PAPER

# Complex Frequency Domain Analysis of Memristor Based on Volterra Series

**Qinghua WANG**† ***and*** **Shiying JIA**†a)**,** ***Nonmembers***

**SUMMARY** At present, the application of different types of memristors in electronics is being deeply studied. Given the nonlinearity characterizing memristors, a circuit with memristors cannot be treated by classical circuit analysis. In this paper, memristor is equivalent to a nonlinear dynamic system composed of linear dynamic system and nonlinear static system by Volterra series. The nonlinear transfer function of memristor is derived. In the complex frequency domain, the n-order complex frequency response of memristor is established by multiple Laplace transform, and the response of MLC parallel circuit is taken as an example to verify. Theoretical analysis shows that the complex frequency domain analysis method of memristor transforms the problem of solving nonlinear circuit in time domain into n times complex frequency domain analysis of linear circuit, which provides an idea for nonlinear dynamic system analysis.

***key words:*** *Volterra series, memristor, complex frequency domain analysis, Laplace transform*

## 1. Introduction

The concept of memristor was first proposed by Mr. Cai Shaotang based on the completeness of the combination of basic variables in the circuit [1]. The characteristic of memristor is defined as the relationship between charge and flux. In 2008, with the implementation of memristor devices in HP lab [2], the research and application of memristors have rapidly become the research focus of scholars. At present, there are two aspects in the research of memristor: one is to research and manufacture devices with memristor characteristics; the other is to study its potential applications in various fields. A nanoscale silicon-based memristor device was demonstrated, which could support important synaptic functions such as spike timing dependent plasticity [3]. A drift type three terminal gated memristor device was established, which has the same memory characteristics, current characteristics and hysteresis effect as the two terminal device. The device can be used to realize various novel digital and analog circuits [4]. In the aspect of simulation experiment and device design, many researchers have also focused on SPICE circuit simulation model of memristor [5], [6] and have made some achievements. In application, memristor is used in intelligent computing, secure communication and other fields. Chaos is a special phenomenon widely existing in nonlinear circuits. A novel simple chaotic circuit with a memristor, a memcapacitor and a linear inductor in parallel is proposed. Nineteen types of different chaotic attractors and rich dynamical characteristics are found in the circuit [7]. A chaotic circuit based on the mathematical realistic model of the HP memristor was introduced. The circuit made use of two HP memristors in antiparallel [8]. Some of the chaotic attractors generated by this circuit and the behavior with respect to changes in its component values are described in the circuit. The convolutional neural network based on memristor cross array is reported in reference [9], which successfully realizes the complex computation with low power consumption and low cost in the form of hardware. Memristors are used as synapses in a spiking neural network performing unsupervised learning [10]. The system can retain functionality with extreme variations of various memristors' parameters and adjust to stimuli presented with different coding schemes.

Memristor is a nonlinear element with memory characteristics, and the system is a nonlinear dynamic system. The nonlinear system is described by nonlinear differential equations or nonlinear operators, which does not meet the superposition principle. Therefore, it is necessary to find an effective method to analyze the nonlinear circuit with memristor. Volterra series uses multiple convolutions to describe the input-output relationship of nonlinear dynamic systems. Using this method, the nonlinear transfer function can be derived, which makes it possible to analyze the nonlinear system in frequency domain. Many scholars have made great contributions in this regard. The piecewise Volterra series macromodeling method was proposed for the memristor [11]. This approach that combines the piecewise idea with the Volterra series method aims at significantly reducing the complexity of the traditional Volterra series model. In literature [12], the part other than the power and memristor is regarded as a two port, then the Volterra series was used to the general case of a single memristor which is coupled to an arbitrary linear circuit. The aim was to model each element as a Volterra system. On the basis, the reference [13] is extended to multiple memristor circuits. The literature presented the applicability of the Volterra series paradigm to model the nonlinear dynamics of a class of circuits with ideal generic memristors [14].

Complex frequency domain analysis is an important method for solving high-order complex linear dynamic circuits. The concept of transfer function is introduced by using Fourier transform and Laplace transform. The time domain analysis of linear dynamic circuit is transformed into frequency domain, which provides convenience for ana-

Manuscript received May 13, 2021.
Manuscript revised November 3, 2021.
Manuscript publicized December 17, 2021.
†The authors are with the TYUST, China.
a) E-mail: 18903462536@163.com
DOI: 10.1587/transfun.2021EAP1047

lyzing system characteristics. In this paper, the combination of Volterra series and complex frequency domain analysis method is proposed to describe and analyze the memristor circuit. The purpose is to establish a complex frequency domain analysis method for nonlinear circuits with memristors. According to Volterra series method, memristor elements are first decomposed into equivalent systems composed of linear dynamic systems and nonlinear static systems. Then the nonlinear transfer function of memristor is derived by multiple Laplace transform. The time domain analysis is transformed into the complex frequency domain analysis, and the complex frequency domain response characteristics of memristor are obtained. This method can transform the time domain problem of the nonlinear system into the $n^{th}$-order linear complex frequency domain analysis problem.

## 2. Nonlinear Characteristics of Memristor

Memristor is defined as a two terminal element. If at any time t, the relationship between the charge $q$ and the flux $\psi$ can be determined by a curve on the plane, then the two-terminal element is called memristor, and the functional relationship is as follows:

$$f(q,\psi) = 0 \tag{1}$$

Take the memristor as the fiux-controled as an example, if the relationship between the charge and the flux is linear, the Eq. (1) can be expressed as follows:

$$q = W\psi \tag{2}$$

$W$ is a constant, called memductance. The VCR of memristor is:

$$i(t) = \frac{dq}{dt} = \frac{dq}{d\psi}\frac{d\psi}{dt} = W(\psi)u(t) = Wu(t) \tag{3}$$

At this time, the volt-ampere characteristic of memristor is a straight line passing through the origin. That is, it is a linear resistance. When the charge $q$ and flux $\psi$ are nonlinear, memristor becomes nonlinear. There are many definitions about the mathematical relationship between the charge and the flux of memristor [15]. In this paper, the common third-order nonlinear function is used to express [16] (in Fig. 1).

$$q(\psi) = \alpha\psi + \beta\psi^3 \tag{4}$$

Where $\alpha$, $\beta$ is a constant, this function represents the mathematical model of the nonlinear relationship between the charge $q$ and the flux $\psi$. Then the formula of the memductance is as follows:

$$\begin{aligned} W(\psi) &= \mathrm{d}q(\psi)/\mathrm{d}\psi \\ &= \alpha + 3\beta\psi^2 \\ &= \alpha + 3\beta(\int u(t)dt)^2 \end{aligned} \tag{5}$$

In Eq. (5), the memductance $W(\psi)$ depends on the integration of voltage $u(t)$, indicating that the memristor has

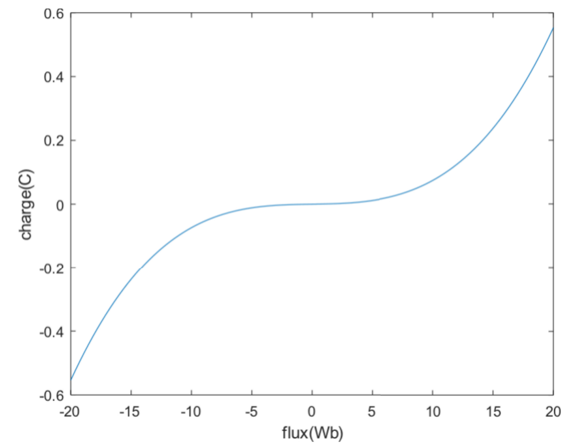



**Fig. 1** Relationship between charge and flux of memristor.

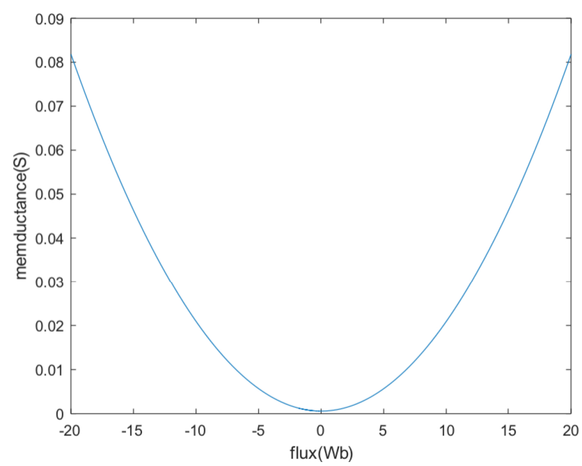



**Fig. 2** The curve between memductance and flux.

memory characteristics. The relation curve between the memductance and the flux is shown in Fig. 2. If the voltage and current of memristor is taken as the associated reference direction, the VCR is obtained.

$$i(t) = W(\psi(t))u(t) = \left(\alpha + 3\beta\psi^2\right)u(t) \tag{6}$$

If a sinusoidal voltage $u = U_m \cos(\omega t + \phi_u)$ is applied to the memristor, the response current can be obtained as follows:

$$\begin{aligned} i =& \left(k_1 + \frac{1}{2}k_2\right)U_m \cos(\omega t + \phi_u) \\ &+\frac{1}{2}k_2 U_m \cos 3(\omega t + \phi_u) \end{aligned} \tag{7}$$

Where $k_1 = \alpha + \frac{3}{2}\beta\left(\frac{U_m}{\omega}\right)^2$, $k_2 = -\frac{3}{2}\beta\left(\frac{U_m}{\omega}\right)^2$. It can be seen from Eq. (7) that when the input signal added to the memristor only contains the fundamental component, the output signal contains not only the fundamental component, but also the second harmonic component. Therefore, for nonlinear systems, the output signal usually produces new harmonic components, and the time domain analysis is very complex.

## 3. Nonlinear Transfer Function of Memristor

The complex frequency domain analysis method based on Laplace transform is used to describe the relationship between input and output of linear system, which simplifies the analysis of linear system. The memristor can be regarded as a nonlinear dynamic system. According to the law of electromagnetic induction, the combination formula (4) takes voltage $u(t)$ as input, current $i(t)$ as output, and the input-output relationship of the memristor is expressed as:

$$\begin{cases} \psi = \int u dt \\ q(\psi) = \alpha\psi + \beta\psi^3 \\ i = dq/dt \end{cases} \tag{8}$$

When the input voltage $\|u(t)\| = [\int_0^\infty u^2(t)dt]^{\frac{1}{2}} < \infty$ is satisfied, the memristor can be cascaded by a linear integrator, a nonlinear static system and a linear differentiator (Fig. 3). The solution of the transfer function of memristor can be transformed into the problem of solving the transfer function of simple nonlinear system and linear system.

The input voltage $u(t)$ is taken as the image function $U(s)$ after Laplace transformation. For subsystem $F_1$, the output flux $\Psi(s)$ in complex frequency domain can be expressed as:

$$\Psi(s) = L[\psi(t)] = L\left[\int u dt\right] = \frac{1}{s}U(s) \tag{9}$$

The subsystem $F_2$ is a nonlinear static system. The out-put charge $q$ by Volterra series expansion is expressed as:

$$\begin{aligned} q(t) &= q_0(t) + q_1(t) + q_2(t) + \ldots + q_n(t) + \ldots \\ &= \sum_{n=0}^{\infty} q_n(t) \end{aligned} \tag{10}$$

Where, $q_n(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_n(\tau_1,\tau_2,\cdots,\tau_n)\psi(t- -\tau_1)$ $\psi(t-\tau_2)\cdots\psi(t-\tau_n)d\tau_1 d\tau_2\cdots d\tau_n$, $h_n(\tau_1,\tau_2,\cdots,\tau_n)$ is the kernel function of Volterra series, which can also be called n-order impulse response of subsystem $F_2$. In subsystem $F_2$, the relation (4) can be regarded as the first three terms of power series expanded at $\psi_0 = 0$ Therefore, when $q_0(t) = 0$ in formula (10), the first three terms of kernel function of Volterra series take multidimensional Dirac function (as formula 11)) [17], and the kernel function of other terms is zero.

$$\begin{cases} h_1(\tau_1) = \alpha\delta(\tau_1) \\ h_2(\tau_1,\tau_2) = 0 \\ h_3(\tau_1,\tau_2,\tau_3) = \beta\delta(\tau_1)\delta(\tau_2)\delta(\tau_3) \end{cases} \tag{11}$$

The Eq. (10) can be expressed as:

$$\begin{aligned} q(t) &= \int_{-\infty}^{\infty} \alpha\delta(\tau_1)\psi(t-\tau_1)d\tau_1 \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \beta\delta(\tau_1)\delta(\tau_2)\delta(\tau_3)\prod_{i=1}^{3}\psi(t-\tau_i)d\tau_i \\ &= \alpha\psi(t) + \beta\psi^3(t) \end{aligned} \tag{12}$$

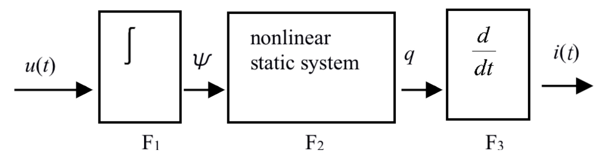



**Fig. 3** Memristor Cascade System.

Obviously, the subsystem $F_2$ can be represented by Volterra series. Therefore, the n-fold Laplace transformation of n-order output charge of subsystem $F_2$ is:

$$\begin{aligned} Q_n(s_1,s_2,\ldots,s_n) &= L[q(t)] \\ &= H_n(s_1,s_2,\ldots,s_n)\prod_{i=1}^{n}\Psi(s_i) \end{aligned} \tag{13}$$

After $n$-fold Laplace transformation, the n-order transfer function of subsystem $F_2$ is $H_n(s_1,s_2,\ldots,s_n)$, Therefore, by taking multiple Laplace transforms in Eq. (11), the frequency domain kernel of Volterra series of memristor is:

$$\begin{cases} H_1(s_1) = \alpha \\ H_2(s_1,s_2) = 0 \\ H_3(s_1,s_2,s_3) = \beta \end{cases} \tag{14}$$

The input of subsystem $F_3$ is charge $q$ and the output is current $i(t)$. In the zero state, the output $I(s)$ obtained by Laplace transformation can be expressed as:

$$I(s) = L[i(t)] = L\left[\frac{dq}{dt}\right] = sQ(s) \tag{15}$$

According to the cascade connection mode in Fig. 3, subsystems $F_1$ and $F_3$ are linear systems with only one order term. In the complex frequency domain, the transfer function of memristor is obtained.
First order term: $Y_1(s_1) = H_1(s_1)\times\frac{1}{s_1}\times s_1 = \alpha$
Second order term: $Y_2(s_1,s_2) = 0$
Third order term:

$$\begin{aligned} Y_3(s_1,s_2,s_3) &= H_3(s_1,s_2,s_3)\times\frac{1}{s_1}\times\frac{1}{s_2}\times\frac{1}{s_3} \\ &\quad\times(s_1+s_2+s_3) \\ &= \beta(s_1+s_2+s_3)/(s_1 s_2 s_3) \end{aligned}$$

Since the input signal is voltage $U(s)$ and the output signal is current $I(s)$, the transfer function of memristor element is also called nonlinear operational admittance, expressed as Eq. (16)

$$\begin{cases} Y_1(s_1) = \alpha \\ Y_2(s_1,s_2) = 0 \\ Y_3(s_1,s_2,s_3) = \beta(s_1+s_2+s_3)/(s_1 s_2 s_3) \end{cases} \tag{16}$$

## 4. Complex Frequency Response Analysis of Memristor

When analyzing the complex frequency domain response of memristor by using the operation method of linearsystem, the input signal can be transformed into the corresponding image function. Combining with the nonlinear operational admittance in the previous section, the problem can be transformed into an n-order linear equation with the image function as the variable. After the complex frequency-domain response is obtained, the inverse Laplace transform is performed to return to the time domain. Let the input signal be a sinusoidal voltage $u(t) = U_m \cos(\omega t + \phi_u)$. According to Euler's formula, the sinusoidal voltage is expressed as a complex exponential function (as Eq. (17))

$$\begin{aligned} u(t) = & \frac{U_m}{2}[e^{j(\omega t+\phi_u)} + e^{-j(\omega t+\phi_u)}]\frac{U_m}{2}e^{j\phi_u}e^{j\omega t} \\ & +\frac{U_m}{2}e^{-j\phi_u}e^{-j\omega t} \end{aligned} \tag{17}$$

Where $A_1 = \frac{U_m}{2}e^{j\phi_u}, A_2 = \frac{U_m}{2}e^{-j\phi_u}$, $\omega_1 = \omega$, $\omega_2 = -\omega$, $A_1$, $A_2$ is obviously a pair of conjugate complex numbers. Then the sinusoidal voltage can be expressed as:

$$u(t) = \sum_{l=1}^{2} A_l e^{j\omega_l t} \tag{18}$$

Taking Laplace transform, the image function is obtained as follows:

$$U_s(s) = A_1\frac{1}{s-j\omega_1} + A_2\frac{1}{s-j\omega_2} \tag{19}$$

For weakly nonlinear systems, the higher-order term of Volterra series decays rapidly, and the requirements can be satisfied only by calculating the second or third-order transfer function [18]. In this section, considering the nonlinear admittance of memristor, Volterra series of output signal current expansion is calculated by tak-ing the third order.
First order output term:

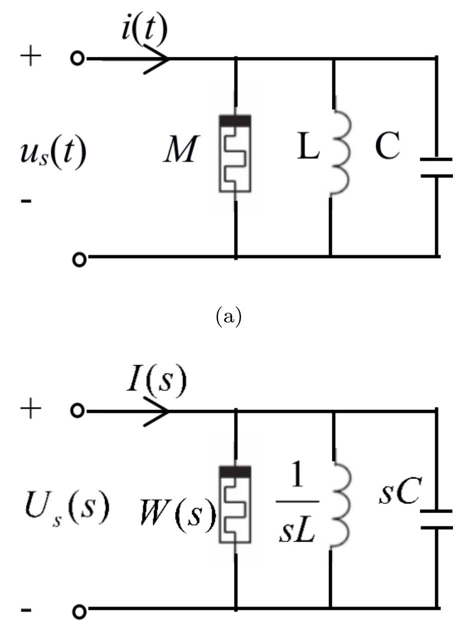
$$I_1(s_1) = Y_1(s_1)U_s(s_1) = \alpha(A_1\frac{1}{s_1-j\omega_1} + A_2\frac{1}{s_1-j\omega_2})$$

Second order output term:

$$I_2(s_1,s_2) = Y_2(s_1,s_2)U_s(s_1)U_s(s_2) = 0$$

Third order output term:

$$\begin{aligned} I_3(s_1,s_2,s_3) = & Y_3(s_1,s_2,s_3)U_s(s_1)U_s(s_2)U_s(s_3) \\ = & Y_3(s_1,s_2,s_3)(A_1\frac{1}{s_1-j\omega_1} + A_2\frac{1}{s_1-j\omega_2}) \\ & (A_1\frac{1}{s_2-j\omega_1} + A_2\frac{1}{s_2-j\omega_2})(A_1\frac{1}{s_3-j\omega_1} \\ & +A_2\frac{1}{s_3-j\omega_2}) \\ = & Y_3(s_1,s_2,s_3) \\ & \{A_1^3\frac{1}{(s_1-j\omega_1)(s_2-j\omega_1)(s_3-j\omega_1)} \\ & +A_1^2A_2[\frac{1}{(s_1-j\omega_1)(s_2-j\omega_1)(s_3-j\omega_2)} \\ & +\frac{1}{(s_1-j\omega_1)(s_2-j\omega_2)(s_3-j\omega_1)} \\ & +\frac{1}{(s_1-j\omega_2)(s_2-j\omega_1)(s_3-j\omega_1)}] \\ & +A_1A_2^2[\frac{1}{(s_1-j\omega_1)(s_2-j\omega_2)(s_3-j\omega_2)} \\ & +\frac{1}{(s_1-j\omega_2)(s_2-j\omega_1)(s_3-j\omega_2)} \\ & +\frac{1}{(s_1-j\omega_2)(s_2-j\omega_2)(s_3-j\omega_1)}] \\ & +A_2^3\frac{1}{(s_1-j\omega_2)(s_2-j\omega_2)(s_3-j\omega_2)}\} \end{aligned}$$




(a)

(b)

**Fig. 4** Memristor inductor and capacitor parallel circuit.

The time domain forms of the output current can be obtained by using the inverse Laplace transform.
Corresponding first-order output in time domain:

$$i_1(t) = \alpha(A_1e^{j\omega_1 t} + A_2e^{j\omega_2 t}) = \alpha U_m\cos(\omega t + \phi_u)$$

Corresponding second-order output in time domain:

$$i_2(t) = 0$$

Corresponding third-order output in time domain:

$$\begin{aligned} i_3(t) = & Y_3(s_1,s_2,s_3)\big|_{s_1,s_2,s_3=j\omega_1} A_1^3e^{j3\omega_1 t} \\ & +Y_3(s_1,s_2,s_3)\big|_{s_1,s_2,s_3=j\omega_2} A_2^3e^{j3\omega_2 t} \\ & +A_1^2A_2(Y_3(s_1,s_2,s_3)\big|_{s_1,s_2=j\omega_1,s_3=j\omega_2} e^{j(2\omega_1+\omega_2)t} \\ & +Y_3(s_1,s_2,s_3)\big|_{s_1=j\omega_2,s_2,s_3=j\omega_1} e^{j(2\omega_1+\omega_2)t}) \\ & +A_1A_2^2(Y_3(s_1,s_2,s_3)\big|_{s_1=j\omega_1,s_2,s_3=j\omega_2} e^{j(\omega_1+2\omega_2)t} \\ & +Y_3(s_1,s_2,s_3)\big|_{s_1,s_3=j\omega_2,s_2=j\omega_1} e^{j(\omega_1+2\omega_2)t} \end{aligned}$$

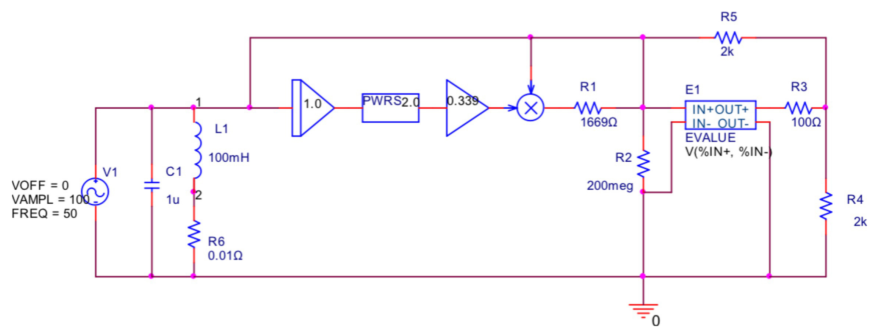



**Fig. 5** Pspice simulation circuit of MLC parallel circuit.

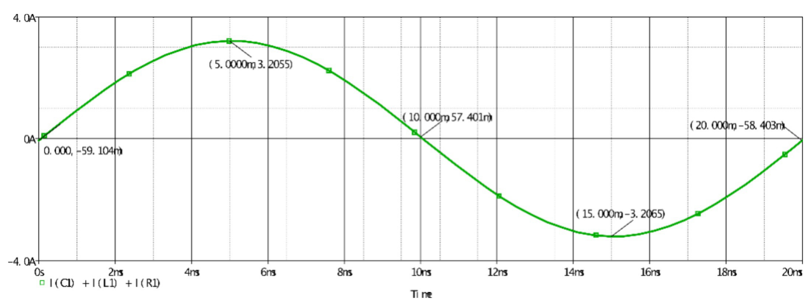



**Fig. 6** MLC parallel circuit response current curve.

$$
\begin{aligned}
&+Y_3(s_1,s_2,s_3)\left|_{s_1,s_2=j\omega_2,s_3=j\omega_1}\right. e^{j(\omega_1+2\omega_2)t})] \\
=\ &\frac{3\beta}{-4}(\frac{U_m}{\omega})^2 U_m \cos 3(\omega t+\phi_u) \\
&+\frac{3\beta}{4}(\frac{U_m}{\omega})^2 U_m \cos(\omega t+\phi_u)
\end{aligned}
$$

Finally, the time domain response of the current is obtained as follows:

$$
\begin{aligned}
i(t) &= i_1(t)+i_2(t)+i_3(t) \\
&= (\alpha+\frac{3\beta}{4}(\frac{U_m}{\omega})^2)U_m \cos(\omega t+\phi_u) \\
&\quad -\frac{3\beta}{4}(\frac{U_m}{\omega})^2 U_m \cos 3(\omega t+\phi_u)
\end{aligned}
\tag{20}
$$

It can be seen from Eq. (20) that the results obtained by the complex frequency domain analysis method of nonlinear systems are consistent with the results of Eq. (7) of time domain analysis, which proves that this method is feasible. In this process, Volterra series and multiple Laplace transform are applied to transform the time domain analysis of non-linear system to n-order linear frequency domain analysis in complex frequency domain, which simplifies the analysis pro-cess.

## 5. Application Example

Taking the parallel circuit of memristor, linear inductor and linear capacitor (MLC) as an example, this circuit is a nonlinear circuit (Fig. 4(a)), and the characteristics of each element are as follows:

$$
\begin{cases}
L : i = L\psi \\
C : q = Cu \\
M : q(\psi) = \alpha\psi + \beta\psi^3
\end{cases}
$$

Assuming that the initial state of each element is zero, the input signal $U_s(t) = 100\cos(314t)\,V$, the corresponding image function is $U_s(s) = a_1\frac{1}{s-j\omega_1} + a_2\frac{1}{s-j\omega_2}$, where, $a_1 = 50e^{j0^\circ}$, $a_2 = 50e^{-j0^\circ}$, $\omega_1 = -\omega_2 = 314rad/s$, L=100 mH, C=1 $\mu$F. The operation circuit is shown in Fig. 4(b). Because the inductance and capacitance are linear elements, there is only one order admittance and the higher order admittance is zero, so the equivalent admittance of each order of the parallel circuit is:

$$
\begin{cases}
Y_1(s_1) = \alpha + s_1 C + 1/s_1 L \\
Y_2(s_1,s_2) = 0 \\
Y_3(s_1,s_2,s_3) = \beta(s_1+s_2+s_3)/(s_1 s_2 s_3)
\end{cases}
$$

The current of each order can be calculated from the excitation voltage and admittance.
First order term:

$$
\begin{aligned}
I_1(s_1) &= Y_1(s_1)\,U_s(s_1) \\
&= (\alpha + s_1 C + 1/s_1 L)\left(a_1\frac{1}{s_1-j\omega_1} + a_2\frac{1}{s_1-j\omega_2}\right)
\end{aligned}
$$

Second order term:

$$
I_2(s_1,s_2) = Y_2(s_1,s_2)U_s(s_1)U_s(s_2) = 0
$$

Third order output term:

$$
I_3(s_1,s_2,s_3) = Y_3(s_1,s_2,s_3)\,U_s(s_1)\,U_s(s_2)\,U_s(s_3)
$$

The response current in time domain can be obtained by multiple Laplace Inverse Transform and bringing the parameters

**Table 1** Comparison of current calculation value and simulation value.

| Time (ms) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|
| caculation value (A) | −0.061 | 1.81 | 2.99 | 3.03 | 1.91 | 0.066 | -1.81 | −2.99 | −3.03 | −1.92 |
| simulation value (A) | −0.059 | 2.24 | 3.03 | 3.06 | 1.93 | 0.057 | −1.84 | −3.03 | −3.07 | −1.93 |
| error | −0.002 | −0.43 | −0.04 | −0.03 | −0.02 | 0.009 | 0.03 | 0.04 | 0.04 | 0.01 |

into time domain.

$$\begin{aligned} i\left(t\right) &= L^{-1}\left[I_1\left(s_1\right) + I_2\left(s_1, s_2\right) + I_3\left(s_1, s_2, s_3\right)\right] \\ &= \left(3.166\cos\left(314t - 1.59\right) + 0.5\times 10^{-3}\cos\left(314t\right) \right. \\ &\quad \left. -0.5\times 10^{-3}\cos\left(942\text{t}\right)\right) A \end{aligned}$$

Figure 5 shows the Pspice simulation circuit of MLC parallel circuit, in which the memristor circuit is realized by the integrator, multiplier, square operation, controlled voltage source and resistance in ABM (analog behavioral modeling) library. In order to avoid short circuit and not affect the performance of the circuit, a small resistance is connected in series on the inductor $L_1$ branch. Setting the circuit simulation type as transient analysis, the response current $i(t)$ of the parallel circuit can be obtained as the curve in Fig. 6. The figure indicates the corresponding current values at 0 ms, 5 ms, 10 ms, 15 ms and 20 ms respectively.

Table 1 lists the current calculation values and simulation values of 10 points between 0~20 ms. It can be seen that the errors are between $(-0.5, 0.5)$. The error indicates that the complex frequency domain analysis method based on Volterra series for nonlinear system is feasible. It can be seen from the above examples that the complex frequency domain analysis method of nonlinear circuit based on Volterra series and multiple Laplace transform transforms the nonlinear analysis in time domain into the analysis of n-order linear circuit in complex frequency domain, which simplifies the analysis process and provides ideas for complex nonlinear circuit analysis.

## 6. Conclusion

With the emergence of more and more new nonlinear electronic devices, the nonlinear phenomena in electronic circuits are difficult to explain by traditional circuit theory. Taking nonlinear memristor as an example, the nonlinear transfer function of memristor is derived by volterra series and multiple Laplace transform. The time domain problem is transformed into complex frequency domain, and the complex frequency domain analysis method of nonlinear circuit is obtained. The characteristics of this method are: (1) Although the third-order nonlinear function of memristor is derived as an example, the method can be extended to the circuit analysis of a class of memristor whose memristor derivative is polynomial $W(\psi) = \sum_{k=1}^{\infty} m_k \psi^k$; (2) If the characteristics of nonlinear components can be expanded by Volterra series, the nonlinear transfer function can be obtained, then the time domain problem of nonlinear circuit can be transformed into the frequency domain problem of n-order linear circuit in complex frequency domain; (3) The nonlinear differential equation in time domain can be transformed into n-order linear algebraic equation in complex frequency domain. The nonlinear transfer function and frequency response of any order can be obtained, which simplifies the analysis of nonlinear system. This transformation method can be applied to solve high-order complex nonlinear systems and become an effective method to analyze nonlinear dynamic circuit systems.

## Acknowledgments

This research was funded by the major project of the Ministry of Science and Technology of Shanxi province, grant number 20191102009 and funded by J2020201.

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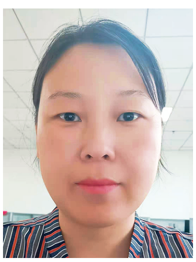

**Qinghua Wang** received the D.E. degrees in mechanical engineering from Taiyuan University of science and technology in 2017. During 2002–2020, she worked in the College of electronic information engineering, Taiyuan University of science and technology.

**Shiying Jia** studied master's degree in control theory and control engineering from Taiyuan University of science and technology from 2019.