

## LETTER

# Improved Source Localization Method of the Small-Aperture Array Based on the Parasitic Fly's Coupled Ears and MUSIC-Like Algorithm

Hongbo LI<sup>†</sup>, Aijun LIU<sup>††a)</sup>, Qiang YANG<sup>†</sup>, Zhe LYU<sup>††</sup>, Nonmembers, and Di YAO<sup>††</sup>, Member

**SUMMARY** To improve the direction-of-arrival estimation performance of the small-aperture array, we propose a source localization method inspired by the *Ormia* fly's coupled ears and MUSIC-like algorithm. The *Ormia* can local its host cricket's sound precisely despite the tremendous incompatibility between the spacing of its ear and the sound wavelength. In this paper, we first implement a biologically inspired coupled system based on the coupled model of the *Ormia*'s ears and solve its responses by the modal decomposition method. Then, we analyze the effect of the system on the received signals of the array. Research shows that the system amplifies the amplitude ratio and phase difference between the signals, equivalent to creating a virtual array with a larger aperture. Finally, we apply the MUSIC-like algorithm for DOA estimation to suppress the colored noise caused by the system. Numerical results demonstrate that the proposed method can improve the localization precision and resolution of the array.

**key words:** small-aperture array, biologically inspired coupled system, modal decomposition method, MUSIC-like algorithm, virtual aperture expansion

## 1. Introduction

Source localization has long been of great research interest in radar array signal processing. Most localization approaches [1]–[3], like the Convention beamforming (CBF) and Multiple signal classification (MUSIC), employ time differences of arrivals for direction-of-arrival (DOA) estimation, and their performance is limited by the array aperture. This means large aperture is required to obtain accurate DOA estimation. However, large-aperture arrays are costly and even be infeasible in many scenarios, so the small-aperture arrays are indispensable.

This paper proposes a source localization method of the small-aperture array. The method is inspired by a parasitic fly named *Ormia Ochracea*. The *Ormia* can local its host cricket precisely despite the tremendous incompatibility between the spacing of its ears (1.5 mm) and the wavelength of the host's sound (70 mm) [4]. This ability benefits from the *Ormia*'s coupled ears which have been modeled as a mechanical coupled model [5], as shown in Fig. 1. The model consists of two spring-damping pairs  $\{(k_i, c_i), i = 1, 2\}$  with

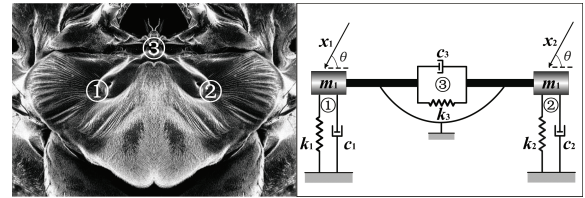


Fig. 1 Mechanical model of the *Ormia*'s ears.

a mass of  $m$  and two rigid bars. The rigid bars are connected by a coupling spring  $k_3$  and damping  $c_3$ .

Inspired by the mechanical model, a circuit model is proposed and a biomimetic antenna array (BMAA) which can amplify the time differences of arrivals is designed [6], [7]. Then, the design schemes of the BMAA are extended by various circuit coupled networks, including T-type network,  $\pi$ -type network, etc [8]–[10]. However, by lack of research on the combination of mechanical coupled model and signal processing algorithms, these papers have not given the localization performance of the BMAA.

In this paper, we propose a improved source localization method of the small-aperture array based on the *Ormia*'s coupled ears and MUSIC-like algorithm. We first implement a biologically inspired coupled (BIC) system by regarding the coupled model of the *Ormia* fly's ears as a two-input two-output filter and applying the received signals of the array as the filter's inputs. Then, by the modal decomposition method, we solve the responses of the BIC system and analyze its influence on the received signals of the array. The research shows that the BIC system can amplify the amplitude ratio and phase difference between the signals, which is equivalent to creating a virtual array with a larger aperture. We also establish the signal model of the virtual array. For the colored noise caused by the BIC system, we apply the MUSIC-like algorithm for the DOA estimation to suppress it. Numerical results show that, compared to the MUSIC algorithm, the proposed method has higher localization precision and resolution, implying a virtual aperture expansion of the array.

## 2. Biologically Inspired Coupled System

### 2.1 Implement of the BIC System

For equal spring-damping pairs, we set  $k = k_1 = k_2, c = c_1 =$

Manuscript received July 12, 2023.

Manuscript revised October 12, 2023.

Manuscript publicized December 8, 2023.

<sup>†</sup>School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin, 150001 China.

<sup>††</sup>School of Information Science and Engineering, Harbin Institute of Technology at Weihai, Weihai, 264209 China.

a) E-mail: liuaijun@hit.edu.cn

DOI: 10.23919/transfun.2023EAL2061

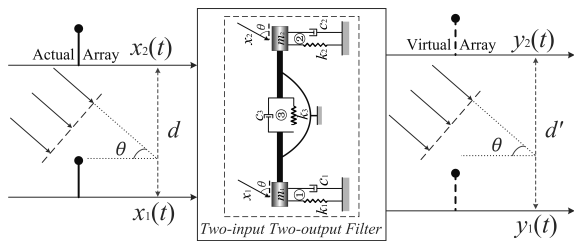


Fig. 2 Complete view of the BIC system.

$c_2$ . According to the Newton's second law, the equation of motion for the coupled model proposed in [5] is

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \ddot{\mathbf{y}} + \begin{bmatrix} c+c_3 & c_3 \\ c_3 & c+c_3 \end{bmatrix} \dot{\mathbf{y}} + \begin{bmatrix} k+k_3 & k_3 \\ k_3 & k+k_3 \end{bmatrix} \mathbf{y} = \mathbf{x} \quad (1)$$

where  $\mathbf{x} = [x_1(t), x_2(t)]^T$  and  $\mathbf{y} = [y_1(t), y_2(t)]^T$  are the input and response vectors of the coupled model, respectively. And  $(\cdot)$  denotes differentiation with respect to time  $t$ .

To implement the BIC system, we regard the coupled model as a two-input two-output filter and apply the received signals of the array as the filter's inputs (see Fig. 2). Consider a two-antenna array with  $d$  antenna spacing,  $x_1(t)$  and  $x_2(t)$  are the received signals of the actual array.  $y_1(t)$  and  $y_2(t)$  are the responses of the BIC system, which are equivalent to the received signals of a virtual array with  $d'$  antenna spacing. Under the far-field narrow-band uncorrelated signal assumption, the received signal vector of the actual array is

$$\mathbf{x} = \mathbf{A}(\theta)\mathbf{S}(t) + \mathbf{e}(t) \quad (2)$$

where

- $\mathbf{s}(t) = [s_1(t), \dots, s_N(t)]^T$  is the incoming signal vector, with  $N$  as the number of sources
- $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_N)]$  is the array manifold, with  $\theta_n$  as the azimuth of the  $n$ -th source
- $\mathbf{a}(\theta_n) = [\exp(j\omega\tau_n/2), \exp(-j\omega\tau_n/2)]^T$  for the array
- $\tau_n = d \sin \theta_n / c$ , with  $d$  as the antenna spacing,  $c$  as the speed of signal propagation
- $\mathbf{e}(t)$  is the Additive White Gaussian Noise (AWGN) with zero mean and variance  $\sigma^2$

## 2.2 Responses of the BIC System

Then the matrix equation of the BIC system is

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{x} \quad (3)$$

where  $\mathbf{M}$ ,  $\mathbf{K}$ , and  $\mathbf{C}$  called the mass matrix, stiffness matrix and damping matrix, respectively.

It is troublesome to solve the responses of the BIC system since (3) is a coupled difference equation, so we first decouple it by the modal decomposition method. It is known that the mode shape has orthogonality concerning the mass matrix and the stiffness matrix [11]. So for the proportional damping system ( $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$ ), the mode shape also has

orthogonality concerning the damping matrix.

$$\mathbf{I} = \Phi^T \mathbf{M} \Phi, \text{diag}(\omega_i^2) = \Phi^T \mathbf{K} \Phi, \text{diag}(2\xi_i \omega_i) = \Phi^T \mathbf{C} \Phi \quad (4)$$

where  $\Phi$  is the modal matrix;  $\mathbf{I}$  is the unit matrix;  $\omega_i$  and  $\xi_i$  are the undamped natural frequency and damping ratio of the  $i$ -th mode, respectively. The BIC system has two degrees of freedom, so  $1 \leq i \leq 2$ , and the modal matrix is

$$\Phi = \frac{1}{\sqrt{2m}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \quad (5)$$

Based on the modal matrix, the physical responses  $\mathbf{y}$  can be transformed into modal responses  $\eta$  ( $\mathbf{y} = \Phi\eta$ ), then (3) is rewritten as

$$\mathbf{M}\Phi\ddot{\eta} + \mathbf{C}\Phi\dot{\eta} + \mathbf{K}\Phi\eta = \mathbf{x} \quad (6)$$

Premultiplying  $\Phi^T$  at both sides of (6) and simplifying it according to (4), we can derive the independent modal coordinate equations which are easier to solve

$$\ddot{\eta}_i + 2\xi_i \omega_i \dot{\eta}_i + \omega_i^2 \eta_i = f_i, \quad i = 1, 2 \quad (7)$$

where  $[f_1; f_2] = \Phi^T \mathbf{x}$ .

Assuming that the incoming signal of the actual array is  $\exp(j\omega t)$ , then the received signals of the array are  $x_1(t) = \exp(j(\omega t + \omega\tau/2))$  and  $x_2(t) = \exp(j(\omega t - \omega\tau/2))$ . Note that the noise is ignored to simplify the solution process. We solve (7) and derive the modal responses as

$$\eta_i(t) = A_i \exp(j(\omega t + \varphi_i)), \quad i = 1, 2 \quad (8)$$

where  $A_i$  and  $\varphi_i$  are the magnitude and phase of the  $i$ -th modal responses, respectively

$$A_1 = \frac{\sqrt{2/m} \sin(\omega\tau/2)}{\sqrt{(\omega_1^2 - \omega^2)^2 + (2\omega_1 \xi_1 \omega)^2}}, \varphi_1 = -\arctan\left(\frac{2\omega_1 \xi_1 \omega}{\omega_1^2 - \omega^2}\right)$$

$$A_2 = \frac{\sqrt{2/m} \cos(\omega\tau/2)}{\sqrt{(\omega_2^2 - \omega^2)^2 + (2\omega_2 \xi_2 \omega)^2}}, \varphi_2 = -\arctan\left(\frac{2\omega_2 \xi_2 \omega}{\omega_2^2 - \omega^2}\right) + \pi/2$$

Accordingly, we can obtain the physical responses of the BIC system as

$$y_1(t) = [\eta_2(t) + \eta_1(t)] / \sqrt{2m}$$

$$y_2(t) = [\eta_2(t) - \eta_1(t)] / \sqrt{2m} \quad (9)$$

Obviously, the physical responses are a composite of the modal responses. Taking the phase of the first modal response as the reference, the graphical explanation of the synthesis process is shown in Fig. 3. Through the triangular operations, we can get the amplitude ratio and phase difference of the responses

$$\Delta A_y = \sqrt{\frac{\sin^2 \Delta\varphi_\eta + (\Delta A_\eta + \cos \Delta\varphi_\eta)^2}{\sin^2 \Delta\varphi_\eta + (\Delta A_\eta - \cos \Delta\varphi_\eta)^2}}$$

$$\Delta\varphi_y = \arctan\left(\frac{2\Delta A_\eta \sin \Delta\varphi_\eta}{1 - \Delta A_\eta^2}\right) \quad (10)$$

where

- $\Delta A_y$  is the amplitude ratio between  $y_1(t)$  and  $y_2(t)$

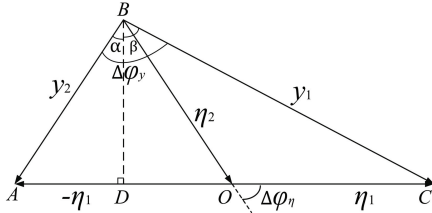


Fig. 3 Process of response synthesis.

- $\Delta\varphi_y$  is the phase difference between  $y_1(t)$  and  $y_2(t)$
- $\Delta A_\eta = A_1/A_2$  is the amplitude ratio between  $\eta_1$  and  $\eta_2$
- $\Delta\varphi_\eta = \varphi_1 - \varphi_2$  is the phase difference between  $\eta_1$  and  $\eta_2$

### 2.3 Effect of the BIC System

To explain the effect of the BIC system, we analyze the value of  $\Delta A_y$  and  $\Delta\varphi_y$ . If the system is uncoupled, the value of  $c_3$  and  $k_3$  should be zero. This results in  $\Delta A_\eta = \tan(\omega\tau/2)$  and  $\Delta\varphi_\eta = \pi/2$ . Then we derive that  $\Delta A_y = \Delta A_x = 1$  and  $\Delta\varphi_y = \Delta\varphi_x = \omega\tau$ . By  $\Delta A_x$  and  $\Delta\varphi_x$  we mean the amplitude ratio and phase difference between  $x_1(t)$  and  $x_2(t)$ .

However, for the coupled system,  $\Delta A_\eta \neq \tan(\omega\tau/2)$  and  $\Delta\varphi_\eta \neq \pi/2$ . And we have  $\Delta A_y > \Delta A_x$  and  $\Delta\varphi_y > \Delta\varphi_x$ , proving that the BIC system can amplify the amplitude ratio and phase difference between the signals.

To describe this amplification quantitatively, we define the amplitude gain  $G_A$  and phase gain  $G_\varphi$  as

$$G_A = \Delta A_y, G_\varphi = \Delta\varphi_y/\omega\tau \quad (11)$$

Then, we regard the response vector  $\mathbf{y} = [y_1(t), y_2(t)]^T$  as the received signals of a virtual array and establish its signal model as

$$\mathbf{y} = \mathbf{H}[\mathbf{x}] = \mathbf{H}[\mathbf{A}(\theta)\mathbf{s}(t)] + \mathbf{H}[\mathbf{e}(t)] \quad (12)$$

where  $\mathbf{H}[\cdot]$  refers to the coupling (filtering) operation of the BIC system to the signals.

Since the signal  $\mathbf{s}(t)$  and noise  $\mathbf{e}(t)$  are independent, the coupling operation satisfies the distributive law. So we divide it into the signal and noise parts for analysis.

**1) Signal Part:** As mentioned above, the BIC system amplifies the amplitude ratio and phase difference between the signals. This means changing the array manifold (or creating a virtual array), so we have

$$\mathbf{H}[\mathbf{A}(\theta)\mathbf{s}(t)] = \tilde{\mathbf{A}}(\theta)\mathbf{s}(t) \quad (13)$$

where

- $\tilde{\mathbf{A}}(\theta) = [\tilde{\mathbf{a}}(\theta_1), \dots, \tilde{\mathbf{a}}(\theta_N)]$  is the virtual array manifold
- $\tilde{\mathbf{a}}(\theta) = [G_A \exp(j\omega\tilde{\tau}/2), \exp(-j\omega\tilde{\tau}/2)]^T$
- $\tilde{\tau} = G_\varphi\tau$  is the virtual time delay

Since  $\tau = d \sin \theta/c$  and the unknown parameter is  $\theta$ , we reasonably define that the coupling operation does not affect the incident angle. Then the virtual time delay as  $\tilde{\tau} = d' \sin \theta/c$  and the virtual antenna spacing as  $d' = G_\varphi d$ .

**2) Noise Part:** As seen in (3), the BIC system is also a linear system. So for the white noise, it will be converted by the BIC system into colored noise.

$$\mathbf{H}[\mathbf{e}(t)] = \tilde{\mathbf{e}}(t) \quad (14)$$

where  $\tilde{\mathbf{e}}(t)$  is the Colored Gaussian Noise.

Therefore, the final signal model of the virtual array is

$$\mathbf{y} = \mathbf{H}[\mathbf{x}(t)] = \tilde{\mathbf{A}}(\theta)\mathbf{S}(t) + \tilde{\mathbf{e}}(t) \quad (15)$$

In localization algorithms, array aperture means localization performance. The virtual array has a larger aperture than the actual array ( $d' > d$ ), so in principle it can improve the localization accuracy. However, the colored Gaussian noise is detrimental to some algorithms and can degrade their performance, such as CBF and MUSIC. In Sect. 3, to suppress the colored noise, we use a MUSIC-like algorithm to estimate the DOA.

### 3. MUSIC-Like Algorithm

The MUSIC-like algorithm is similar to MUSIC, but based on the fourth-order cumulant of the received data, rather than the second-order statistic on which the MUSIC algorithm is based [12]. Compared with the second-order cumulant, the fourth-order cumulant (FOC) has the advantage of suppressing Gaussian noise. According to the properties of cumulants described in [13], the FOCs of signal  $\mathbf{y}$  are

$$\begin{aligned} \text{cum}(\mathbf{y}_i, \mathbf{y}_j, \mathbf{y}_k^*, \mathbf{y}_l^*) \\ = \sum_{n=1}^N \tilde{\mathbf{a}}_n(i) \tilde{\mathbf{a}}_n(j) \tilde{\mathbf{a}}_n^*(k) \tilde{\mathbf{a}}_n^*(l) \gamma_{4s}(n) \end{aligned} \quad (16)$$

where

- $i, j, k, l \in [1, M]$ , with  $M = 2$  is the number of antenna
- $N$  is the number of sources
- $\tilde{\mathbf{a}}_n(i)$  is the  $i$ -th element of the array manifold for the  $n$ -th source
- $\gamma_{4s}(n) = \text{cum}(\mathbf{s}_n(t), \mathbf{s}_n(t), \mathbf{s}_n^*(t), \mathbf{s}_n^*(t)) = E|s|^4 - |E s^2|^2 - 2E|s^2|E|s^2|$  denotes the FOC of the  $n$ -th source

Apparently, (16) has  $M^4$  values for various  $i, j, k, l$ . By setting  $\text{cum}(\mathbf{y}_i, \mathbf{y}_j, \mathbf{y}_k^*, \mathbf{y}_l^*)$  as the  $[(i-1)M+k]$ -th row and  $[(j-1)M+l]$ -th column of a matrix, we construct the FOC matrix  $\mathbf{C}_4$  as (see also [12], [14])

$$\mathbf{C}_4 = \mathbf{B}(\theta)\mathbf{C}_s\mathbf{B}^H(\theta) \quad (17)$$

where

- $\mathbf{B}(\theta) = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \dots, \mathbf{b}(\theta_N)]$
- $\mathbf{b}(\theta) = \tilde{\mathbf{a}}(\theta) \otimes \tilde{\mathbf{a}}^*(\theta) = [G_A^2, G_A \exp(j\omega\tilde{\tau}), G_A \exp(-j\omega\tilde{\tau}), 1]^T$  as the FOC array manifold, with  $\otimes$  represents the kronecker product
- $\mathbf{C}_s = \text{diag}(\gamma_{4s}(1), \gamma_{4s}(2), \dots, \gamma_{4s}(N))$  is the FOC matrix of the sources

Compared with the covariance matrix of the receive signals  $\mathbf{R} = \mathbf{A}(\theta)\mathbf{R}_s\mathbf{A}^H(\theta) + \sigma^2\mathbf{I}$ , the FOC matrix  $\mathbf{C}_4$  has

eliminated the influence of Gaussian noise. Analogous to the MUSIC algorithm, we perform eigenvalue decomposition on  $\mathbf{C}_4$  and obtain

$$\mathbf{C}_4 = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Lambda}_n \mathbf{U}_n^H \quad (18)$$

where

- $\mathbf{U}_s = [u_1, \dots, u_N]$  is the FOC signal subspace
- $\mathbf{U}_n = [u_{N+1}, \dots, u_{M^2}]$  is the FOC noise subspace
- $\mathbf{\Lambda}_s = \text{diag}(\lambda_1, \dots, \lambda_N)$  is the diagonal matrix composed of  $N$  large eigenvalues
- $\mathbf{\Lambda}_n = \text{diag}(\lambda_{N+1}, \dots, \lambda_{M^2})$  is the diagonal matrix composed of  $M^2 - N$  small eigenvalues

Then based on the orthogonality between the FOC array manifold  $\mathbf{b}(\theta)$  and the FOC noise subspace  $\mathbf{U}_n$ , the spatial spectrum of the MUSIC-like algorithm is derived as

$$P_{\text{MUSIC-like}}(\theta) = \frac{1}{\|\mathbf{b}^H(\theta)\mathbf{U}_n\|^2} \quad (19)$$

Finally, the DOA estimation can be obtained by performing a spectral peak search on the spatial spectrum.

#### 4. Numerical Results

We compare the DOA estimation performance of our proposed method and MUSIC algorithm under the standard array. The scenarios used are as follows: Single signal incoming from  $45^\circ$  with 5 kHz frequency;  $d = \lambda/20$  or  $d = \lambda/10$  element distances; standard coupling parameter referred from [5].

The precision and resolution results of DOA estimation are demonstrated in Figs. 4 and 5. In Fig. 4, for a given number of snapshots,  $L = 500$ , we obtain the root mean square error (RMSE) of the DOA estimation. The RMSE decreases as the Signal-to-Noise Ratio (SNR) increases. Under the same aperture and SNR, the RMSE of the proposed method is smaller, means that the estimation error is reduced and the localization precision is improved. We also combine the BIC system and MUSIC algorithm for comparison (the purple curve) with  $d = \lambda/10$ . The high RMSE validates that the performance of MUSIC algorithm degrades when combine with the BIC system.

In Fig. 5, fixed SNR = 10, we plot the probability of detection (Pd) for the proposed method and MUSIC algorithm. We consider the detection successful if the estimation error is less than  $2^\circ$ . Under the same aperture and snapshot number, the Pd of the proposed method is higher than that of the MUSIC algorithm. Figures 4 and 5 confirm that the combined use of BIC system and MUSIC-like algorithm can improve the performance of DOA estimation. The principle of this improvement is that the coupling operation amplifies the phase difference between the received signals of the actual array, which is equivalent to a virtual aperture expansion for the array. It is worth noting that this improvement is more pronounced at small element spacing  $d$ .

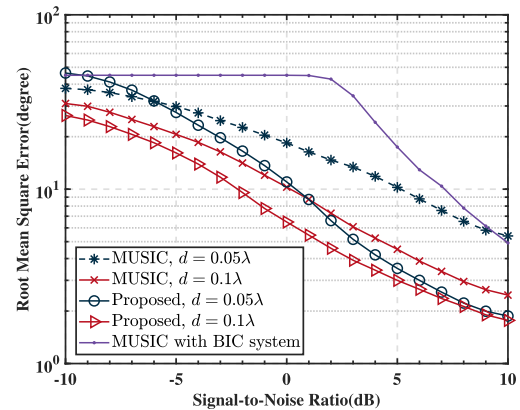


Fig. 4 RMSE of the DOA estimation.

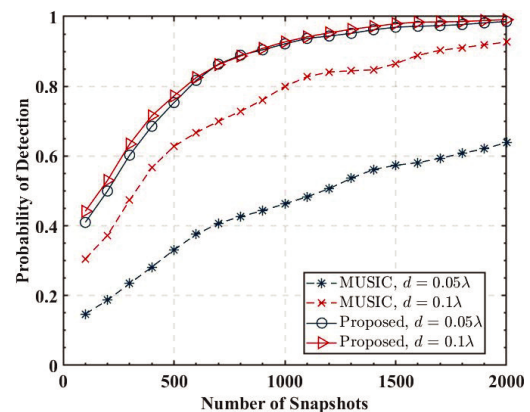


Fig. 5 Pd of the DOA estimation.

#### 5. Conclusion

In this paper, we propose a improved source localization method of the small-aperture array based on the *Ormia* fly's coupled ears and MUSIC-like algorithm. We first implement the BIC system by regarding the coupled model of the *Ormia* fly's ears as a two-input two-output filter and applying the received signals of the array as the filter's inputs. Then, we solve the responses of the BIC system by the modal decomposition method and analyze the effect of the BIC system on the the received signals. It is proved that the BIC system can amplify the amplitude ratio and phase difference between the signals, which is equivalent to creating a virtual array with a larger aperture. We then derive the received signal model of the virtual array. To suppress the colored noise caused by the BIC system, we introduce the MUSIC-like algorithm to estimate DOA. Numerical results show that, the RMSE and Pd of the proposed method are greater than those of the MUSIC algorithm. This means that the DOA estimation performance of the array has improved by our proposed method, which also means an enlargement of the array aperture.

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