LETTER

A Feasible Scheme for the Backward Transmission in the Three-User X Channel with Reciprocal Propagation Delay

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SUMMARY This letter proposes a scheme for the backward transmission of the propagation-delay based three-user X channel, which is reciprocal to the forward transmission. The given scheme successfully delivers 10 expected messages in 6 time-slots by cyclic interference alignment without loss of degrees of freedom, which supports efficient bidirectional transmission between the two ends of the three-user X channel.

key words: *backward transmission, forward transmission, three-user X channel, propagation delay, reciprocal model*

1. Introduction

Recently, the propagation-delay (PD) based X channels have been studied for high transmission efficiency, which is suitable for scenarios with different path PD among links such as the satellite or underwater acoustic communications. Most existing works focus on unidirectional transmission to maximize the achievable degrees of freedom (DoF), including the unicast message case [1]–[5] and the multicast case [6].

However, bidirectional transmission is often involved in practical communications. Given a scheme for forward transmission, it is important to provide the corresponding scheme for the backward transmission. Unfortunately, the reciprocity property shown in [7] cannot be applied to the PD-based X model.

In this letter, we address the reciprocal design for the PD-based three-user X channel [8]. The reciprocity of the PD structure and the message configuration is exploited to maximize the achievable DoF.

2. System Model

The system model for the three-user X channel is shown in Fig. 1. On the left, the forward transmission for messages $\{\overrightarrow{W}_{ij}\}$ happens from three transmitters S_j, $j = 1, 2, 3$ to three receivers T_i, $i = 1, 2, 3$, where τ_{ij} denotes the PD between receiver T_i and transmitter S_j. The backward transmission on the right shows the reciprocal model of the forward transmission, i.e., T_i sends message $\overleftarrow{W}_{ji}$ to S_j via the PD $\tau_{ji} = \tau_{ij}$.

To maximize the achievable DoF, we consider the forward transmission scheme given in [8]. The transmitted polynomial from S_j is

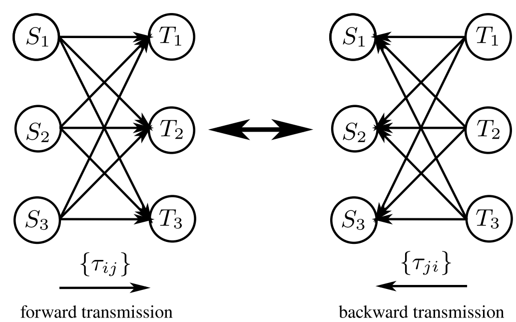

Fig. 1 System model of forward and backward transmissions with reciprocal PD for the three-user X channel.

$$\overrightarrow{v}_1(x) = \overrightarrow{W}_{11} + x^2\overrightarrow{W}_{21} + x^3\overrightarrow{W}^1_{31} + x^5\overrightarrow{W}^2_{31} \mod (x^6 - 1) \quad (1)$$

$$\overrightarrow{v}_2(x) = \overrightarrow{W}_{12} + x^2\overrightarrow{W}_{22} + x^4\overrightarrow{W}_{32} \mod (x^6 - 1) \quad (2)$$

$$\overrightarrow{v}_3(x) = \overrightarrow{W}_{23} + x^1\overrightarrow{W}_{33} + x^2\overrightarrow{W}_{13} \mod (x^6 - 1) \quad (3)$$

The corresponding PD polynomial matrix is

$$\overrightarrow{\boldsymbol{D}} = \begin{pmatrix} x^{\tau_{11}} & x^{\tau_{12}} & x^{\tau_{13}} \\ x^{\tau_{21}} & x^{\tau_{22}} & x^{\tau_{23}} \\ x^{\tau_{31}} & x^{\tau_{32}} & x^{\tau_{33}} \end{pmatrix} = \begin{pmatrix} x^0 & x^1 & x^2 \\ x^0 & x^5 & x^4 \\ x^0 & x^0 & x^0 \end{pmatrix} \quad (4)$$

With the above configuration, each receiver T_i obtains its signals by computing

$$\overrightarrow{r}_i(x) \equiv \overrightarrow{\boldsymbol{D}}(i,:)\overrightarrow{\boldsymbol{v}}^T \mod (x^n - 1) \quad (5)$$

and selects its required messages via cyclic interference alignment. Here $\overrightarrow{\boldsymbol{v}} = (\overrightarrow{v}_1(x), \overrightarrow{v}_2(x), \overrightarrow{v}_3(x))$. In this way, ten messages are forwardly sent in six time-slots, which gives the DoF of 5/3 as shown in [8].

For the backward transmission of the above scheme, its PD matrix is

$$\overleftarrow{\boldsymbol{D}} = \overrightarrow{\boldsymbol{D}}^T = \begin{pmatrix} x^0 & x^0 & x^0 \\ x^1 & x^5 & x^0 \\ x^2 & x^4 & x^0 \end{pmatrix} \quad (6)$$

Our goal is to find a feasible scheme for the backward transmission by determining its transmit polynomials $\{\overleftarrow{v}_i\}$ with the same DoF as the forward transmission.

3. Proposed Scheme

Since the backward transmission is the reciprocal model of

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the forward one, the backward messages $\{\overleftarrow{W}_{ji}\}$ include $\overleftarrow{W}_{11}$, $\overleftarrow{W}_{21}$, $\overleftarrow{W}_{31}$ from T_1, $\overleftarrow{W}_{12}$, $\overleftarrow{W}_{22}$, $\overleftarrow{W}_{32}$ from T_2, and $\overleftarrow{W}^1_{13}$, $\overleftarrow{W}^2_{13}$, $\overleftarrow{W}_{23}$, $\overleftarrow{W}_{33}$ from T_3.

The transmit polynomials are set as

$$\overleftarrow{v}_1(x) = x\overleftarrow{W}_{31} + x^4\overleftarrow{W}_{21} + x^5\overleftarrow{W}_{11} \mod (x^6-1) \quad (7)$$

$$\overleftarrow{v}_2(x) = x\overleftarrow{W}_{32} + x^3\overleftarrow{W}_{12} + x^4\overleftarrow{W}_{22} \mod (x^6-1) \quad (8)$$

$$\overleftarrow{v}_3(x) = \overleftarrow{W}^2_{13} + x\overleftarrow{W}_{23} + x^2\overleftarrow{W}^1_{13} + x^4\overleftarrow{W}_{33} \mod (x^6-1) \quad (9)$$

Then, we check the receive polynomial at S_j by

$$\overleftarrow{r}_j(x) \equiv \overleftarrow{\boldsymbol{D}}(j,:)\overleftarrow{\boldsymbol{v}}^T \mod (x^n-1) \quad (10)$$

where $\overleftarrow{\boldsymbol{v}} = (\overleftarrow{v}_1(x), \overleftarrow{v}_2(x), \overleftarrow{v}_3(x))$.

By (6)–(10) and $n = 6$, we obtain

$$\begin{aligned}\overleftarrow{r}_1(x) \equiv& \overleftarrow{\boldsymbol{D}}(1,:)\overleftarrow{\boldsymbol{v}}^T \mod (x^6-1)\\ \equiv& x^0\overleftarrow{v}_1(x) + x^0\overleftarrow{v}_2(x) + x^0\overleftarrow{v}_3(x) \mod (x^6-1)\\ \equiv& x^5\overleftarrow{W}_{11} + x^3\overleftarrow{W}_{12} + x^2\overleftarrow{W}^1_{13} + \overleftarrow{W}^2_{13}\\ &+ x^1(\overleftarrow{W}_{31} + \overleftarrow{W}_{32} + \overleftarrow{W}_{23})\\ &+ x^4(\overleftarrow{W}_{21} + \overleftarrow{W}_{22} + \overleftarrow{W}_{33}) \mod (x^6-1)\end{aligned} \quad (11)$$

It shows that at S_1 the four required messages $\overleftarrow{W}_{11}$, $\overleftarrow{W}_{12}$, $\overleftarrow{W}^1_{13}$ and $\overleftarrow{W}^2_{13}$ are located at the sixth, fourth, third and first time-slots in each cycle without any interference, respectively. Similarly, there is

$$\begin{aligned}\overleftarrow{r}_2(x) \equiv& \overleftarrow{\boldsymbol{D}}(2,:)\overleftarrow{\boldsymbol{v}}^T \mod (x^6-1)\\ \equiv& x^1\overleftarrow{v}_1(x) + x^5\overleftarrow{v}_2(x) + x^0\overleftarrow{v}_3(x) \mod (x^6-1)\\ \equiv& x^5\overleftarrow{W}_{21} + x^3\overleftarrow{W}_{22} + x\overleftarrow{W}_{23}\\ &+ x^0(\overleftarrow{W}_{11} + \overleftarrow{W}_{32} + \overleftarrow{W}^2_{13})\\ &+ x^2(\overleftarrow{W}_{31} + \overleftarrow{W}_{12} + \overleftarrow{W}^1_{13})\\ &+ x^4\overleftarrow{W}_{33} \mod (x^6-1)\end{aligned} \quad (12)$$

Thus S_2 respectively obtains its three desired messages $\overleftarrow{W}_{21}$, $\overleftarrow{W}_{22}$ and $\overleftarrow{W}_{23}$ from the sixth, fourth, and second time-slots in each cycle with no interference. Finally, the receiver T_3 has

$$\begin{aligned}\overleftarrow{r}_3(x) \equiv& \overleftarrow{\boldsymbol{D}}(3,:)\overleftarrow{\boldsymbol{v}}^T \mod (x^6-1)\\ \equiv& x^2\overleftarrow{v}_1(x) + x^4\overleftarrow{v}_2(x) + x^0\overleftarrow{v}_3(x) \mod (x^6-1)\\ \equiv& x^3\overleftarrow{W}_{31} + x^5\overleftarrow{W}_{32} + x^4\overleftarrow{W}_{33}\\ &+ x^0(\overleftarrow{W}_{21} + \overleftarrow{W}^2_{13}) + x^1(\overleftarrow{W}_{11} + \overleftarrow{W}_{12} + \overleftarrow{W}_{23})\\ &+ x^2(\overleftarrow{W}_{22} + \overleftarrow{W}^1_{13}) \mod (x^6-1)\end{aligned} \quad (13)$$

which indicates that the fourth, sixth, and fifth time-slots respectively contain the three interference-free messages $\overleftarrow{W}_{31}$, $\overleftarrow{W}_{32}$, and $\overleftarrow{W}_{33}$, while the remaining time-slots are occupied by aligned interferences.

In summary, all ten messages are successfully delivered through the backward transmission in six time-slots, which obtains the same DoF as the forward channel. Therefore, the goal has been achieved.

4. Conclusion

Under the reciprocal message and PD structures, the proposed scheme for the backward transmission achieves the same DoF as the forward model of the three-user X channel. It easily enables bidirectional communications in practical scenarios.

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