Doppler Ambiguity Compensation within the Batch for Weak Moving Target Detection in Passive Bistatic Radar

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SUMMARY We consider the Doppler ambiguity compensation problem for weak moving target detection in passive bistatic radar. Detecting an unknown high-speed weak target has a high probability of the presence of Doppler ambiguity, which will decrease the integration performance and accordingly make the target detection difficult under low signal-to-noise ratio (SNR) environments. Resorting to the well-known keystone transform (KT) method, an approach to compensate for the Doppler ambiguity within the batch is proposed for the first time. The proposed approach establishes a good coupling between the reference and echo signals by adding a frequency shift related to the Doppler frequency in the procedure of computing the cross ambiguity function (CAF). Simulation results show that the coherent integration gain of our approach is close to the theoretical upper bound even in the presence of Doppler ambiguity.

key words: Doppler ambiguity compensation, passive bistatic radar, keystone transform, weak moving target

1. Introduction

Passive bistatic radar (PBR) systems have been under extensive research for a few decades due to their excellent properties such as anti-jamming, anti-radiation missiles, and anti-stealth compared to the active radar systems [1]-[3]. As for a low radar cross section (RCS) moving target at long distance, by exploiting illuminators of opportunity such as frequency modulation (FM), digital video broadcasting terrestrial (DVB-T), digital video broadcasting satelllite (DVB-S), cell phone tower [4], PBR systems might suffer from the problem of detecting moving targets under low signal-to-noise ratio (SNR) environments. An effective way to cope with the low SNR problem is increasing the observation time. Inspired by the long-time integration technique particularly designed for active pulsed radar systems, many coherent and non-coherent integration methods have been proposed for PBR systems. They segment the received continuous waveform into multiple batches at the receiver such that the classical slow-time/fast-time framework of a traditional pulsed radar can be established, and accordingly the long-time integration methods defined for the active radar systems, where a batches algorithm architecture [5], [6] is employed, can be applied into PBR systems.

The keystone transform (KT) [7]-[9] and Radon-Fourier transform (RFT) [10] methods are two typical coherent integration strategies used to compensate for the linear range migration of a moving target, which can acquire good integration performance by well compensating for the phase fluctuation and become more suitable for low environments compared to the incoherent integration method [11]. For a PBR system exploiting the slow-time/fast-time framework, to achieve a satisfactory level of performance, the duration of each batch should be larger than the maximum time delay between the reference and echo signals of a moving target of interest, which will lead to a relative low pulse repetition frequency (PRF) when a long-distance target is considered. When the target of interest is moving at a high speed, there is a high likelihood of Doppler ambiguity occurring due to the limitations of PRF.

All the above-mentioned methods [7]-[11] for PBR systems assume that there does not exist a Doppler ambiguity. The coherent integration gain will significantly decrease when the Doppler ambiguity is present [12], and accordingly, it is difficult to detect the moving target under low SNR environments. As for active radar systems, a Doppler ambiguity compensation algorithm between the batches has been proposed in [13]. However, to the best knowledge of the authors, the compensation of Doppler ambiguity within the batch in PBR systems has not been discussed in the literature yet. In this paper, we propose an approach to compensate for the Doppler ambiguity within the batch by establishing a good coupling between the reference and echo signals by adding a frequency shift related to the Doppler frequency in the procedure of computing the cross ambiguity function (CAF).

As well known, the theoretical upper bound on the coherent integration gain is given as $G = BT$ for a PBR system, where $B$ and $T$ are the signal bandwidth and the coherent integration time, respectively. Simulation results show that our proposed approach significantly improves the coherent integration performance compared to the ones without Doppler ambiguity compensation within the batch, and the resulting coherent integration gain is very close to the theoretical upper bound $G$.

In this paper, the organization is as follows. In Sect. 2, we give the signal model, and the KT method is reviewed. The Doppler ambiguity compensation along the slow time and the proposed method of Doppler ambiguity compensation within the batch are given in Sect. 3. In Sect. 4, we evaluate the performance of our method via several simulations. Finally, we make a conclusion in Sect. 5.

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2. Signal Model And Range Migration Compensation via KT

Figure 1 illustrates a simplified PBR system, where the DVB-S is used as an illuminator. The illuminator signal transmits isotropically in all directions. The ground station is equipped with the co-located reference antenna and surveillance antenna, which receive the reference signal and the target echo signal, respectively. L denotes the baseline distance. The target is initially located at position O and then flies along a horizontal direction with a constant velocity. Thus the bistatic range \( R(t) \) is time-varying, where \( R(t) = R_i(t) + R_R(t) \).

The time delay between the reference signal and the echo signal is represented as \( \tau(t) = (R(t) - L)/c \) and the Doppler frequency is defined as \( f_d = -dR(t)/(\lambda dt) \) [7], where \( \lambda \) and \( c \) are the signal wavelength and the speed of light, respectively. Resorting to Taylor expansion and ignoring the quadratic and higher terms, the time delay can be well approximated as [7]

\[
\tau(t) = \tau_0 + \alpha_t t
\]

where \( \tau_0 \) denotes the initial time delay of the echo signal, \( \alpha_t \) is the coefficient of time delay variation which is related to the target motion. Many recent researches focus on the high-order corrections [9], [14] wherein the complex moving target detection problem is considered. To simplify our analysis, we only take into account the first-order case.

Ignoring the channel noise temporally, let \( s_{ref}(t) = s(t) \) denote the baseband reference signal. Then the baseband waveform of the target echo signal can be expressed as

\[
s_{echo}(t) = s(t - \tau_0 - \alpha_t t)e^{-j2\pi f_c \tau_0}e^{-j2\pi f_c \alpha_t t} \tag{2}
\]

where \( f_c \) is the center frequency of the signal, \( e^{-j2\pi f_c \tau_0} \) the constant phase factor caused by the initial time delay, \( e^{-j2\pi f_c \alpha_t} \) the time-varying phase factor caused by the target motion, and \( f_c \).

Here we refer to the method in [7] to reduce the computational complexity of computing the CAF under a long integration time or a large number of sample conditions, where the input continuous-wave signals are segmented into \( M \) batches with the length of each batch being \( L \) samples such that the time delay remains quasi-constant during each batch. The batches are treated as pulses, and the PRF is \( f_s = f_s/L \), where \( f_s \) is the sampling frequency.

Let \( t_m = mL/f_s, m = 1, 2, ..., M \) and \( t_f = t - t_m \) denote the slow time (ST) and the fast time (FT), respectively. We can deduce from Eq. (2):

\[
s_{echo}(t_m + t_f) = s(t_m + t_f - \tau_0 - \alpha_t (t_m + t_f))
+ e^{-j2\pi f_c \tau_0}e^{-j2\pi f_c \alpha_t (t_m + t_f)} \tag{3}
\]

For the sake of subsequent convenience in representation, the time variable of the signal function is decomposed into slow time variable and fast time variable. This expression can be represented as \( s(t_f, t_m) = (t_m + t_f) \), we can obtain

\[
s_{echo}(t_f, t_m) = s(t_f - \tau_0 - \alpha_t (t_m + t_f), t_m)
+ e^{-j2\pi f_c \tau_0}e^{-j2\pi f_c \alpha_t t_m} \tag{4}
\]

Due to the significantly smaller magnitude of the \( \alpha_t t_f \) term within the parentheses compared to other variables in the formula, this term is omitted in subsequent calculations. Assume that the linear phase term within each batch can be neglected. The segmented echo signal of the target can be approximated as [7]

\[
s_{echo}(t_f, t_m) = s(t_f - \tau_0 - \alpha_t t_m, t_m)
+ e^{-j2\pi f_c \tau_0}e^{-j2\pi f_c \alpha_t t_m} \tag{5}
\]

According to [7] the CAF can be quickly calculated as

\[
\chi(t, f_d) = \mathcal{F}^{-1}\{\mathcal{F}\{s_{ref}(t_f, t_m)\}\}^\ast \times
\mathcal{F}\{s_{echo}(t_f, t_m)\}\} \tag{6}
\]

where \( \mathcal{F}\{\cdot\}_1 \) denotes FFT along the FT domain, \( \mathcal{F}\{\cdot\}_2 \) the FFT along the ST domain, \( \mathcal{F}^{-1}\{\cdot\}_1 \) the IFFT along the FT domain, \( (\cdot)^\ast \) the conjugation operator. The term \( \langle\mathcal{F}\{s_{ref}(t_f, t_m)\}\rangle^\ast \times
\mathcal{F}\{s_{echo}(t_f, t_m)\}\} \) is known as frequency-domain pulse compression (FDPC) [7], which can be easily obtained as

\[
S_{ref}(f, t_m) = |S(f)|^2 e^{-j2\pi f_f \tau_0}e^{-j2\pi f_f \alpha_t t_m} \tag{7}
\]

where \( S(f) \) is the spectrum of \( s(t) \). The first exponential term in Eq. (7) indicates the target position at the initial time, whereas the second exponential term reflects the variation of phase along the ST. The term \( \alpha_t t_m \) will lead to the range migration problem and can be efficiently solved by using the KT or RFT methods [7], [10]. As well known, the KT method compensates for this term by rescaling the ST according to the following rule.
\[ i'_m = \frac{f_c + f}{f_c} \times t_m \]  

(8)

where \( i'_m \) is defined as the virtual ST. Substituting Eq. (8) into Eq. (7) yields

\[ S_{re}(f, i'_m) = |S(f)|^2 e^{-j2\pi(f+f_c)\tau_0} e^{-j2\pi f_c \alpha_t i'_m} \]  

(9)

We can get the CAF from (9) as follows

\[ \chi(\tau, f_d) = F^{-1}(F\{S_{re}(f, i'_m)\}^2) \]  

(10)

3. Doppler Ambiguity Compensation

For fast-moving targets, Doppler ambiguity is likely to occur due to the limited radar PRF [12, 15, 16]. Suppose that the Doppler ambiguity factor is \( F \). The Doppler frequency can be expressed as

\[ f_d = f'_d + F \times \text{PRF} \]  

(11)

where \( f'_d \) is the apparent frequency at the sampling frequency. The Doppler ambiguity factor \( F \) can be obtained through the Chinese remainder theorem (CRT) method [17]. Two close PRFs, \( f_1 \) and \( f_2 \), can be obtained by setting different values of batch length \( L \). Their corresponding apparent Doppler frequencies are determined as \( f_{d1} \) and \( f_{d2} \) respectively. Therefore, the following relationships hold

\[ f_d = f_{d1} + F_1 f_1 = f_{d2} + F_2 f_2 \]  

(12)

where \( F_1 \) and \( F_2 \) are integers, and \( F_1 \geq F_2 \). Furthermore, since \( f_1 \) and \( f_2 \) are adjacent PRFs, we can deduce that \( F_1 = F_2 \) or \( F_1 = F_2 + 1 \). Due to the coprimality of \( f_1 \) and \( f_2 \), the solution to Eq. (12) can be obtained using the CRT.

3.1 Doppler ambiguity compensation along the ST

Substituting (11) into (9), we obtain

\[ S_{re}(i'_m) = |S(f)|^2 e^{-j2\pi(f+f_c)\tau_0} e^{-j2\pi f_c \alpha_t i'_m} \]  

(13)

The Doppler ambiguity will reduce the coherent integration gain. We can define a compensation factor \( H(i'_m) = e^{-j2\pi F \times \text{PRF} i'_m} \) to compensate for the Doppler ambiguity along the ST as follows

\[ S'_{re}(f, i'_m) = S_{re}(f, i'_m) H(i'_m) \]  

(14)

\[ = |S(f)|^2 e^{-j2\pi(f+f_c)\tau_0} e^{-j2\pi f_c \alpha_t i'_m} \]

As mentioned in Sect. 2, we can get the CAF by

\[ \chi(\tau, f_d) = F^{-1}\{F\{S'_{re}(f, i'_m)\}^2\} \]  

(15)

3.2 Doppler ambiguity compensation within the batch

Although the coherent integration gain can be improved by compensating for the Doppler ambiguity along the ST as mentioned above, there is still a large SNR loss because the linear phase term with the batch \( e^{-j2\pi \alpha_t f_d} \) has been ignored from Eq. (2) to Eq. (5). The resulted in SNR loss has been expressed as [15]

\[ \text{SNR}_{\text{loss}} = -20 \log_{10} |\text{sinc}(\frac{f_d L}{f_c})| \]  

(16)

where \( \text{sinc}(\cdot) \) denotes the sinc function. The SNR loss will increase with the presence of Doppler ambiguity.

Differing from Eq. (5), we have taken into account linear phase terms in each batch, solely neglecting the \( \alpha_t f_d \) term within the parentheses in Eq. (4). Then the new segmented echo signal is given by

\[ s_{\text{echo}}(f, t_m) = s(f,f_0-\alpha_t t_m,t_m) e^{-j2\pi f_c \tau_0} e^{-j2\pi f_c \alpha_t i'_m} \]  

(17)

Substituting Eq. (11) into Eq. (17) and ignoring the term \( f_{d1} \), we obtain

\[ s_{\text{echo}}(f, t_m) = s(f,f_0-\alpha_t t_m,t_m) e^{-j2\pi f_c \tau_0} e^{-j2\pi f_c \alpha_t (i'_m + t'_m)} \]  

(18)

Performing the FFT along the FT domain for Eq. (18)

\[ S_{\text{echo}}(f, t_m) = F\{s_{\text{echo}}(f,t_m)\} = S(f-f \times \text{PRF}) e^{-j2\pi f_0 \tau_0} e^{-j2\pi f_0 \alpha_t i'_m} \]  

(19)

We can see that the Doppler ambiguity will lead to a frequency shift in the frequency domain, and therefore we add the same frequency shift in the reference signal, i.e., \( S_{\text{echo}}(f-f \times \text{PRF}, t_m) \), to establish a good coupling between the reference signal and echo signal. We can obtain the new FDPC

\[ S_{\text{re}}(f, t_m) = S_{\text{re}}(f, t_m) \times e^{-j2\pi (f_0+f_c) \tau_0} e^{-j2\pi (f_0+f_c) \alpha_t i'_m} \]  

(20)

Here the KT method is used to compensate for the range migration

\[ \tilde{S}_{\text{re}}(f, t'_m) = |S(f-f \times \text{PRF})|^2 \times e^{-j2\pi (f_0+f_c) \tau_0} e^{-j2\pi (f_0+f_c) \alpha_t i'_m} \]  

(21)

Since the Doppler ambiguity still exists in ST domain, we compensate for it with \( H(i'_m) \) as follows

\[ \tilde{S}_{\text{re}}(f, t'_m) = \tilde{S}_{\text{re}}(f, t'_m) H(i'_m) \]  

(22)

Finally, the CAF after Doppler ambiguity compensation within the batch can be obtained by

\[ \chi(\tau, f_d) = F^{-1}\{F\{\tilde{S}_{\text{re}}(f, t'_m)\}^2\} \]  

(23)
Since the apparent frequency $f'_d$ is ignored in Eq. (18), there still exists a small loss in our proposed method. However, it is quite trivial and will not materially affect the target detection performance, which can be shown in the simulation results.

The flowchart of the above steps is illustrated in Fig. 2. The main steps of the proposed method are summarized as follows:

1. Segment the reference and echo signals into multiple batches.
2. Perform the FFT along the FT domain for the reference and echo signals, respectively.
3. Perform the Doppler ambiguity compensation within the batch, i.e., $S_{ref}(f - F \times PRF, t_m)$.
4. Calculate the FDPC, see Eq. (20).
5. Use the KT method to compensate for the range migration, see Eq. (21).
6. Perform the Doppler ambiguity compensation in ST domain, see Eq. (22).
7. Perform the FFT along the virtual ST $t'_m$, and the IFFT along the FT domain to obtain the CAF, see Eq. (23).

4. Simulation Results

Simulations are conducted to investigate the coherent integration performance of the proposed algorithm in the presence of Gaussian noise. In the Doppler ambiguity calculation process of this paper, the batch length $L$ was set to 4000 and 3998, resulting in two approximate PRFs of 13750 Hz and 13757 Hz. The Doppler ambiguity factor can be calculated as $F = 1$. The parameters of the simulation are listed in Table 1. It can be calculated from Table 1 that the Doppler frequency of the target is $f_d = 11900$ Hz and the PRF is PRF $= f_s/L = 13750$ Hz. Since the Doppler frequency is larger than PRF/2, the Doppler ambiguity problem occurs. The apparent frequency $f'_d = -1850$ Hz. The CAF results are shown in Fig 3, 4 and 5 where the SNR of the echo signal is set to $SNR_{in} = -37$ dB. Figure 3 gives the CAF result without Doppler ambiguity compensation. We can see that the echo signal is submerged in the noise background, and no obvious peak at the corresponding target position is generated. Fig. 4 and Fig. 5 give the CAF results with Doppler ambiguity compensation along the ST domain and within the batch, respectively. It is observed that a well-focused peak is generated for both Fig. 4 and Fig. 5. However, the peak value shown in Fig. 5 is much higher than that shown in Fig. 4, which demonstrates that compared to the Doppler ambiguity compensation along the ST domain, the coherent integration performance can be significantly improved by compensating for the Doppler ambiguity within the batch. Regarding the peak of the CAF as the output signal and the other components as the output noise, we can get the output SNR by performing 100 Monte-Carlo experiments as $SNR = 12.82$ dB for Fig. 4 and $SNR = 24.3$ dB for Fig. 5, respectively. The coherent integration gain ($SNR - SNR_{in}$) is easily calculated as 49.82 dB for Fig. 4 and 61.3 dB for Fig. 5. The theoretical gain formula for the system can be expressed as $Gain = 10 \log (B \times T)$. It can be easily obtained from Table 1 that the theoretical upper bound on the coherent integration gain is $Gain = 62.7$ dB. We can see that the coherent integration gain of our method is very close to the theoretical upper bound, and there only exists a gap of about 1.4 dB between them.

To further assess the performance of our method, the target detection performance against the input SNR of the echo signal is shown in Fig. 6. According to the results in [18], the detection probability is related to the output SNR as follows.

$$P_d^N = \frac{1 + \frac{\alpha}{N(1 + SNR_{in})}}{N - 1}$$

where $\alpha = N((P_{f_a}^N)^{-1/N} - 1)$, $N = 32$ represents the reference unit number, and $P_{f_a} = 1 \times 10^{-6}$ are the reference cells number and the false alarm rate, respectively. As above-mentioned simulation, we get the output SNR at the peak of the CAF result by performing 100 Monte-Carlo experiments under different input SNR scenarios. Then the detection probability can be calculated directly from (24) once the output SNR has been obtained. Regarding the sum of the

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<th>Table 1 Parameters of simulation.</th>
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<td>Parameters</td>
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<tr>
<td>Carrier frequency $f_c$</td>
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<tr>
<td>Signal bandwidth $B$</td>
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<tr>
<td>Sampling frequency $f_s$</td>
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<tr>
<td>Initial time delay $\tau_0$</td>
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<td>Coefficient of time delay variation $\alpha$</td>
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<td>The number of batches $M$</td>
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<td>Coherent time $T$</td>
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input SNR and the theoretical upper bound on the coherent integration gain as the output SNR, we can easily get the theoretical upper bound on the detection probability. It can be seen from Fig. 6 that poor detection performance is present under low input SNR scenarios for the cases of Doppler ambiguity without compensation and Doppler ambiguity compensation along the ST domain, whereas our method has a much better performance and very close to the theoretical upper bound on the detection probability.

5. Conclusion

This paper presents an effective algorithm for Doppler ambiguity compensation within the batch by establishing a good coupling between the reference and echo signals. Simulation results demonstrate that the presented algorithm has a good coherent integration performance in the presence of Doppler ambiguity, which is of great benefit for the detection of a high-speed target under low SNR environments.

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References


