LETTER

Peak-to-Average Power Ratio Reduction Scheme in DCO-OFDM with a Combined Index Modulation and Convex Optimization

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SUMMARY Direct-current biased optical orthogonal frequency division multiplexing (DCO-OFDM) exhibits a high peak-to-average power ratio (PAPR), which leads to nonlinear distortion in the system. In response to the above, the study proposes a scheme that combines direct-current biased optical orthogonal frequency division multiplexing with index modulation (DCO-OFDM-IM) and convex optimization algorithms. The proposed scheme utilizes partially activated subcarriers of the system to transmit constellation modulated symbol information, and transmits additional symbol information of the system through the combination of activated carrier index. Additionally, a dither signal is added to the system's idle subcarriers, and the convex optimization algorithm is applied to solve for the optimal values of this dither signal. Therefore, by ensuring the system's peak power remains unchanged, the scheme enhances the system's average transmission power and thus achieves a reduction in the PAPR. Experimental results indicate that at a system's complementary cumulative distribution function (CCDF) of 10^{-4}, the proposed scheme reduces the PAPR by approximately 3.5 dB compared to the conventional DCO-OFDM system. Moreover, at a bit error rate (BER) of 10^{-3}, the proposed scheme can lower the signal-to-noise ratio (SNR) by about 1 dB relative to the traditional DCO-OFDM system. Therefore, the proposed scheme enables a more substantial reduction in PAPR and improvement in BER performance compared to the conventional DCO-OFDM approach.

key words: *DCO-OFDM, index modulation, convex optimization, peak-to-average power ratio*

1. Introduction

Visible light communication (VLC) has become one of the key technologies for today's wireless communication systems due to its advantages, such as large bandwidth, absence of electromagnetic interference, and high security [1]. OFDM is an efficient transmission modulation technique that is widely used in VLC systems due to its resistance to intersymbol interference and high spectral efficiency [2].

In VLC systems, the information transmitted using intensity modulation/direct detection (IM/DD) techniques must be positive real signals. Early scholars solution was DC-biased optical OFDM (DCO-OFDM) [3], which introduces a DC bias to transform bipolar OFDM signals into non-negative signals, but excessive DC biasing leads to low energy efficiency. Asymmetrically clipped optical OFDM (ACO-OFDM) [4] was also proposed, where the negative part of the signal is directly clipped to produce non-negative signals at the cost of sacrificing spectral efficiency.

To further improve the energy efficiency or spectral efficiency (SE) of optical OFDM (O-OFDM) systems, an index-modulated OFDM (OFDM-IM) technique has been employed [5]. This technique indexes subcarriers in the frequency domain, utilizes subcarrier indexing patterns to transmit additional symbol bits, and achieves a favorable trade-off between power efficiency and spectral efficiency by adjusting the number of active subcarriers [6]. Indexed modulation in O-OFDM has been a hot research topic in recent years. In [7], indexed modulation was used earlier in DCO-OFDM systems, and it was confirmed that the DCO-OFDM-IM system can obtain better BER performance than the conventional system. Subsequently, scholars have proposed an O-OFDM-IM system scheme based on the discrete hartley transform (DHT), which achieves better spectral efficiency and bit error rate performance [8]. However, the current study of the proposed O-OFDM-IM technique reveals that these systems still exhibit high PAPR characteristics and a large number of idle subcarriers are generated during the indexed modulation process, leading to the problem of underutilization of the system carriers.

In this study, we propose a DCO-OFDM-IM system, add dither signals to the idle subcarriers generated by the system, and apply convex optimization to solve the best dither signal scheme. The scheme utilizes a combination of activated subcarriers and their carrier indexes to transmit the symbol information of the system, while dither signals are added to the idle subcarriers and a convex optimization algorithm is applied to solve for the optimal value of the dither signals. The objective is to increase the average transmit power without changing the peak power of the system, which effectively reduces the system PAPR and makes full use of the idle carriers. The results show that the proposed scheme not only reduces the PAPR of the DCO-OFDM-IM system but also improves the BER performance of the system and, at the same time is able to freely regulate the number of activated subcarriers of the system.

2. DCO-OFDM System Based on Indexed Modulation

2.1 DCO-OFDM-IM System Model

As shown in Fig. 1, in the DCO-OFDM-IM system, assume that a randomly generated M-bit binary bit stream enters the DCO-OFDM-IM transmitters. The M bits of data are evenly divided into g groups, with each group containing p bits, i.e., $p = M/g$. In the frequency domain, each group of p bits is mapped onto an OFDM sub-block of length n, where

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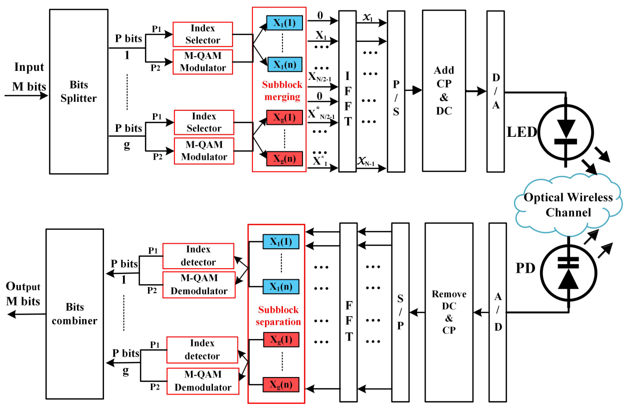

Fig. 1 Block diagram of DCO-OFDM-IM system.

$n = N/g$ and N is the number of subcarriers in the system. Finally, the g OFDM sub-blocks are combined to form one OFDM transmission block.

Since all OFDM sub-blocks are independent of each other and their operations at the transmitters are identical, In this study, we will take the α sub-block as an example, $\alpha \in \{1, 2 \cdots, g\}$. Where each sub-block can transmit p bits information is divided into index bits p_1 and symbol bits p_2. Here, the index bits are calculated as $p_1 = \lfloor \log_2(C(n,k)) \rfloor$, where $\lfloor . \rfloor$ the largest integer not larger than the independent variable, and $C(n,k)$ denotes the number of k activated subcarrier mapping combinations in subblock n. Therefore, the combination of indices for the active subcarriers can be represented as

$$I_\alpha = \{i_\alpha(1), i_\alpha(2), \ldots, i_\alpha(k)\} \tag{1}$$

where $i_\alpha(\beta) \in \{1, 2, \ldots, n\}, \beta = 1, 2, \cdots, k$. Correspondingly, the symbol bits $p_2 = k \bullet \log_2(M)$ represent a vector with k-modulated symbols obtained from M-order modulation mapping. Therefore, the modulated symbol vector transmitted by the active subcarriers is denoted as

$$s_\alpha(\beta) = [s_\alpha(1), s_\alpha(2), \cdots, s_\alpha(k)]^T \tag{2}$$

where $s_\alpha(\beta) \in S, \beta = 1, 2 \cdots, k$, S represents the set of M-order modulation symbols. Thus, the α-th OFDM sub-block can be represented as

$$X_\alpha = [X_\alpha(1) \cdots X_\alpha(n)], X_\alpha(\eta) = \begin{cases} s_\alpha(\eta) & \eta \in I_\alpha \\ 0 & \eta \notin I_\alpha \end{cases} \tag{3}$$

For systems with N subcarriers, the transmitted symbols need to satisfy Hermitian symmetry to obtain a positive real unipolar OFDM signal after inverse fast fourier transform (IFFT) operation. Subsequently, the g OFDM sub-blocks are combined into a complete OFDM transmission block containing the frequency domain signal with $\frac{(N-2)}{2}$ subcarriers denoted as

$$X_F = \left[X_1, \cdots, X_g\right]^T = [X(1), \cdots, X(N/2-1)]^T \tag{4}$$

Therefore, the total number of bits carried by one OFDM transmission block is $M = g \times (p_1 + p_2)$, $g = \frac{N-2}{2n}$. The spectral efficiency of the system can be expressed as

$$\eta = \frac{M}{N} \approx \frac{\lfloor \log_2(C(n,k)) \rfloor + k\log_2(M)}{2n} \tag{5}$$

The X_F is processed using Hermitian symmetry, and the symbols are transformed into real-valued signals suitable for transmission in visible light. It can be expressed as

$$X_U = \left[0, X_1, \cdots, X_{\frac{N}{2}-1}, 0, X^*{}_{\frac{N}{2}-1}, \cdots, X^*{}_1\right] \tag{6}$$

Afterward, X_U is subjected to an IFFT, thus obtaining the time-domain signal expression for the OFDM block as

$$x_T = \frac{1}{\sqrt{N}} F^H\{X_U\}, \{X_U\} = [X(1), \ldots, X(\frac{N}{2}-1)]^T \tag{7}$$

where F denotes fast fourier transform (FFT) and F^H denotes fast fourier inverse transform (IFFT). Subsequently, the cyclic prefix (CP) and DC bias (DC) are added to x_T, and a digital-to-analog conversion (D/A) operation is performed to transmit the OFDM "real and positive" signs in the visible channel. At the receiver of the system, after analog-to-digital conversion (A/D), removing DC and CP and performing FFT operation, the frequency domain signal is obtained, which can be expressed as

$$\mathrm{Y}_\alpha = X_\alpha + N_\alpha \tag{8}$$

where X_α is the α-th element of X, and N_α denotes additive white Gaussian noise (AWGN) with variance σ^2.

The detection algorithm at the receiver end of the DCO-OFDM-IM system often adopts Maximum Likelihood (ML) detection, but their high complexity makes them more difficult to implement in practical systems. Therefore, the paper utilizes a low-complexity Greedy Detection (GD) algorithm [9]. It first detects the active sub-carriers in a sub-block and calculates the energy carried by each subcarrier in the current sub-block, denoted as $E_\alpha(\lambda) = |Y_\alpha(\lambda)|^2$, $\lambda \in \{1, 2 \cdots, n\}$, n is the sequence number of the subcarrier of block α. Finally, the index bit p_1 and the symbol bit p_2 contained therein are demodulated to combine the information and output the original data.

2.2 Index Mapping Method

In the transmitters of the DCO-OFDM-IM system, the indexed bits at the transmitters are mapped into the activated subcarrier index combinations using an index mapper. Therefore, it is necessary to create and store tables of the same size $c = 2^{p_1}$ at both the transmitters and receiver sides of the system. Simultaneously, the table needs to detail the mapping relationship between the index bits p_1 and the active subcarrier index combinations [5]. As shown in Table 1 below, an index mapping lookup table is provided for $n = 4, k = 2$, where $s_1, s_2 \in S$. Since $C(4,2) = 6$, the amount of index bits of information that each sub-block can transmit

Table 1 A look-up table of index modulation for $p_1 = 2, n = 4, k = 2$.

Index bits	Index combinations	Subblocks
[0 0]	$\{1,2\}$	$[s_1, s_2, 0, 0]$
[0 1]	$\{1,3\}$	$[s_1, 0, s_2, 0]$
[1 0]	$\{1,4\}$	$[s_1, 0, 0, s_2]$
[1 1]	$\{2,3\}$	$[0, s_1, s_2, 0]$

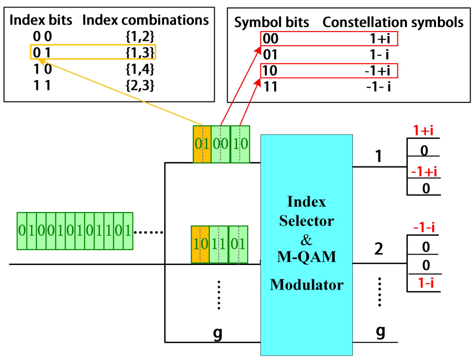

Fig. 2 Block diagram of the DCO-OFDM-IM system index structure.

is $p_1 = \lfloor \log_2 6 \rfloor = 2$ with its corresponding number of activated subcarrier combinations $c = 2^{p_1} = 4$, so there will be two sets of combinations unused in all available combination cases.

Based on the index modulation principle described earlier and the lookup table shown in Table 1, Fig. 2 illustrates the index structure block diagram of the DCO-OFDM-IM system when the system input data is M bits and 4-QAM constellation mapping is used.

3. Using Convex Optimization to Reduce PAPR Scheme

It is first investigated that the PAPR in the DCO-OFDM-IM system is defined as

$$PAPR\{x_T\} = \frac{\max\limits_{i=0,\cdots,(N-2)/2} |x_i|^2}{E\left[|x_i|^2\right]} \tag{9}$$

In the DCO-OFDM-IM system, the set of all g sub-block activation modes for the frequency domain signal X_F is denoted as $I = \bigcup\limits_{\partial=1}^{g} I_\partial$. Where the number of I is L, the complement of I is denoted as $\bar{I} = \{1, 2, 3, \ldots, \frac{(N-2)}{2}\}\backslash I$, whose number is $N_1 = \frac{N-2}{2} - L$. $\bar{I}$ is the set of idle subcarrier indices. In addition, we denote the frequency domain dither vector as $\delta' = \left(\delta'_1, \delta'_2 \cdots, \delta'_{(N-2)/2}\right)^T$, and when $\delta'_\partial = 0$, $\partial \in I$. Then, in order to further simplify the formula, we remove the L zero elements from the frequency domain dither vector δ' and pick them out in the original order to constitute the N_1-dimensional dither signal δ, where δ represents the dither signal value added in the idle carrier of the system, and use the convex optimization algorithm to solve for the optimal dither signal. Therefore, reducing the PAPR of the DCO-OFDM-IM system can be expressed as

$$\begin{aligned} &\min_{\delta} \left\| F_{\bar{I}}^H \delta + x_T \right\|_\infty^2 \\ &s.t. \|\delta\|_\infty \le R \end{aligned} \tag{10}$$

where $x_T = F_{(N-2)/2}^H X_F$ denotes the initial time-domain signal transmitted by the activated subcarriers of the system, and $F_{\bar{I}}^H \in C^{(\frac{N-2}{2}) \times N_1}$ consists of the N_1 columns selected in the original order of the idle subcarrier positions of the frequency-domain signal X_F in $F_{(N-2)/2}^H$. The operation of $F_{\bar{I}}^H \delta$ is to transform the frequency-domain dither signal δ into a time-domain signal. The objective function in Eq. (10) denotes minimizing the time domain signal x_T, and then by using the method of adding the dither signal δ to the system idle subcarriers, the scheme results in a further increase in the system signal power, which is further maximized by the method proposed in this paper compared to the scheme where the value of the system power generated by the idle carriers of the original OFDM-IM system is zero. The constraint inequality is used to limit the amplitude of the dither signal δ. The purpose of the amplitude constraint is to determine the effect of the dither signal on the BER performance of the system.

The objective function and the constraint function are both convex functions, and to solve the convex optimization problem [10], the CVX toolbox in MATLAB simulation is used to solve the problem. According to Eq. (9), this paper utilizes the transmission of initial symbol information x_T by the system's active subcarriers, which, without altering the initial signal's peak power, enhances the system's average power by adding δ to the idle subcarriers, thereby further reducing the system's PAPR.

4. Simulation Results

In this section, the system performance using convex optimization methods in the idle carriers of DCO-OFDM-IM is verified through computer simulation. The experimental simulation environment is in an indoor additive white Gaussian noise visible light channel, using 9.6×10^4 OFDM symbols, $N = 128$ subcarriers, and 4-QAM modulation.

As shown in Fig. 3, the simulation results of the PAPR performance for the proposed scheme, DCO-OFDM-IM(n, k), and DCO-OFDM are presented when the number of sub-block carriers, $n = 4$. From the figure, it can be observed that when the complementary cumulative distribution function (CCDF) is 10^{-4}, the smaller the value of k in the DCO-OFDM-IM(n, k) system, the better the PAPR performance. However, the downside is that many idle carriers are wasted, resulting in a very low spectral efficiency of the system. Therefore, the proposed scheme in the study is based on the structure of DCO-OFDM-IM$(4, 2)$ structure, which adds δ to the idle subcarrier and combines with a convex optimization algorithm to solve for the optimal dither signal value and obtains the optimal value of PAPR of 5.7 dB for $\delta = 0.18$. Compared to the OFDM-IM$(4, 2)$ system with a PAPR of 7.2 dB, this system can effectively lower the PAPR value while increasing the spectral efficiency. Compared to

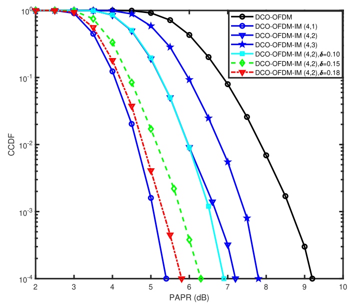

Fig. 3 Comparison of PAPR in the proposed scheme, DCO-OFDM-IM(n,k), and DCO-OFDM systems.

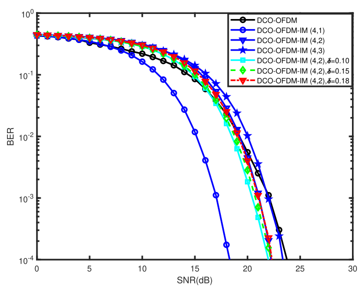

Fig. 4 Comparison of BER in the proposed scheme, DCO-OFDM-IM(n,k), and DCO-OFDM systems.

the traditional DCO-OFDM system with a PAPR of 9.2 dB, PAPR has been reduced by about 3.5 dB. The experimental results show that the scheme proposed in the study can enhance the system's PAPR performance.

Figure 4 shows the simulation results of the BER performance for the proposed scheme, DCO-OFDM-IM(n,k), and DCO-OFDM are presented when the number of sub-block carriers, $n = 4$. It is evident from the figure that the proposed scheme is based on the DCO-OFDM-IM(4, 2) structure. By adding δ to the idle subcarriers, as δ increases, the system's bit error rate performance deteriorates. However, the optimal dither signal $\delta = 0.18$ was obtained after incorporating constraints from a convex optimization algorithm. The experimental results show that when the BER is 10^{-3}, the signal-to-noise ratio (SNR) of the proposed scheme is approximately close to that of DCO-OFDM-IM(4, 2) at 21.1 dB. This indicates that introducing a small-scale δ into the system after applying constraints through the convex optimization algorithm has a minimal impact on the system's bit error rate performance. Therefore, compared to the traditional DCO-OFDM system with an SNR of 22.1 dB, the proposed scheme shows an average reduction of 1 dB in SNR. The experimental results prove that the proposed scheme can better enhance the bit error rate performance of the DCO-OFDM system.

Figure 5 shows the simulation results of the spectral efficiency (SE) performance for DCO-OFDM-IM and DCO-OFDM systems are presented when the number of sub-block carriers n and the number of active subcarriers k vary. It can be observed from the figure that the spectral efficiency of

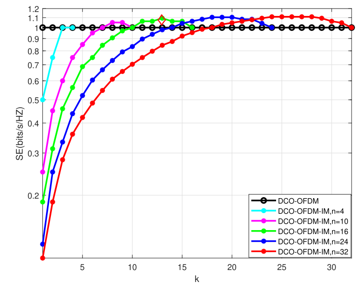

Fig. 5 Comparison of SE (spectral efficiency) in the proposed scheme, DCO-OFDM-IM(n,k), and DCO-OFDM systems.

DCO-OFDM is 1. However, the spectral efficiency of the DCO-OFDM-IM system varies with the number of active subcarriers k. Particularly when n is large, at certain values of k, the spectral efficiency of the DCO-OFDM-IM system can exceed that of the traditional DCO-OFDM system. Based on the calculation using Eq. (5), when $n = 16$ and $k = 13$, the spectral efficiency of the DCO-OFDM-IM system is, on average, 10% higher than that of the traditional DCO-OFDM system. Therefore, as the values of n and k increase, the spectral efficiency advantage of the DCO-OFDM-IM system becomes increasingly apparent. The experimental results demonstrate that the spectral efficiency of the DCO-OFDM-IM system can be adjusted through the parameters n and k.

5. Conclusion

In this study, we have proposed a method that combines DCO-OFDM with Index Modulation (IM), establishing a DCO-OFDM-IM system. Dither signals are introduced into the idle subcarriers of the system, and the optimal dither signal is determined through a convex optimization algorithm. The results indicate that this method significantly reduces the system's PAPR while also effectively enhancing the system's bit error performance. Moreover, when n and k values are large, the DCO-OFDM-IM system can achieve higher spectral efficiency.

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References

[1] S. Zhang, P. Du, H. Yang, R. Zhang, C. Chen, and A. Alphones, "Recent progress in visible light positioning and communication systems," IEICE Trans. Commun., vol.E106-B, no.2, pp.84–100, Feb. 2023.

[2] J. Armstrong, "OFDM for optical communications," J. Lightw. Technol., vol.27, no.3, pp.189–204, 2009.

[3] H. Elgala, R. Mesleh, and H. Haas, "Indoor optical wireless communication: Potential and state-of-the-art," IEEE Commun. Mag., vol.49, no.9, pp.56–62, 2011.

[4] S. Ma, R. Yang, X. Deng, X. Ling, X. Zhang, F. Zhou, S. Li, and D.W. K. Ng, "Spectral and energy efficiency of ACO-OFDM in visible light communication systems," IEEE Trans. Wireless Commun., vol.21, no.4, pp.2147–2161, 2021.
[5] E. Basar, M. Wen, R. Mesleh, M. Di Renzo, Y. Xiao, and H. Haas, "Index modulation techniques for next-generation wireless networks," IEEE Access, vol.5, pp.16693–16746, 2017.
[6] E. Başar, Ü. Aygölü, E. Panayırcı, and H.V. Poor, "Orthogonal frequency division multiplexing with index modulation," IEEE Trans. Signal Process., vol.61, no.22, pp.5536–5549, 2013.
[7] E. Başar and E. Panayırcı, "Optical OFDM with index modulation for visible light communications," 2015 4th International Workshop on Optical Wireless Communications (IWOW), IEEE, pp.11–15, 2015.
[8] X.-Y. Xu, Q. Zhang, and D.-W. Yue, "Orthogonal frequency division multiplexing with index modulation based on discrete hartley transform in visible light communications," IEEE Photon. J., vol.14, no.3, pp.1–10, 2022.
[9] J. Crawford and Y. Ko, "Low complexity greedy detection method with generalized multicarrier index keying OFDM," 2015 IEEE 26th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC), IEEE, pp.688–693, 2015.
[10] A. Aggarwal and T.H. Meng, "Minimizing the peak-to-average power ratio of OFDM signals using convex optimization," IEEE Trans. Signal Process., vol.54, no.8, pp.3099–3110, 2006.