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LETTER

Zero-Order-Hold Triggered Control of a Chain of Integrators with an Arbitrary Sampling Period*

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SUMMARY We propose a zero-order-hold triggered control for a chain of integrators with an arbitrary sampling period. We analytically show that our control scheme globally asymptotically stabilizes the considered system. The key feature is that the pre-specified sampling period can be enlarged as desired by adjusting a gain-scaling factor. An example with various simulation results is given for clear illustration.

key words: *chain of integrators, zero-order-hold triggered, arbitrary sampling period*

1. Introduction

We consider a global asymptotic stabilization problem for a chain of integrators by a zero-order-hold triggered (ZOHT) control. The key feature with our control scheme is that the pre-specified sampling period of ZOHT control can be made arbitrarily large by utilizing a gain-scaling factor of our controller.

Note that the ZOHT control method discretely updates the control input at each sampling period. So, the ZOHT control has a similar feature of the traditional event-triggered (ET) control in the sense of discrete updates of the control input, which leads to the similar effective usage of the communication resources [2], [5], [7]–[10]. Both ZOHT and ET control have less number of control input updates than the continuous-time control in general.

However, our proposed ZOHT control method has some clear benefits over the traditional ET control method in the following aspects. First, the ET control requires a somewhat complicated design of a triggering condition which must be engaged both in the system analysis and Zeno behavior analysis at the same time [6]. Second, in terms of controller implementation, the ET control requires a memory to store some past state/output values in order to check the event-triggering condition in real time.

Thus, our ZOHT control method is much simpler in terms of control implementation and it does not require any extra analytical effort to prove the avoidance of the Zeno behavior. As the title indicates, our control method can stabilize the considered system with an arbitrary sampling period. So, by setting a large sampling period, we can reduce the number of control input updates as desired, which leads to the same efficient usage of communication resources as the ET control [2] with a easier implementation.

2. System and Problem Formulation

We consider a chain of integrators given by

$$\dot{x} = Ax + Bu \tag{1}$$

where $x = [x_1, \cdots, x_n]^T \in \mathcal{R}^n$ is the state, $u \in \mathcal{R}$ is the input, and (A, B) is a Brunovsky canonical pair, i.e., $A = [a_{ij}]$, $1 \leq i, j \leq n$, where if $j = i + 1$, $a_{ij} = 1$, else $a_{ij} = 0$ and $B = [0, \cdots, 0, 1]^T$.

Remark 1. *A chain of integrators may seem to be very simple at first glance. However, there are two notable related aspects: (i) in a practical aspect, a satellite attitude control model is represented by a second-order chain of integrators [3]; (ii) In an extensional aspect, the control results on a chain of integrators can be extended to a class of feedforward systems. This aspect will be briefly addressed in Sect. 4.*

Regarding the system (1), we first state our control problem: *Globally stabilize the system* (1) *by a state feedback controller with an arbitrary sampling period* T.

To introduce our controller, we define some notations:

- t_k, $k = 0, 1, \cdots$: Controller update time with $t_0 = 0$.
- $T = t_{k+1} - t_k > 0$: Pre-specified sampling period.

In solving our control problem, the following ZOHT controller is introduced.

$$u(t) = K(\gamma)x(t_k), \quad t \in [t_k, t_{k+1}) \tag{2}$$

where $K(\gamma) = [k_1/\gamma^n, \cdots, k_n\gamma]$, $\gamma \geq 1$ to be chosen.

Remark 2. *The proposed controller resembles the ET controller [9] as the control input is updated discretely. However, unlike the ET controller [9], the proposed controller does not require a triggering condition which means that its implementation is simpler because any memory storage to monitor the state deviation from the past state values recorded at the previous execution time is not needed. Moreover, there is no additional need to check the avoidance of Zero behavior [6] because the positive lower bound of T is guaranteed by default.*

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3. Main Results

Before we state our main theorem, we define some notations for proof: let $A_{K(\gamma)} = A + BK(\gamma)$, $K = K(1)$, $A_K = A_{K(1)}$, $\bar{K} = [k_1^2, \cdots, k_n^2]$, $E_\gamma = \text{diag}[1, \gamma, \cdots, \gamma^{n-1}]$, $I = E_1$, and $\|\cdot\|$ denotes the Euclidean norm.

Theorem 1. *Suppose that K is selected such that A_K is Hurwitz. Then, the closed-loop system* (1) *with the controller $\hat{u}$ in* (2) *is globally asymptotically stable. Moreover, the sampling period T can be enlarged by increasing γ.*

Proof. For $t \in [t_k, t_{k+1})$, the closed-loop system is

$$\begin{aligned}\dot{x} =& Ax + BK(\gamma)x(t_k)\\ =& Ax + BK(\gamma)[x + x(t_k) - x]\\ =& A_{K(\gamma)}x - BK(\gamma)[x - x(t_k)]\\ =& A_{K(\gamma)}x - BK(\gamma)\int_{t_k}^{t} \dot{x}(s)ds\\ =& A_{K(\gamma)}x - BK(\gamma)A\int_{t_k}^{t} xds\\ &- (BK(\gamma))^2\int_{t_k}^{t} x(t_k)ds \end{aligned} \tag{3}$$

Here, similarly to the analysis in [2], we set $V(x) = x^T P_\gamma x$ where $P_\gamma = E_\gamma P E_\gamma$ and $A_K^T P + PA_K = -I$. Then, along the trajectory of (3), we have

$$\begin{aligned}\dot{V}(x) =& -\gamma^{-1}\|E_\gamma x\|^2 - 2x^T E_\gamma P E_\gamma BK(\gamma)A\int_{t_k}^{t} xds\\ &- 2x^T E_\gamma P E_\gamma (BK(\gamma))^2\int_{t_k}^{t} x(t_k)ds\end{aligned} \tag{4}$$

Regarding the second term of (4), we have

$$\begin{aligned}&2x^T E_\gamma P E_\gamma BK(\gamma)A\int_{t_k}^{t} xds\\ &= 2x^T E_\gamma P \underbrace{E_\gamma BK(\gamma)AE_\gamma^{-1}}_{\le \gamma^{-2}\|K\|}\int_{t_k}^{t} E_\gamma xds\\ &\le 2\gamma^{-2}\|P\|\|K\|\|E_\gamma x\|\int_{t-T}^{t}\|E_\gamma x\|ds\\ &\le 2\gamma^{-2}\|P\|\|K\|\|E_\gamma x\|\cdot T\sup_{-T\le\theta\le 0}\|E_\gamma x(t+\theta)\|\end{aligned} \tag{5}$$

Regarding the third term of (4), we have

$$\begin{aligned}&2x^T E_\gamma P E_\gamma (BK(\gamma))^2\int_{t_k}^{t} x(t_k)ds\\ &= 2x^T E_\gamma P \underbrace{E_\gamma (BK(\gamma))^2 E_\gamma^{-1}}_{\le \gamma^{-2}\|\bar{K}\|}\int_{t_k}^{t} E_\gamma x(t_k)ds\\ &\le 2\gamma^{-2}\|P\|\|\bar{K}\|\|E_\gamma x\|\cdot T\sup_{-T\le\theta\le 0}\|E_\gamma x(t+\theta)\|\end{aligned} \tag{6}$$

Here, we note that the second and third terms of (4) are upper bounded by some delayed terms. In order to treat these terms, we utilize the Razumikhin theorem [4]. We set $V(x(t+\theta)) \le qV(x)$, $-T \le \theta \le 0$, which leads to

$$\sup_{-T\le\theta\le 0}\|E_\gamma x(t+\theta)\| \le \bar{q}\|E_\gamma x\| \tag{7}$$

where $\bar{q} = \sqrt{q\lambda_{\max}(P)/\lambda_{\min}(P)}$.

Now, gathering (4)–(7) together, we have

$$\dot{V}(x) \le -\gamma^{-2}\underbrace{\left\{\gamma - 2\bar{q}\|P\|(\|K\| + \|\bar{K}\|)T\right\}}_{=:\Delta(\gamma,T)}\|E_\gamma x\|^2 \tag{8}$$

Thus, the closed-loop system becomes globally asymptotically stable when $\Delta(\gamma, T) > 0$. Moreover, it is clear by observing $\Delta(\gamma, T)$ that the pre-specified sampling period T can be enlarged by increasing γ. □

Remark 3. *The proposed ZOHT control method updates the control input discretely like the ET controller [9]. Moreover, the pre-specified samapling period T can be further enlarged by using the gain-scaling factor γ. So, the advantage of saving communication resource by the traditional event-triggered control method is retained. Again, the proposed control method does not require any triggering condition, which leads to the simpler implementation of the controller. Also, extra analytical effort showing the avoidance of Zeno behavior is not necessary.*

4. Extension to a Class of Feedforward Nonlinearity

Here, we briefly show that our proposed control scheme can be extended to a more generalized nonlinear systems, namely a class of feedforward systems. First, the system (1) is extended as [1]

$$\dot{x} = Ax + Bu + \delta(t, x, u) \tag{9}$$

where $\delta(t, x, u) = [\delta_1(t, x, u), \cdots, \delta_{n-1}(t, x, u), 0]^T$ which satisfies $\delta_i \le c(|x_{i+2}| + \cdots + |x_n|)$, $i = 1, \cdots, n-1$.

Considering the derivation given in [1], without showing the tedious details, the inequality (8) can be modified into the following

$$\dot{V}(x) \le -\gamma^{-2}\bar{\Delta}(\gamma, T)\|E_\gamma x\|^2 \tag{10}$$

where

$$\bar{\Delta}(\gamma, T) = \gamma - 2\bar{q}\|P\|(\|K\| + \|\bar{K}\|)T - 2\|P\|\bar{c} \tag{11}$$

where the additional term $2\|P\|\bar{c}$ is independent of γ.

Thus, the main results of Theorem 1 remain the same in principle except some necessary changes in allowable range of γ corresponding to T due to the additional nonlinearity. This is just one brief sketch of many potential extensions of our current work. There can be much more extensions such as delays, uncertain parameters, and so forth.

5. Illustrative Example

We consider a case of $n = 2$ and address the following two cases.

Case 1: The enlargement of T by utilizing γ.

We select $K = [-1, -2]$ which gives us $\Delta(\gamma, T) \simeq \gamma - 22T > 0$. Here, we observe three cases:

(i) We let $\gamma = 1$. Then, $\Delta(\gamma, T) > 0$ for $0 < T < 0.0455$. We choose $T = 0.04$. As shown in Fig. 1, the system is asymptotically stabilized with 25 control input updates.

(ii) For the time-being, fix $\gamma = 1$. By increasing T further, we observe that the system starts showing unstable behavior when T surpasses about 1.03 as shown in Fig. 2.

(iii) With $T = 1.03$, we increase γ to be 23. Then, the system becomes asymptotically stabilized again as shown in Fig. 3. So, the enlargement of T by our ZOHT control is clear shown.

Case 2: Comparison with the ET controller [2].

Following [2], their ET controller is given as

$$u(t) = K(\gamma)x(t_i), \ \ \forall \in [t_k, t_{k+1}), \ \ t_0 = 0 \qquad (12)$$

$$t_{k+1} = \inf\{t > t_k : \gamma^n |e(t)| > \sigma \|E_\gamma x(t)\|\} \qquad (13)$$

where the parameters σ and γ are to be selected from the following sufficient condition:

$$1 - 2\|PBK\|\|K\|\gamma^{-1} - 2\|P\|\sigma > 0 \qquad (14)$$

Again, we select $K = [-1, -2]$. This gives us $\gamma = 11$ and $\sigma = 0.098$. For our proposed controller, the condition is $\gamma - 22T > 0$. So, we get $\gamma = 11$ and $T = 0.499$.

For both control methods, the simulations are performed with the same initial conditions and obtained results are shown in Figs. 4 and 5. The system is similarly asymptotically stabilized by both methods. The ET control method updates the control input 49 times and the proposed ZOHT control method updates the control input 60 times. Thus, our newly proposed ZOHT controller achieves the similar performance to the event-triggered controller while it is memoryless(without any triggering conditions) and its number of

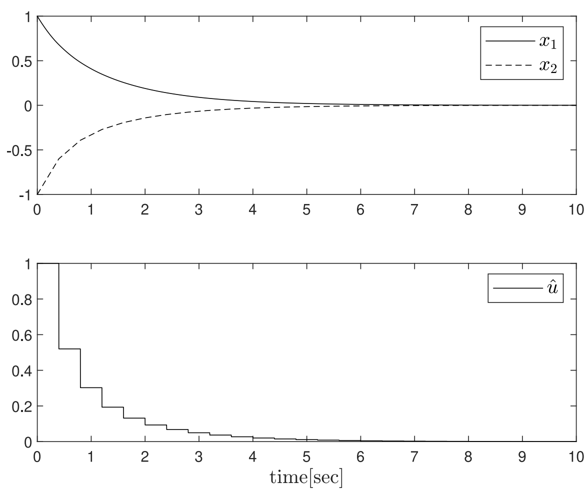

Fig. 1 Trajectories of states and input when $\gamma = 1$ and $T = 0.4$.

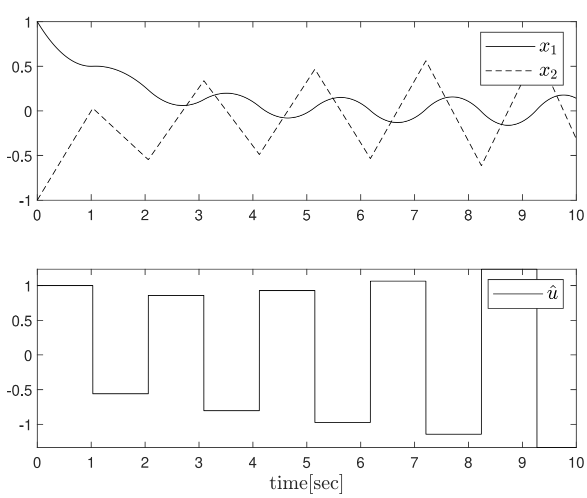

Fig. 2 Trajectories of states and input when $\gamma = 1$ and $T = 1.03$.

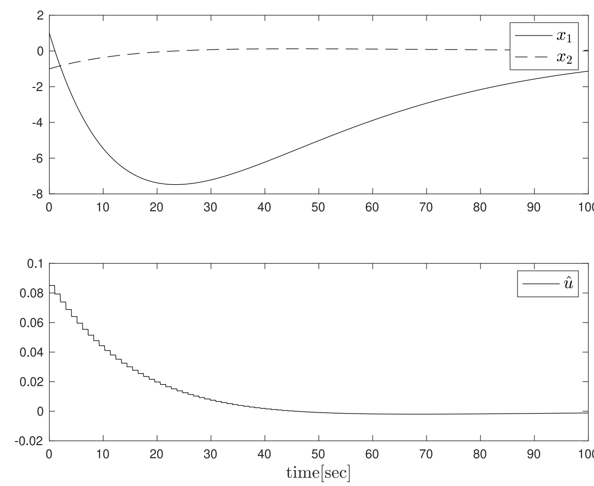

Fig. 3 TTrajectories of states and input when $\gamma = 23$ and $T = 1.03$.

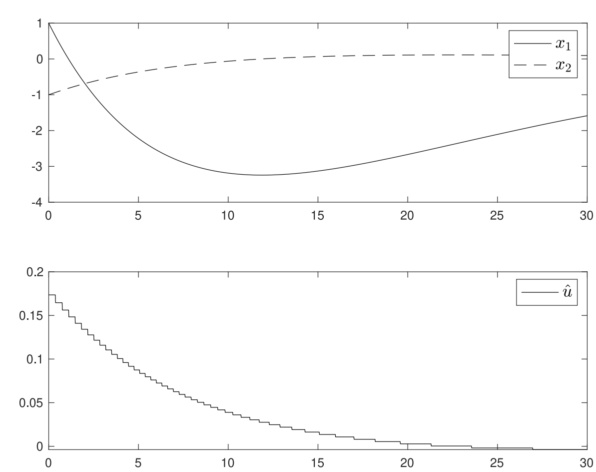

Fig. 4 Trajectories of states and input by the ET control [2].

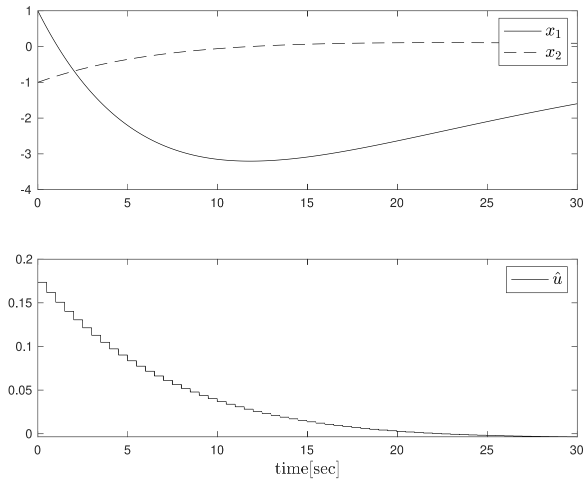

Fig. 5 Trajectories of states and input by the proposed ZOHT control.

input dates remains similar.

6. Conclusions

We have provided a new ZOHT controller with an arbitrary sampling period for a chain of integrators. We have shown that our proposed control method stabilizes the considered system. Also, we have shown that the pre-specified sampling period can be enlarged by using a gain-scaling factor. Various simulation results confirm the advantages of the proposed method over the traditional ET control method.

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