IEICE TRANSACTIONS

on Fundamentals of Electronics, Communications and Computer Sciences

DOI:10.1587/transfun.2024EAL2018

Publicized:2024/08/06

This advance publication article will be replaced by the finalized version after proofreading.

A PUBLICATION OF THE ENGINEERING SCIENCES SOCIETY
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LETTER

New Bounds for Aperiodic Wide-Gap Frequency Hopping Sequences

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SUMMARY Frequency hopping sequences (FHSs) play a significant role in modern frequency hopping spread spectrum communication and radar systems. In terms of application, the aperiodic Hamming correlation (HC) holds greater significance compared to the periodic HC as it directly impacts the communication performance. In addition, it is crucial for each user's FHS to have a substantial wide-gap (WG) in order to prevent the received signals from interfering with each other. In this letter, we obtain a new bound by extending the aperiodic bound proposed by Peng-Fan and the WG FHS bound introduced by Li-Fan-Yang-Wang. The proposed bound is strict since they can be verified using specific parameters of aperiodic WG FHSs.

key words: *Peng-Fan bounds, Aperiodic wide-gap, Hamming correlation, Frequency hopping sequences.*

1. Introduction

Frequency hopping (FH) multiple-access (MA) spread spectrum systems are widely applicable in military radar communications, mobile communications, and modern radar and sonar applications [1], [2]. In such system, each frequency hopping sequence (FHS) correspond to one user. It generates mutual interference (MI) when more than one user sends signals at the same time. This MI needs to be as little as possible to mitigate MA interference and enhance overall system performance [2], [3].

The aperiodic Hamming correlation (HC) property of FHS is more significant than the periodic HC property, due to aperiodic HC is more accurate to evaluate system performance in real-world applications [4], [5], and the periodic HC mainly has been considered in most of the papers [6]–[11]. In contrast to periodic HC, aperiodic HC has received little attention in the literature. In 2004, Peng and Fan [12] derived the aperiodic bound on the FHS set.

However, the rapid development of modern information technology poses many challenges to the traditional frequency hopping communication technology. These challenges include the scarcity of available spectrum resources, the continuous upgrade of complex interference sources, and the complexities of variable communication link environments. One of the issues encountered in FH systems is the interference that arises when there is a small gap between neighbouring frequency hops of an FHS. This interference can significantly affect the received signal quality. To address this problem, researchers have proposed FHSs with wide gaps between adjacent frequency hops, referred to as wide-gap (WG) FHSs [13], [14]. By ensuring a sufficient spectrum span between adjacent frequency hops, even if a particular hop signal falls within an interference band, the adjacent hop signals remain unaffected, and the interfered information can be recovered through encoding. In 2019, Li, Fan, Yang and Wang [15] derived the WG's bound on the FHS set.

In summary, this paper introduces the theoretical bound for designing aperiodic WG FHSs. For the construction of aperiodic WG FHSs, this theoretical bound that provide a theoretical basis for constructing FHSs. The outline of this paper is as follows. In Section II, we review some preliminaries. In Section III, we generalize the bound on FHS to the case of aperiodic WG FHS. We generalize the bound on FHS set to the case of aperiodic WG FHS set in Section IV. An example of aperiodic WG FHSs, whose parameters are met with the proposed bound, are given in Section V.

2. Preliminaries

Let $\mathcal{S}$ denote a set of FHSs defined on the frequency slot set F, with parameters L, M, and q. Here, L represents the period of each FHS, M represents the number of FHSs in set $\mathcal{S}$, and q represents the size of the frequency slots in the set F. At time delay τ, the periodic HC and the aperiodic HC between any two FHSs $X = \{x_j\}_{j=0}^{L-1}$ and $Y = \{y_j\}_{j=0}^{L-1}$ in the set $\mathcal{S}$ are respectively defined as follows.

$$H_{X,Y}(\tau) = \sum_{j=0}^{L-1} h[x_j, y_{j+\tau}], \quad 0 \le \tau \le L-1,$$

$$\bar{H}_{X,Y}(\tau) = \sum_{j=0}^{L-\tau-1} h[x_j, y_{j+\tau}], \quad 0 \le \tau \le L-1,$$

where the modulo L is performed to calculate the index $j+\tau$, and $h[x_j, y_{j+\tau}]$=1 if $x_j = y_{j+\tau}$, and 0 otherwise.

When $X = Y$, we obtain the periodic Hamming autocorrelation (HAC) value $H_X(\tau)$ of the FHS X at time delay τ. When $X \neq Y$, we obtain the periodic Hamming cross-correlation (HCC) value $H_{X,Y}(\tau)$ between X and Y at time delay τ. The maximum HAC value of $\mathcal{S}$, the maximum

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HCC value, and the maximum periodic Hamming correlation (MHC) value are respectively denoted as

$$H_a(\mathcal{S}) = \max_{X\in\mathcal{S}} \{H_X(\tau) \mid \tau = 1,2,\dots,L-1\},$$
$$H_c(\mathcal{S}) = \max_{X,Y\in\mathcal{S},X\neq Y} \left\{H_{X,Y}(\tau) \mid \tau = 0,1,\dots,L-1\right\},$$
$$H_m(\mathcal{S}) = \max\{H_a(\mathcal{S}), H_c(\mathcal{S})\}.$$

When $X = Y$, we obtain the aperiodic Hamming autocorrelation (AHAC) value $\bar{H}_X(\tau)$ of the FHS X at time delay τ. When $X \neq Y$, we obtain the aperiodic Hamming cross-correlation (AHCC) value $\bar{H}_{X,Y}(\tau)$ between X and Y at time delay τ. Furthermore, the maximum AHAC value of $\mathcal{S}$, the maximum AHCC value, and the maximum aperiodic Hamming correlation (MAHC) value are respectively denoted as

$$\bar{H}_a(\mathcal{S}) = \max_{X\in\mathcal{S}} \left\{\bar{H}_X(\tau) \mid \tau = 1,2,\dots,L-1\right\},$$
$$\bar{H}_c(\mathcal{S}) = \max_{X,Y\in\mathcal{S},X\neq Y} \left\{\bar{H}_{X,Y}(\tau) \mid \tau = 0,1,\dots,L-1\right\},$$
$$\bar{H}_m(\mathcal{S}) = \max\left\{\bar{H}_a(\mathcal{S}), \bar{H}_c(\mathcal{S})\right\}.$$

In 2004, the lower bound of the aperiodic HC FHS set was established by Peng, Fan as follows.

Lemma 1 (Peng-Fan bound [12]): Let $\mathcal{S}$ be a set of M FHSs of period L over an frequency slot set with size q, and $I = \lfloor \frac{2ML}{q+M} \rfloor$. We have

$$\bar{H}_m(\mathcal{S}) \geq \left\lceil \frac{(3ML - qL - L - M - q)L}{(2ML - M - 1)(M+q)} \right\rceil \tag{1}$$

and

$$\bar{H}_m(\mathcal{S}) \geq \left\lceil \frac{(4I - L - 1)ML - I(I+1)(M+q+2)}{(2ML - M - 1)M} \right\rceil. \tag{2}$$

It is claimed that the set is an optimal FHS set if its relevant parameters meet the Peng-Fan bound with equality.

Definition 1: For any given $X \in \mathcal{S}$, and a positive integer D, if

$$\min_{0\leq j\leq L-1} \{|x_{j+1} - x_j|, |x_{L-1} - x_0|\} > D, D > 0,$$

then X is called a WG FHS with a minimum FH gap D.

In 2019, the lower bound of a WG FHS set was established by Li, Fan, Yang and Wang as follows.

Lemma 2 (Li-Fan-Yang-Wang bound [15]): Let $\mathcal{S}$ be a set of M WG FHSs of period L over an frequency slot with size q, $I = \lfloor \frac{ML}{q} \rfloor$, and the maximum periodic Hamming correlation $H_m(\mathcal{S})$. Then we have

$$H_m(\mathcal{S}) \geq \left\lceil \frac{(LM - q)L}{(LM - 3)q} \right\rceil, \tag{3}$$

$$H_m(\mathcal{S}) \geq \left\lceil \frac{2ILM - (I+1)Iq}{(LM - 3)M} \right\rceil. \tag{4}$$

A WG FHS set $\mathcal{S}$ is optimal if it achieves the equal sign of Eq.(3) or Eq.(4).

Let $\bar{F}$ be a new frequency slot set obtained by adding M distinct frequency slots $q_1, q_2, ..., q_M$ to the original frequency slot set F.

Expanding the period length of $X^i = \{x_0^i, x_1^i, \dots, x_{L-1}^i\} \in \mathcal{S}$ to $2L$ obtains the sequence $\bar{X}^i = \{x_0^i, x_1^i, \dots, x_{L-1}^i, q_i, q_i, \dots, q_i\} \in \bar{\mathcal{S}}$, where $1 \leq i \leq M$. $\bar{\mathcal{S}}$ is a set of M FHSs of length $2L$ over $\bar{F}$ of size $q + M$.

For any aperiodic FHSs $\bar{X} = \{\bar{x}_j\}_{j=0}^{L-1}, \bar{Y} = \{\bar{y}_j\}_{j=0}^{L-1} \in \bar{\mathcal{S}}$, we have

$$H_{\bar{X}}(\tau) = \begin{cases} \bar{H}_X(\tau) + L - \tau, & 0 \leq \tau \leq L-1 \\ 0, & \tau = L \\ \bar{H}_X(2L - \tau) + \tau - L, & L+1 \leq \tau \leq 2L-1. \end{cases}$$
$$H_{\bar{X},\bar{Y}}(\tau) = \begin{cases} \bar{H}_{X,Y}(\tau), & 0 \leq \tau \leq L-1 \\ 0, & \tau = L \\ \bar{H}_{X,Y}(2L - \tau), & L+1 \leq \tau \leq 2L-1. \end{cases} \tag{5}$$

For any frequency slot $f \in F$ and FHS $X = \{x_j\}_{j=0}^{L-1}$, we have

$$u(X, f) = \sum_{j=0}^{L-1} h(x_j, f),$$

which is the number of occurrences that the frequency slot f appeared in X.

Lemma 3 ([12]): Let frequency slot $f \in F$, then we have

$$\sum_{\tau=0}^{L-1} H_X(\tau) = \sum_{X\in\mathcal{S}} u(X, f)^2.$$

3. New Bound on Aperiodic Wide-Gap Frequence-Hopping Sequence

Theorem 1: Let X be a WG FHS of period L over F. Then one has

$$\bar{H}_m(X) \geq \left\lceil \frac{L^2 - r^2 + qr - 2qL}{q(2L - 3)} \right\rceil, \tag{6}$$

where $r = L - \left\lfloor \frac{L}{q} \right\rfloor \cdot q$.

Definition 2: A WG FHS $X \in \mathcal{S}$ is said to be optimal if X reaches the equal sign of Eq.(6).

Proof: From the definition of WG FHS, it's clear that there are

$$H_{\bar{X}}(1) = \sum_{j=0}^{L-1} h\left[\bar{x}_j, \bar{x}_{j+1}\right] + L - 1 = L - 1.$$

Then, we have

$$
\begin{aligned}
\sum_{\tau=0}^{2L-1} H_{\bar{X}}(\tau) &= 2L + L - 1 + \sum_{\tau=2}^{2L-1} H_{\bar{X}}(\tau) \\
&= 3L - 1 + \sum_{\tau=2}^{L-1}\{\bar{H}_X(\tau) + L - \tau\} + 0 \\
&+ \sum_{\tau=L+1}^{2L-1}\{\bar{H}_X(2L-\tau) + \tau - L\} \\
&= 3L - 1 + (L-1)^2 + \sum_{\tau=2}^{L-1}\bar{H}_X(\tau) \\
&+ \sum_{\tau=L+1}^{2L-1}\bar{H}_X(2L-\tau). \\
&= L + L^2 + \sum_{\tau=2}^{L-1}\bar{H}_X(\tau) + \sum_{\tau=1}^{L-1}\bar{H}_X(\tau).
\end{aligned}
$$

Let

$$\bar{PH}(X) = \frac{1}{2L-3}\sum_{\tau=2}^{2L-1}\bar{H}_X(\tau).$$

Then, we have

$$\bar{PH}(X) = \frac{\sum_{\tau=0}^{2L-1} H_{\bar{X}}(\tau) - L^2 - L}{2L-3}. \tag{7}$$

Let $k_{\bar{X}}(\tau|e) = \sum_{j=e}^{e+L-1}\sum_{\tau=0}^{2L-1} h[\bar{x}_j, \bar{x}_{j+\tau}]$, then

$$
\begin{aligned}
k_{\bar{X}}(\tau|0) &= \sum_{j=0}^{L-1}\sum_{\tau=0}^{2L-1} h[\bar{x}_j, \bar{x}_{j+\tau}] \\
&= \sum_{j=0}^{L-1}\sum_{\tau=0}^{L-1} h[\bar{x}_j, \bar{x}_{j+\tau}] + \sum_{j=0}^{L-1}\sum_{\tau=L-1}^{2L-1} h[\bar{x}_j, \bar{x}_{j+\tau}] \\
&= H_X(\tau) = \alpha
\end{aligned}
$$

According to the Lemma 3, $\alpha = \sum_{X\in\mathcal{S}} u(X,f)^2$. $\sum_{X\in\mathcal{S}} u(X,f)^2$ is rewritten as $u_X(f)^2$. It is clear that the optimal solution of the problem is the distribution of $u_X(f)$ that minimizes the value of α if and only if $u_X(f)$ is as nearly uniform as possible.

It is well known that integer L can be expressed in terms of a positive integer I of the form $L = Iq + r, 0 \le r < q$, Thus, the value of α is found to be

$$
\begin{aligned}
\min\alpha &= (q-r)I^2 + r(I+1)^2 \\
&= \frac{L^2 - r^2 + qr}{q}.
\end{aligned} \tag{8}
$$

And it is clear that $k_{\bar{X}}(\tau|L+1) = L^2 - L, 0 \le \tau \le 2L-1$. Then,

$$
\begin{aligned}
\min\left\{\sum_{\tau=0}^{2L-1} H_{\bar{X}}(\tau)\right\} &= \min\{k_{\bar{X}}(\tau|0)\} + \min\{k_{\bar{X}}(\tau|L)\} \\
&+ \min\{k_{\bar{X}}(\tau|L+1)\} \\
&= \alpha + L^2 - L.
\end{aligned} \tag{9}
$$

Combining (7), (8), and (9), one arrive at

$$\bar{PH}(X) \ge \frac{\min\alpha - 2L}{2L-3} = \frac{L^2 - r^2 + qr - 2qL}{q(2L-3)}.$$

Hence, for every aperiodic WG FHS X, one has

$$\bar{H}_m(X) \ge \bar{PH}(X) \ge \frac{L^2 - r^2 + qr - 2qL}{q(2L-3)}.$$

Q.E.D.

4. New Bound on Aperiodic Wide-Gap Frequence-Hopping Sequence set

Lemma 4 ([12]): Let $\bar{\mathcal{S}}$ be a set of M FHSs of period $2L$ over $\bar{F}$. Then we have

$$\sum_{\tau=0}^{2L-1} H_{\bar{X},\bar{Y}}(\tau) \ge \frac{4L^2M^2}{M+q}$$

and

$$\sum_{\tau=0}^{2L-1} H_{\bar{X},\bar{Y}}(\tau) \ge 2(2I+1)LM - I(I+1)(M+q),$$

where $I = \lfloor \frac{2LM}{M+q} \rfloor$.

Lemma 5: For any given aperiodic WG FHS set $\bar{\mathcal{S}}$, we have

$$
\begin{aligned}
\sum_{\bar{X},\bar{Y}\in\bar{\mathcal{S}}}\sum_{\tau=0}^{2L-1} H_{\bar{X},\bar{Y}}(\tau) &\le M(L^2+L) + (L-2)M\bar{H}_{\mathrm{a}}(\mathcal{S}) \\
&+(L-1)M\bar{H}_a(\mathcal{S}) + (M-1)ML\bar{H}_c(\mathcal{S}) \\
&+M(M-1)(L-1)\bar{H}_c(\mathcal{S}).
\end{aligned}
$$

Proof: For any two sequences $\bar{X}, \bar{Y} \in \bar{\mathcal{S}}$ and any two sequences $X, Y \in \mathcal{S}$,

$$
\begin{aligned}
\sum_{\tau=0}^{2L-1} H_{\bar{X},\bar{Y}}(\tau) &= \sum_{\tau=0}^{2L-1} H_{\bar{X}}(\tau) + \sum_{\bar{X}\ne\bar{Y}}\sum_{\tau=0}^{2L-1} H_{\bar{X},\bar{Y}}(\tau) \\
&= H_{\bar{X}}(0) + H_{\bar{X}}(1) + \sum_{\tau=2}^{L-1} H_{\bar{X}}(\tau) + \sum_{\tau=L+1}^{2L-1} H_{\bar{X}}(\tau) \\
&+ \sum_{\bar{X}\ne\bar{Y}}\sum_{\tau=0}^{L-1} H_{\bar{X},\bar{Y}}(\tau) + \sum_{\bar{X}\ne\bar{Y}}\sum_{\tau=L+1}^{2L-1} H_{\bar{X},\bar{Y}}(\tau)
\end{aligned}
$$

$$
\begin{aligned}
&= 2LM + M(L-1) + \sum_{\tau=2}^{L-1} \{\bar{H}_X(\tau) + L - \tau\} \\
&+ \sum_{\tau=L+1}^{2L-1} \{\bar{H}_X(2L-\tau) + \tau - L\} \\
&+ \sum_{X\neq Y}\sum_{\tau=0}^{L-1} \bar{H}_{X,Y}(\tau) + \sum_{X\neq Y}\sum_{\tau=1}^{L-1} \bar{H}_{X,Y}(L-\tau) \\
&\leq 3LM - M + (L-2)M\bar{H}_{\mathrm{a}}(\mathcal{S}) + M\frac{(1+L-2)(L-2)}{2} \\
&+ (L-1)M\bar{H}_a(\mathcal{S}) + M\frac{(1+L-1)(L-1)}{2} \\
&+ (M-1)ML\bar{H}_c(\mathcal{S}) + M(M-1)(L-1)\bar{H}_c(\mathcal{S}).
\end{aligned}
$$

$$
\begin{aligned}
&\leq M(L^2+L) + (L-2)M\bar{H}_{\mathrm{a}}(\mathcal{S}) + (L-1)M\bar{H}_a(\mathcal{S}) \\
&\quad +(M-1)ML\bar{H}_c(\mathcal{S}) + M(M-1)(L-1)\bar{H}_c(\mathcal{S}).
\end{aligned}
$$

Q.E.D.

Combining Lemmas 4 and 5, we can generalize the Peng-Fan bound and Li-Fan-Yang-Wang bound for a aperiodic WG FHS set.

Theorem 2: Let $\mathcal{S}$ be a set of M aperiodic WG FHSs of period L over F, and $I = \lfloor \frac{2LM}{M+q} \rfloor$. Then we have

$$\bar{H}_m(\mathcal{S}) \geq \frac{3ML^2 - ML - qL^2 - q}{(2ML - M - 2)(M+q)} \tag{10}$$

and

$$\bar{H}_m(\mathcal{S}) \geq \frac{ML(4I - L + 1) - (I^2 + I)(M+q)}{M(2ML - M - 2)}. \tag{11}$$

Definition 3: A aperiodic WG FHS set $\mathcal{S}$ is optimal if it achieves the equal sign of Eq.(10) or Eq.(11).

5. Example

Example 1: Let frequency slot set $F = \{0,1,2,3,4,5,6,7,8,9\}$, FHS set $\mathcal{S} = \{X^1, X^2, X^3, X^4\}$, where

$$
\begin{aligned}
X^1 &= [0,2,4,6,8], X^2 = [1,3,5,7,9], \\
X^3 &= [0,4,8,2,6], X^4 = [1,5,9,3,7].
\end{aligned}
$$

It can be verified from the above that the sequence set $\mathcal{S}$ is a WG FHS set where $D = 2$.

As shown in Fig.1, the MAHC value for time delay $0 < \tau \leq L-1$ is 1. By substituting Theorem 2, we get

$$\bar{H}_m(\mathcal{S}) = \lceil \frac{3ML^2 - ML - qL^2 - q}{(2ML - M - 2)(M+q)} \rceil = \lceil \frac{20}{476} \rceil = 1.$$

That is, $\mathcal{S}$ is an optimal set of aperiodic WG FHS set.

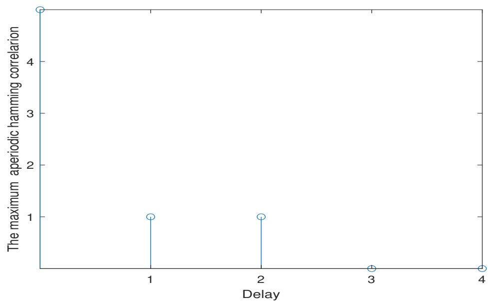

Fig. 1 The maximum aperiodic HC of $\mathcal{S}$

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