{2m-1} , which is a half of the code-length. The proposed scheme can increase the number of users without MUI.

5. Bit Error Rate Performance

In this section, we investigate the BER performance and energy efficiency of the proposed scheme comparing with those of the conventional optical CDMA schemes under atmospheric propagation environment using the log-normal turbulence model [1], [19]. The conventional schemes evaluated in this section are optical CDMA schemes with CSK (EWO) signaling using GMPSCs, bi-orthogonal codes and MPOMs with code-lengths of 16 and 64. Before the evaluation under atmospheric propagation environment, we have confirmed by computer simulation that both of the conventional and proposed schemes achieve error-free performance for an ideal link in which the noise can be considered negligible.

5.1 Atmospheric Propagation Environment

BERs of the proposed and conventional schemes are evaluated by computer simulation considering the effect of scintillation, background light, and avalanche photo diode (APD) noise including shot noise and thermal noise. The scintillation causes the fluctuation of the received light power in the atmospheric optical communications.

The received light power P(t) at the APD can be denoted by $P(t) = X(t)P_s$, where P_s is the received light power without scintillation at the time t [19]. X(t) is the scintillation characterized by the stationary probability process and its probability density function p(X) is described as

$$p(X) = \frac{1}{\sqrt{2\pi\sigma_s^2}X} \exp\left\{-\frac{\left(\ln X + \frac{\sigma_s^2}{2}\right)^2}{2\sigma_s^2}\right\},$$
 (16)

where the average of scintillation X is normalized to unity, and σ_s^2 is logarithm variance of X. The variance σ_s^2 is determined by the atmospheric state.

We consider that the receiver uses APD to convert an optical signal into an electrical signal at every chip duration T_c . The probability that a specified number of photons is absorbed from an incident optical field by an APD detector over a chip duration of T_c is given by a Poisson distribution [26]. The average number of absorbed photons over a chip duration is $T_c\lambda_s$, where λ_s is the photon absorption rate by the signal at the time *t* and given by

$$\lambda_s = \frac{\eta P(t)}{hf}.\tag{17}$$

In Eq. (17), η is the APD quantum efficiency, *h* is Planck's constant, and *f* is the optical frequency.

According to Refs. [26] and [27], we assume that the APD output is a Gaussian random variable. The average μ and variance σ^2 of the APD output are written, respectively, as

 Table 4
 Nominal parameters in system model.

Name	Parameter	Value
APD gain	G	100
Chip duration	T_c	0.5 ns
Background light power	P_b	-45 dBm
Equivalent receiver noise temperature	T_r	1,100 K
Receiver load resistance	R_L	1,030 Ω
Optical wavelength	1/f	540 nm
APD quantum efficiency	η	0.6
APD effective ionization ratio	k_{eff}	0.02
APD bulk leakage current	Ib	0.1 nA
APD surface leakage current	I_s	10 nA
Modulation extinction rate	M_e	100
Logarithm variance of scintillation	σ_s^2	0.1

$$\mu = GT_c(\overline{\lambda_s} + \lambda_b + I_b/e) + I_sT_c/e, \qquad (18)$$

$$\sigma^2 = \sigma_{sh}^2 + \sigma_{th}^2, \tag{19}$$

where G is the average APD gain, $\overline{\lambda_s}$ is the average of λ_s , λ_b is the photon absorption rate due to the background light, I_b is the APD bulk leakage current, e is the elementary charge, I_s is the APD surface leakage current, F_e is the excess noise factor, σ_{sh}^2 is the variance of shot noise, and σ_{th}^2 is the variance of thermal noise. λ_b , F_e , σ_{sh}^2 and σ_{th}^2 are given by

$$\lambda_b = \frac{\eta P_b}{hf},\tag{20}$$

$$F_e = k_{eff}G + (1 - k_{eff})(2G - 1)/G,$$
(21)

$$\sigma_{sh}^2 = G^2 F_e T_c (\overline{\lambda_s} + \lambda_b + I_b/e) + I_s T_c/e, \qquad (22)$$

$$\sigma_{th}^2 = 2k_B T_r T_c / (e^2 R_L), \qquad (23)$$

where P_b is the received background light power, k_{eff} is the APD effective ionization ratio, k_B is Boltzmann's constant, T_r is the equivalent receiver noise temperature, and R_L is the receiver load resistance. The nominal values of the parameters used in our computer simulation are given in Table 4.

5.2 P_w versus BER Performance

In our computer simulation, we assume that the frequency of the scintillation is much lower than the bit rate. We also assume that the received light power without scintillation from every emitting light source is equal. We represent the received light power without scintillation from each emitting light source is represented by P_w . The actual received light power P_s without scintillation is a total power of the optical signals transmitted from all the light sources at each chip.

In this section, the BER performances are evaluated for each user group classified by the following way:

- G1: Group of users using a GMPSC in the proposed scheme
- G2: Group of users using an extended bi-orthogonal code (EBOC) in the proposed scheme
- G3: Group of users in the conventional scheme using a GMPSC
- G4: Group of users in the conventional scheme using a



Fig. 4 P_w versus BER performances (Code-length n = 16).



Fig. 5 P_w versus BER performances (Code-length n = 64).

bi-orthogonal code (BOC)

G5: Group of users in the conventional scheme using MPOMs

Figures 4 and 5 show P_w versus BER performances for G1, G2, G3, G4 and G5. Figure 4 shows the performances when the code-length *n* is 16, and Fig. 5 shows the performances when the code-length *n* is 64. The simulation results show that the group G2, which is the group of users using an extended bi-orthogonal code in the proposed scheme, achieves the best performance, and the group G3, the group of users in the conventional scheme using a GMPSC, achieves the second best among the five groups. The performances of the remaining three groups are almost the same.

As mentioned in Sect. 3.2, the decision distance at the EWO decoder for the GMPSC is $2\sqrt{n}$ and that for an extended bi-orthogonal code is *n* in the proposed scheme when their code-length is *n*. In addition, the decision distances at the EWO decoders for the MPOMs and bi-orthogonal code

are *n* when their code-lengths are *n*. This means that the distances for G2, G4 and G5 are $\sqrt{n}/2$ times as large as those for G1 and G3. If the thermal noise is dominant at the receiver, the BERs for G2, G4 and G5 are lower than those for G1 and G3. However, the variance of APD output is determined by not only the thermal noise but also the shot noise. The shot noise is mainly affected by the signal light intensity, when the background light intensity is low. The signal light intensities for G4 and G5 are the greatest and that for G3 is the least among the five groups. As a result, we obtain the BER performances shown in Figs. 4 and 5.

5.3 E_b versus BER Performance

In this section, we introduce the measure *average received light energy per bit* E_b , and evaluate E_b versus BER performances. Energy per bit is a widely used metric to evaluate energy efficiency in digital communication systems, and it is defined as power per data rate [28], [29]. By using this metric, it is possible to compare the performances of several systems having various data rates.

In optical CDMA systems under consideration, the total data rate is determined by the three parameters, which are the chip duration, the code-length and the number of simultaneous users. In this section, in order to evaluate energy efficiency of the proposed and conventional schemes with different parameters, we define E_b as the ratio of the total average received power to the total data rate as follows:

$$E_{b} = \frac{\text{Total received light power}}{\text{Total data rate}}$$
$$= \frac{N_{\text{user}} P_{w} w_{\text{H}} / n}{N_{\text{user}} / (n T_{c})}$$
$$= w_{\text{H}} P_{w} T_{c}, \qquad (24)$$

where *n* is a code-length, $w_{\rm H}$ is a code-weight, $N_{\rm user}$ is a number of simultaneous users, and T_c is a chip duration. The unit of E_b is joules/bit (J/bit or W·sec/bit), when the units of P_w and T_c are watts (W) and seconds (sec), respectively.

Figures 6 and 7 show E_b versus BER performances for G1, G2, G3, G4 and G5. Figure 6 shows the performances when the code-length n is 16, and Fig. 7 shows the performances when the code-length n is 64. The simulation results show that the group G3, which consists of the users in the conventional scheme using GMPSC, achieves the best performance. They also show that G1 and G2, which consist of the users using GMPSC and extended bi-orthogonal code in the proposed scheme, respectively, achieve the second best among the five groups. The performances of G4 and G5, which consist of the users in the conventional schemes using bi-orthogonal code and MOPMs, respectively, are the worst. In general, the scheme with less code-weight achieves better energy efficiency in optical CDMA systems. Hence, G3 has the best performance in energy efficiency. The performance for the proposed scheme is worse than that for G3 and better than those for G4 and G5. However, the total number of users for the proposed scheme is larger than that for G3.



Fig. 6 E_b versus BER performances (Code-length n = 16).



6. Conclusions

In this paper, we proposed a new optical CDMA scheme using a GMPSC and an extended bi-orthogonal code simultaneously. The proposed scheme accommodates a larger number of users, and consequently achieves higher total data rate without increasing the chip rate comparing with the conventional scheme using GMPSC. We evaluated BER performance and energy efficiency of the proposed scheme comparing with those of the conventional schemes under the atmospheric propagation environment.

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Appendix A: Cancellation of Background Light Varying by Block in the Conventional Scheme Using GMPSC

It has been reported that the EWO scheme cancels background light and MUI simultaneously, when background light intensity is constant during a slot and a GMPSC is employed as a signature code [25]. In this appendix, we show that the EWO decoder in the conventional scheme using a GMPSC removes background light and MUI simultaneously, even if the intensity of background light varies by block.

We consider an optical CDMA employing a GMPSC with $q = 2^m$ and EWO signaling. This scheme is referred to as the conventional scheme in this paper. We suppose a link where other performance-degrading factors than MUI and background light are negligible. We also suppose that intensity of background light is constant during a block, and the intensity at the *i*th block is $L_b^{(i)}$ for $i = 0, 1, \dots, q-1$. The background light is received at PD together with the optical sequences from a desired and interfering users. Figure A·1 shows an example of received signals including background light and users' sequences.

GMPSC codewords are assigned to $q^2/2$ users in total. From Eqs. (1), (14) and (15), the correlation values Γ_1 and Γ_0 calculated at the *k*th user's decoder ($0 \le k \le q^2/2 - 1$) are represented as follows:



Fig. A•1 An example of received signals including background light and users' sequences in the conventional scheme using GMPSC (q = 4).

$$\Gamma_1 = qI_k L_s + (q^2 - q)L_s/2 + \sum_{i=0}^{q-1} L_b^{(i)}, \qquad (A \cdot 1)$$

$$\Gamma_0 = q(1 - I_k)L_s + (q^2 - q)L_s/2 + \sum_{i=0}^{q-1} L_b^{(i)}, \qquad (A \cdot 2)$$

where I_k ($I_k \in \{0, 1\}$) is a datum transmitted, and L_s is the received light intensity of each mark chip. In Eqs. (A·1) and (A·2), $(q^2 - q)L_s/2$ and $\sum_{i=0}^{q-1} L_b^{(i)}$ are the terms caused by MUI and background light, respectively. When the decoder calculates the difference $\Gamma_1 - \Gamma_0$, these two terms are eliminated completely. Thus, the EWO decoder removes background light and MUI simultaneously, even if the intensity of background light varies by block.

Appendix B: Cancellation of MUI and Background Light Varying By Slot in the Proposed Scheme

In this appendix, it is shown that each EWO decoder in the proposed scheme cancels MUI and background light varying by slot, even if two kinds of signature codes are used simultaneously. We suppose a link where the noise can be considered negligible. The received light intensities of mark chips in GMPSC codewords and those in extended biorthogonal codewords are supposed to be L_s . N_1 and N_2 are the numbers of users employing a GMPSC and an extended bi-orthogonal code, respectively. As mentioned in Sect. 3, $N_1 = q^2/2 = 2^{2m-1}$, $N_2 = q - 1 = 2^m - 1$, and $N_0 = N_1 + N_2$. Figure A·2 shows an example of received signals including background light and users' sequences for the proposed scheme.

From Eqs. (1), (8) and (9), the correlation values Γ_1 and Γ_0 calculated at the *k*th user's decoder ($0 \le k \le N_1 - 1$) are represented as follows:

where I_k ($I_k \in \{0, 1\}$) is a datum transmitted. In Eqs. (A·3) and (A·4), the second terms, that is, $(q^2 - q)L_s/2$, are the MUI terms caused by the cross-correlation between codewords in a GMPSC, the third terms, that is, $(q^2 - q)L_s/2$, are the MUI terms caused by the cross-correlation between a GMPSC codeword and extended bi-orthogonal codewords, and the last terms, that is, qL_b , are the terms caused by the



Fig. A \cdot **2** An example of received signals including background light and users' sequences in the proposed scheme (q = 4).

background light. When the decoder calculates the difference $\Gamma_1 - \Gamma_0$, these terms are canceled completely.

On the other hand, the correlation values Γ_1 and Γ_0 calculated at the *k*th user's decoder $(N_1 \le k \le N_0 - 1)$ are represented as follows:

$$\begin{split} \Gamma_1 &= q^2 I_k L_s / 2 + q^3 L_s / 4 + (q^3 - 2q^2) L_s / 4 + q^2 L_b / 2, \quad (A \cdot 5) \\ \Gamma_0 &= q^2 (1 - I_k) L_s / 2 + q^3 L_s / 4 + (q^3 - 2q^2) L_s / 4 + q^2 L_b / 2, \\ (A \cdot 6) \end{split}$$

where I_k ($I_k \in \{0, 1\}$) is a datum transmitted. In Eqs. (A· 5) and (A· 6), the second terms, that is, $q^3L_s/4$, are the MUI terms caused by the cross-correlation between GMPSC codewords and an extended bi-orthogonal codeword, the third terms, that is, $(q^3 - 2q^2)L_s/4$, are the MUI terms caused by the cross-correlation between codewords in an extended bi-orthogonal code, and the last terms, that is, $q^2L_b/2$, are the terms caused by the background light. When the decoder calculates the difference $\Gamma_1 - \Gamma_0$, these terms are canceled completely.

It should be noted that the received light intensities are not required to be equal for N_0 users theoretically in the proposed scheme in order to cancel MUI and the background light. This fact can be explained by a similar discussion to that described in Ref. [25]. It is also noted that background light whose intensity varies by block affects the performance for the N_2 users with an extended bi-orthogonal code, although it does not affect the performance for the N_1 users with a GMPSC.

Thus, the EWO decoder in the proposed scheme removes MUI and background light varying by slot, even if two kinds of signature codes are used simultaneously.



Shoichiro Yamasaki received B.E., M.E. and Dr.Eng. degrees in Electrical Engineering from Keio University, Yokohama, Japan, in 1980, 1982 and 1985, respectively. He was with Toshiba Corporation from 1985 to 2004. He was a Professor at Polytechnic University, Japan, from 2004 to 2021, and is currently an Emeritus Professor there. He is also a Visiting Researcher at Hiroshima City University from 2021. His research interests include coding, signal processing, and security in wireless and mul-

timedia communications. Dr. Yamasaki is a member of the IEEE.



Shinichiro Miyazaki received B.E. degree from Polytechnic University, Kanagawa, Japan, in 1996. He received Dr.E. degree from Yokohama National University, Japan, in 2012. He joined Employment and Human resources Development Organization of Japan, in 1996. He was an assistant since 2005, an assistant professor since 2012, and he has been an associate professor since 2017 at Polytechnic University, Japan. His research interests are wireless communication systems.



Tomoko K. Matsushima received B.E., M.E. and Dr.E. degrees from Waseda University, Tokyo, Japan, in 1985, 1987 and 1999, respectively. She was with Toshiba Corporate R&D Center from 1987 to 1994. She has been with Polytechnic University, Japan, since 1994, and is currently an Emerita Professor. She is also a Professor at Yokohama College of Commerce from 2021. From 2001 to 2002, she was a Visiting Researcher at the University of Hawaii, U.S.A. Her research interests are coding theory

and its applications. She received the 1991 Shinohara Memorial Young Engineer Award, and the 2008 Best Paper Award from IEICE. Dr. Matsushima is a member of the Information Processing Society of Japan and the IEEE.



Kyohei Ono received B.S. degree in Manufacturing Technology from National Institution for Academic Degrees and Quality Enhancement of Higher Education, Tokyo, Japan, in 2019. He was a student of Graduate Course of Master of Science in Manufacturing Engineering at Polytechnic University. He is currently with Japan Organization for Employment of the Elderly, Persons with Disabilities and Job Seekers. His research interests include optical communication systems, secret sharing and physical

layer security.