

# Spatial Vectors Effective for Nakagami- $m$ Fading MIMO Channels

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**SUMMARY** This letter proposes a transmission scheme called spatial vector (SV), which is effective for Nakagami- $m$  fading multiple-input multiple-output channels. First, the analytical error rate of SV is derived for Nakagami- $m$  fading MIMO channels. Next, an example of SV called integer SV (ISV) is introduced. The error performance was evaluated over Nakagami- $m$  fading from  $m = 1$  to  $m = 50$  and compared with spatial modulation (SM), enhanced SM, and quadrature SM. The results show that for  $m > 1$ , ISV outperforms the SM schemes and is robust to  $m$  variations. **key words:** *spatial vector, millimeter wave, MIMO, Nakagami- $m$  fading, Rician fading*

## 1. Introduction

Fifth generation (5G) and beyond wireless networks use millimeter-wave (mmWave) frequencies. Large-scale path loss at mmWave frequencies is well known. In contrast, small-scale propagation behaviors at mmWave frequencies have only recently been reported by [1] and [2]. For directional antennas, Rappaport et al. showed that at 73 GHz in urban micro-cell environments, fading in both line-of-sight (LOS) and non-LOS (NLOS) environments follows Rician distributions. The Rice  $K$  factor varies from 7 to 17 dB for LOS channels and from 9 to 21 dB for NLOS channels. This depends on the receiver orientation and the environment, where the  $K$  factor denotes the ratio of the power of the dominant component to the total power of the scattered components [1]. Similarly, at 60 GHz in small-cell networks, Yoo et al. revealed that the Rician fading model corresponds to the measurement of data on both LOS and NLOS channels, and the  $K$  factor changes from approximately 5 to 10 dB depending on the position of the user equipment, such as head, hands, and pockets [2].

Spatial modulation (SM) and its derivatives, enhanced SM (ESM) and quadrature SM (QSM) are multiple-input multiple-output (MIMO) communication technologies used in 5G networks [3]–[6]. SM only activates a single antenna, and thereafter, transmits information using the active antenna index in conjunction with conventional signal constellations. Activating only a single antenna reduces interference between channels, synchronization requirements between an-

tennas, and receiver complexity. In addition, the use of a transmitter allows the use of a single radio-frequency (RF) chain [3]. ESM is an improved SM that uses multiple active transmitter antennas and multiple signal constellations for Rayleigh fading channels. It conveys information bits not only via the index of the active antennas but also via the constellation of the transmitted signals [4]. QSM separately transmits the real and imaginary parts of a data symbol. QSM outperforms SM, while maintaining a single RF chain over Rayleigh fading channels [5]. However, increasing  $K$ , that is, increasing  $m$  of Nakagami- $m$  fading, degrades the error performance of these SM methods [7].

This letter proposes a transmission scheme called spatial vector (SV), which is robust to  $m$  variations in Nakagami- $m$  fading MIMO channels. Similar to SM, SV sends different information for each symbol time but does not carry information using the active antenna index. The analytical error rate of SV is derived for Nakagami- $m$  fading channels. In addition, as an example of SV, integer SV (ISV) is introduced. The bit error rate (BER) of ISV over Nakagami- $m$  fading channels is compared with those of SM, ESM, and QSM. The remainder of this letter is organized as follows. In Sect. 2, SV is defined and its approximate analysis error rate over Nakagami- $m$  fading MIMO channels is derived. In Sect. 3, ISV is introduced, and the error performance is evaluated. In Sect. 4, the BER of ISV is compared to that of three different SM methods. This letter is concluded in Sect. 5.

## 2. Spatial Vector

We consider a MIMO system using SV with  $M_t$  transmit and  $M_r$  receive antennas. The  $M_r \times 1$  received signal vector  $\mathbf{Y}$  is expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{V} + \mathbf{N}, \quad (1)$$

where  $\mathbf{V}$  is an  $M_t \times 1$  SV codeword,  $\mathbf{N}$  is an  $M_r \times 1$  complex white noise vector with independent and identically distributed (i.i.d.) entries, and  $\mathbf{H}$  is an  $M_r \times M_t$  channel matrix. The codeword  $\mathbf{V}$  is represented as

$$\mathbf{V}_g = \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_{M_t-1} \end{bmatrix} = \begin{bmatrix} \mu_0\{f_0(g)\} \\ \mu_1\{f_1(g)\} \\ \vdots \\ \mu_{M_t-1}\{f_{M_t-1}(g)\} \end{bmatrix}, \quad (2)$$

Manuscript received December 9, 2020.

Manuscript revised June 3, 2021.

Manuscript publicized August 3, 2021.

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DOI: 10.1587/transfun.2021TAL0001

where  $g$  denotes an integer that is  $0 \leq g < M$ , which is determined by  $\log_2 M$  information bits. In this study, the value of  $M$  is set to the number of signal points mapped by  $\mu$ , for example,  $M$ -QAM signal constellation. Other cases will be considered in the future.  $f_i(g)$  ( $0 \leq i < M_t$ ) denotes a function of  $g$ , and the  $\mu_i\{\cdot\}$  term denotes a mapping from  $f_i(g)$  to a signal point on a signal constellation.

$f_i(g)$  and  $\mu_i\{\cdot\}$  should be selected so as to satisfy  $d = \sum_{i=0}^{M_t-1} |v_i - v'_i|^2 \neq 0$  for any pairwise codewords  $\mathbf{V}$  and  $\mathbf{V}'$ . In addition,  $d_{com} = |\sum_{i=0}^{M_t-1} v_i - \sum_{i=0}^{M_t-1} v'_i|^2 \neq 0$  must be satisfied in order to be decodable for Nakagami- $m$  fading with a large  $m$ . The main difference between SV and spatial multiplexing [8] is that SV transmits information using multiple antennas, whereas spatial multiplexing sends different information from each antenna.

Next, we consider the error performance of SV for Nakagami- $m$  fading MIMO channels using the following approximation [9]:

$$m = \frac{(1+K)^2}{1+2K}. \quad (3)$$

The error performance of SV needs to be calculated for the distance between each pairwise codeword, taking into account  $\mu_i$  and  $f_i(g)$ . Therefore, we start with the BER expression of binary phase-shift keying (BPSK), which can directly indicate the distance between two signal points.

In [10], Yu et al. provided an approximate BER  $P_{b,p}$  for PSK space-time block code (STBC) in Rician fading channels with perfect channel state information (CSI) at the receiver for a low signal-to-noise (SNR) [10, Eq. (24)].  $P_{b,p}$  for BPSK is given by

$$P_{b,p} \simeq \frac{\Gamma(mM_tM_r + 1/2)}{2\Gamma(mM_tM_r + 1)} \frac{\sqrt{\alpha/\pi}}{(1+\alpha)^{mM_tM_r+1/2}} \times {}_2F_1\left(1, mM_tM_r + 1/2; mM_tM_r + 1; \frac{1}{1+\alpha}\right), \quad (4)$$

where  $\Gamma(\cdot)$  is the gamma function,  ${}_2F_1(a, b; c; d)$  is the Gauss hypergeometric function [11],  $\rho$  is the average SNR per received antenna,  $r$  is the code rate of STBC, and  $\alpha = (1+K)\rho \sin^2(\pi/2)/(rmM_t) = (1+K)\rho/(rmM_t)$ .

The squared Euclidean distance between two symbols on BPSK is represented as  $4 \sin^2(\pi/2)$ . Thus, this term can be replaced by the squared distance  $d$  between  $\mathbf{V}_g$  and  $\mathbf{V}_{g'}$  of SV and  $r = 1$  in SV. Therefore,  $\alpha$  in Eq. (4) for SV becomes  $(1+K)\rho d/(4mM_t)$ .

Next, we consider the diversity gain of SV. Equation (4) shows that the diversity gain of STBC is  $mM_tM_r$ . SV does not transmit mutually orthogonal sequences from each antenna. Therefore, SV does not provide space diversity gain  $M_t$  or spatial multiplexing [8, p.196].

In addition, Zhao et al. revealed that Rayleigh fading and Nakagami- $m$  fading MIMO channels have equivalent diversity gains when the block length of the codeword is only one [12]. Rayleigh fading is equivalent to Nakagami- $m$  fading

with  $m = 1$ . Thus, the diversity gain of SV for Nakagami- $m$  fading is not multiplied by  $m$ . From the information above, the total diversity gain of SV is  $M_r$ . SV provides a large Euclidean distance at the expense of the advantages of conventional STBC. Because of this, SV provides an effective property for Nakagami- $m$  fading channels, as shown in the following sections.

As a result, an approximate pairwise error probability (PEP) between  $\mathbf{V}_g$  and  $\mathbf{V}_{g'}$  of SV for a Nakagami- $m$  fading channel is given by

$$P(\mathbf{V}_g \rightarrow \mathbf{V}_{g'}) \simeq \frac{\Gamma(M_r + 1/2)}{2\Gamma(M_r + 1)} \frac{\sqrt{c/\pi}}{(1+c)^{M_r+1/2}} \times {}_2F_1\left(1, M_r + 1/2; M_r + 1, \frac{1}{1+c}\right), \quad (5)$$

where  $c = (1+K)\rho d/(4mM_t)$ . The sum of PEPs between a codeword and other codewords provides the union bound of symbol error rate (SER)  $P_e$  as follows:

$$P_e \leq \frac{1}{M} \sum_{i=0}^{M-1} \sum_{j=0, j \neq i}^{M-1} P(\mathbf{V}_i \rightarrow \mathbf{V}_j). \quad (6)$$

In the following sections,  $P_e$  is called the average SER.

The shape of a Nakagami- $m$  distribution with  $m > 1$  is similar to that of a Rice distribution, except for the distribution slope, the amplitude of which is close to zero. The difference between the low-amplitude slope of the Nakagami- $m$  distribution and that of the Rice distribution affects the error probability, especially for high SNRs. Thus, Yu et al. applied [10, Eq. (24)] using the Nakagami- $m$  distribution to Rician fading by limiting to low SNR. In this study, we use the Nakagami- $m$  distribution directly to determine the PEP of the proposed SV for Nakagami- $m$  fading channels. Thus, the derived PEP is not limited to low SNR for SV. However, the SER of SV can only be given by the union upper bound. Therefore, especially for low SNR, the SER is different from the true value.

### 3. Integer Spatial Vector

In this section, an example of SV called ISV is provided. The derived average SER is shown to be almost correct for ISV. The function  $f_i(g)$  of ISV is defined as  $f_i(g) = \beta_i \cdot g$ , which is calculated using a multiplication operation over  $Z_M$ , where  $Z_M$  denotes the ring of integer modulo  $M$ . In this study, ISV uses a natural mapping  $\mu\{\cdot\}$  for all  $\mu_i\{\cdot\}$ , which is applied between the elements of  $Z_M$  and the signal points on  $M$ -QAM. An example of natural mapping  $\mu$  for 16-QAM is shown in Fig. 1, where numbers from 0 to 15 are sequentially mapped from the lower-left symbol to the upper-right symbol. Similarly, the natural mapping  $\mu'$  for 64-QAM is decided as shown in Fig. 2. In [13], the author revealed that the combination of natural mapping and STBC over  $Z_M$  provides good performance. Therefore, this study also uses natural mapping and  $Z_M$ .

For ISV of  $[\beta_0, \beta_1] = [1, 6]$  with 16-QAM and  $M_t = 2$ ,

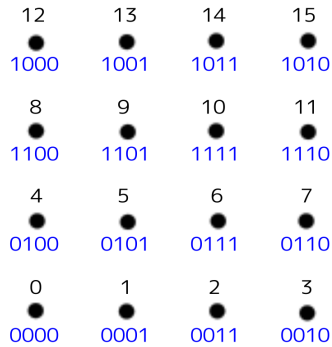


Fig. 1 Natural mapping  $\mu$  for 16-QAM.

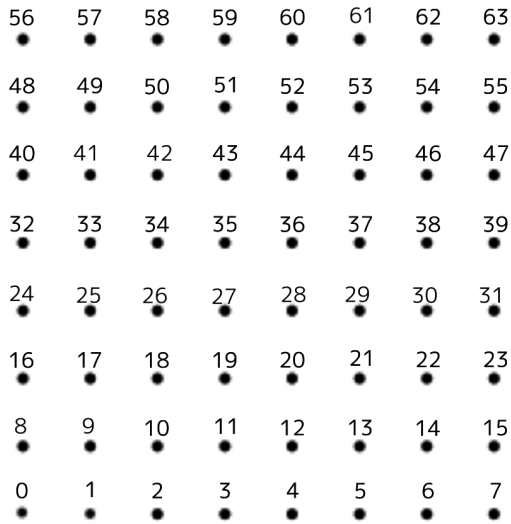


Fig. 2 Natural mapping  $\mu'$  for 64-QAM.

if the information bits are 0101, then  $g$  becomes 5, and the codeword  $\mathbf{V}_5$  is given by

$$\mathbf{V}_5 = \begin{bmatrix} \mu\{\beta_0 \cdot 5\} \\ \mu\{\beta_1 \cdot 5\} \end{bmatrix} = \begin{bmatrix} \mu\{1 \cdot 5 \bmod 16\} \\ \mu\{6 \cdot 5 \bmod 16\} \end{bmatrix} = \begin{bmatrix} \mu\{5\} \\ \mu\{14\} \end{bmatrix}.$$

Thus, the two signal points labeled 5 and 14 in Fig. 1 were simultaneously transmitted from the two antennas.

Next, we present optimal ISVs with four and six bits per channel use (bpcu) with  $M_t = 2$  and evaluate their error performance. The optimal  $\beta_i$  of ISV was selected by comparing the average SER in Eq. (6) for various values of  $\beta_i$ . Figure 3 shows the average SER of ISV [1,6] for  $m = 1, 5, 10$  with 16-QAM for  $M_r = 2$ . In this figure, the analytical lines depict the calculation result of Eq. (6), and the simulation lines show the result of the maximum likelihood (ML) decoding with perfect CSI at the receiver

$$\hat{\mathbf{V}} = \arg \min_{\mathbf{V}_k} \|\mathbf{Y} - \mathbf{H}\mathbf{V}_k\|, \quad (7)$$

where the abscissa is the average SNR per received antenna.

Similarly, Fig. 4 shows the average SER of 64-QAM ISV with  $[\beta_0, \beta_1] = [1, 20]$ . ISV adopts a natural mapping  $\mu'$  in Fig. 2. ISV of 64-QAM can transmit six bits at a time.

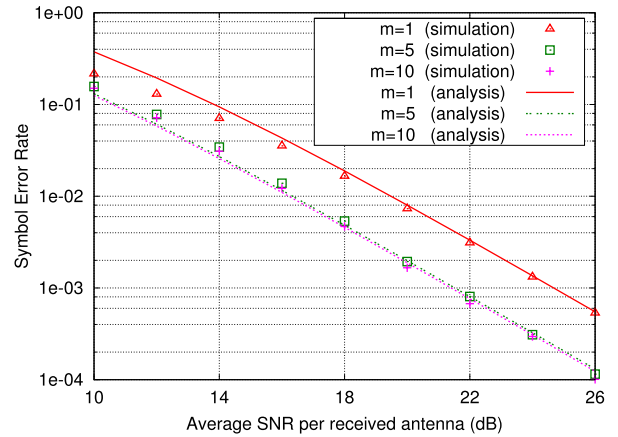


Fig. 3 SER of 16-QAM integer spatial vector (ISV) [1,6] with  $M_t = 2$ ,  $M_r = 2$  and four bpcu.

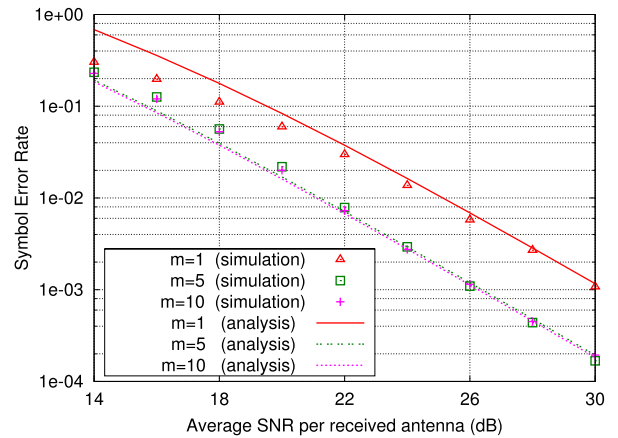


Fig. 4 SER of 64-QAM integer spatial vector (ISV) [1,20] with  $M_t = 2$ ,  $M_r = 2$  and six bpcu.

Interestingly, the SERs of ISVs for  $m = 5$  and  $m = 10$  are almost the same. As  $m$  is increased, that is, as  $K$  is increased,  $(1 + K)/m = (2K + 1)/(K + 1)$  approaches two, and thus,  $c$  becomes nearly  $\rho d/(2M_t)$ . For example, if  $m = 5$ , then  $(1 + K)/m = 1.89$ , and if  $m = 10$ , then  $(1 + K)/m = 1.95$ . This property provides robustness against variations in  $m$ . In fact, these ISVs can be decoded correctly without fading  $m \rightarrow \infty$ . The optimal code for the additive white Gaussian noise is  $[\beta_0, \beta_1] = [1, 1]$  for  $M_t = 2$ , which is a repetition code. However, the repetition code is unsuitable for  $m < \infty$ . The set of symbols transmitted from other than the first antenna of ISV is a subset of the  $M$ -QAM signal constellation, which increases the Euclidean distances between the codewords. If a valid ISV is required only for Rayleigh fading of  $m = 1$ , then [1,19] is better than [1,20] for six bpcu.

#### 4. Performance Comparison

In this section, we demonstrate a comparison of the BER of the proposed ISV [1,6] for four bpcu with the BERs of SM, ESM, and QSM using computer simulations with an ML decoder and  $M_r = 4$ . As mentioned in Sect. 1, the  $K$

**Table 1** Transmission vectors of integer spatial vector (ISV) [1,6], spatial modulation (SM), enhanced spatial modulation (ESM), and quadrature spatial modulation (QSM) [6].

Bits	$v_1, v_2$	SM	ESM	QSM
0000	0, 0	1, 0	$\frac{1+j}{\sqrt{2}}, 0$	$\frac{1+j}{\sqrt{2}}, 0$
0001	1, 6	$\frac{1+j}{\sqrt{2}}, 0$	$\frac{-1+j}{\sqrt{2}}, 0$	$\frac{-1+j}{\sqrt{2}}, 0$
0010	2, 12	$j, 0$	$\frac{-1-j}{\sqrt{2}}, 0$	$\frac{-1-j}{\sqrt{2}}, 0$
0011	3, 2	$\frac{-1+j}{\sqrt{2}}, 0$	$\frac{1-j}{\sqrt{2}}, 0$	$\frac{1-j}{\sqrt{2}}, 0$
0100	4, 8	-1, 0	0, $\frac{1+j}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}, \frac{j}{\sqrt{2}}$
0101	5, 14	$\frac{-1-j}{\sqrt{2}}, 0$	0, $\frac{-1+j}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}, \frac{j}{\sqrt{2}}$
0110	6, 4	-j, 0	0, $\frac{-1-j}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}, \frac{-j}{\sqrt{2}}$
0111	7, 10	$\frac{1-j}{\sqrt{2}}, 0$	0, $\frac{1-j}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}, \frac{-j}{\sqrt{2}}$
1000	8, 0	0, 1	$\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	$\frac{j}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
1001	9, 6	0, $\frac{1+j}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$	$\frac{j}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$
1010	10, 12	0, j	$\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$	$\frac{-j}{\sqrt{2}}, \frac{1}{\sqrt{2}}$
1011	11, 2	0, $\frac{-1+j}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$	$\frac{-j}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$
1100	12, 8	0, -1	$\frac{j}{\sqrt{2}}, \frac{j}{\sqrt{2}}$	0, $\frac{1+j}{\sqrt{2}}$
1101	13, 14	0, $\frac{-1-j}{\sqrt{2}}$	$\frac{j}{\sqrt{2}}, \frac{-j}{\sqrt{2}}$	0, $\frac{-1+j}{\sqrt{2}}$
1101	14, 4	0, -j	$\frac{-j}{\sqrt{2}}, \frac{j}{\sqrt{2}}$	0, $\frac{-1-j}{\sqrt{2}}$
1111	15, 10	0, $\frac{1-j}{\sqrt{2}}$	$\frac{-j}{\sqrt{2}}, \frac{-j}{\sqrt{2}}$	0, $\frac{1-j}{\sqrt{2}}$

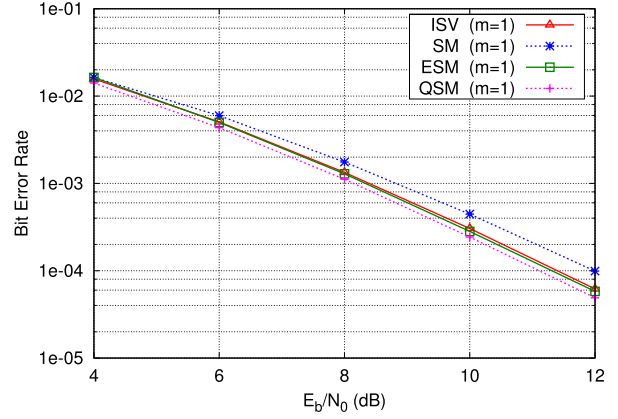
factor shown in [1] varies from 7 to 21 dB, depending on the orientation of the receiver. If  $K$  is 10 dB, then  $m$  of Nakagami- $m$  fading is approximately 5.8, and if  $K$  is 20 dB, then  $m$  is approximately 50.8. Therefore, we evaluated the BERs for  $m = 1, 5, 20, 50$  in this simulation.

Basar provided SM, ESM, and QSM transmission vectors for four bpcu and two transmission antennas in [6, Table 1]. In this study, we used these provided values. The transmission vectors [6] and the codewords of ISV  $[\beta_0, \beta_1] = [1, 6]$  are presented in Table 1.

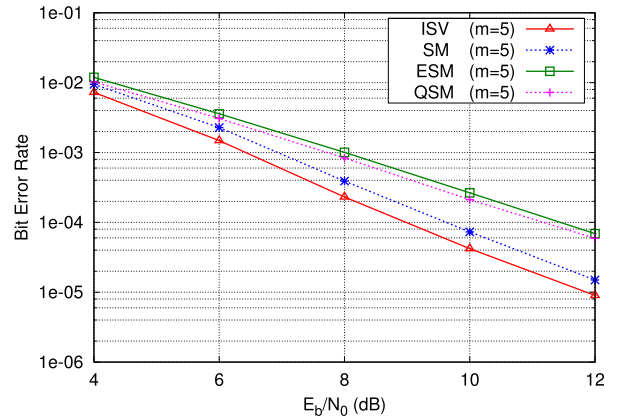
As shown in Table 1, for the four bpcu, SM, ESM, and QSM, an 8-PSK signal constellation is utilized. The active antenna index is applied to carry the remaining information bits that are not sent as signal points. SM always adopts only one transmitting antenna, whereas ESM and QSM transmit by employing one or two antennas. In addition, QSM uses only a single RF chain to simplify the transmitter [6]. In contrast, ISV always uses a 16-QAM signal constellation and two transmit antennas.

Figure 5 shows the BER for the  $m = 1$  Nakagami- $m$  fading channel, that is, the Rayleigh fading channel. In this figure, QSM achieves the best performance. The performance of QSM does not differ significantly from those of ISV and ESM but differs considerably from that of SM. Figures 6, 7, and 8 show comparisons of the BERs for  $m = 5, 20, 50$ , respectively. The BERs of ISV are almost stable as  $m$  is changed, whereas those of the SM methods deteriorate. ISV outperforms the SM methods for these values of  $m$ . For example, for  $m = 50$ , ISV is better than SM by approximately 2 dB at BER =  $10^{-4}$ .

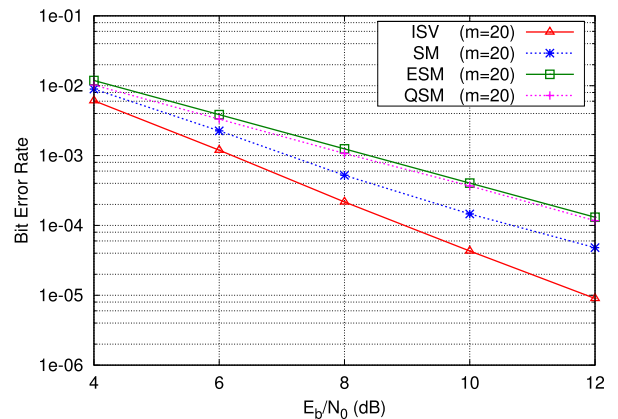
From comparison, increasing the value of  $m$  results in worse ESM and QSM performances than that of SM, even though ESM and QSM perform better than SM for Rayleigh



**Fig. 5** BER Comparison of integer spatial vector (ISV) with spatial modulation (SM), enhanced spatial modulation (ESM), and quadrature spatial modulation (QSM) for  $m = 1$ .

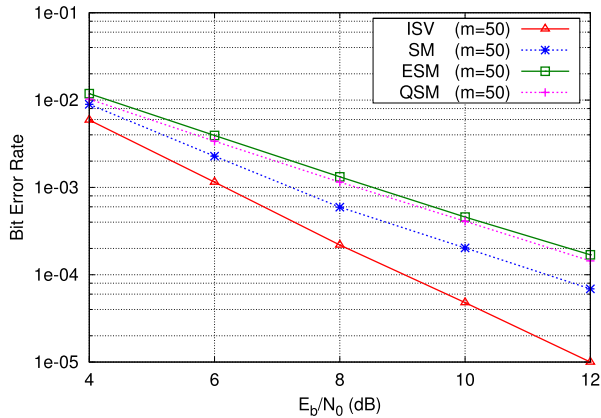


**Fig. 6** BER comparison of integer spatial vector (ISV) with spatial modulation (SM), enhanced spatial modulation (ESM), and quadrature spatial modulation (QSM) for  $m = 5$ .



**Fig. 7** BER comparison of integer spatial vector (ISV) with spatial modulation (SM), enhanced spatial modulation (ESM), and quadrature spatial modulation (QSM) for  $m = 20$ .

fading channels  $m = 1$ . The spatial correlation between different channel paths increases with the value of  $m$ . As a result, detecting the active antenna index becomes difficult.



**Fig. 8** BER comparison of integer spatial vector (ISV) with spatial modulation (SM), enhanced spatial modulation (ESM), and quadrature spatial modulation (QSM) for  $m = 50$ .

SM employs only one active antenna. Thus, SM deteriorates less than ESM and QSM, which require strict distinctions between active multiple antennas [7].

As mentioned in Sect. 1, the SM methods have several advantages. However, if an effective transmission MIMO system for Nakagami- $m$  fading is required, ISV is a viable option.

## 5. Conclusion

We proposed a MIMO transmission scheme for Nakagami- $m$  fading channels called SV. As an example of SV, ISV was provided and compared to three different SM schemes for Nakagami- $m$  fading MIMO channels from  $m = 1$  to  $m = 50$ . The results showed that for  $m > 1$ , ISV outperformed the SM schemes and was robust to fading variations. In the future, we will investigate the performance of SV to which forward error correcting coding is applied.

## Acknowledgments

This work was supported by JSPS KAKENHI (Grant

Number JP20K04492).

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