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PAPER

A Common Lyapunov Function Approach to Event-Triggered Control with Self-Triggered Sampling for Switched Linear Systems

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SUMMARY In this paper, a common Lyapunov function approach to event-triggered control with self-triggered sampling for switched linear systems is proposed. A switched system is a system where the dynamics can be switched by a switching signal (mode). In the proposed method, based on the upper bound of the common Lyapunov function, the update time of the control input and the mode, and the next sampling time of the state are determined. As a control specification, it is guaranteed that the closed-loop system is uniformly ultimately bounded. Finally, the proposed method is demonstrated by a numerical example.
key words: *common Lyapunov function, event-triggered control, self-triggered sampling, switched linear systems*

1. Introduction

With the development of information and communication technology, a cyber-physical system (CPS) has been attracting attention. A CPS is a system where physical subsystems and information subsystems are deeply connected through a communication network [1], [2]. Much attention has been paid to control methods that reduce the amount of communication and save the energy. In particular, control methods that focus on reducing the number of communications between sensors and controllers or between controllers and actuators have been actively researched.

For a CPS, event-triggered control and self-triggered control have been proposed as the typical control methods [3]. Event-triggered control is a method in which a sensor sends a measured value only when the measured state changes significantly (i.e., event-triggering condition is satisfied) [4]–[8]. As a result, it is possible to reduce the number of updates of the control input thus the amount of communication between the controller and actuator can be reduced. However, if the event-triggering condition is not satisfied for an extended period, it may not always be necessary to take measurements at each time. Self-triggered control is a control method that determines the next sampling time based on the current state and predictions [9]–[12]. This method reduces the number of measurements thus the amount of communication between the sensor and the controller. However, disturbances and uncertainties can unnecessarily shorten the time until the next state is measured. In this paper, we focus on "sampling" in self-triggered control. We introduce the term "self-triggered sampling", in which the next measurement time is calculated using the current measured value [13].

A control method that combines event-triggered control and self-triggered sampling has been proposed [13]. In this method, the next sampling time is calculated based on the policy of self-triggered control. In the conventional self-triggered control method, the control input is necessarily updated if the state is measured. In the method in [13], the controller decides at the sampling time if the control input is updated based on an event-triggering condition. Even if the state is measured, the control input may not be necessarily updated. As a result, the number of communications can be reduced, comparing this method with conventional event-triggered and self-triggered control methods. In [14], we have proposed a new method of event-triggered control with self-triggered sampling based on Lyapunov functions. However, in [13], [14], a linear system is considered as a plant, and may be inadequate when dealing with complex CPSs.

In this paper, we propose a new method of event-triggering control with self-triggered sampling for switched linear systems. A switched system is one of the hybrid systems in which the dynamics are switched according to a switching law. By introducing switched systems, it becomes possible to handle more complex and real-world systems. In our proposed method, we use a common Lyapunov function [15] for the calculation of the next sampling time and the mode, where the state-feedback controller is associated with the mode. Using a common Lyapunov function, we prove that the closed-loop system driven by the proposed method is uniformly ultimately bounded.

This paper is organized as follows. In Sect. 2, the problem formulation on switched systems and event-triggered control with self-triggered sampling is given. In Sect. 3, the upper bounds of a common Lyapunov function is derived. Based on the upper bounds, a procedure of event-triggered control with self-triggered sampling is proposed In Sect. 4, a numerical example is presented to demonstrate the proposed method. In Sect. 5, we conclude this paper.

Notation: Let $\mathcal{R}$ denote the set of real numbers. For the matrix M, the transpose matrix of M is denoted by $M^\top$. For the matrix M, the minimum and maximum eigenvalues of M are denoted by $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$, respectively. For the vector x, let $\|x\|$ denote the Euclidean norm (2-norm) of x.

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2. Problem Formulation

As a plant, consider the following discrete-time switched linear system:

$$x(t+1) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + Dw(t), \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state, $u(t) \in \mathcal{R}^m$ is the control input, $w(t) \in \mathcal{R}^l$ is the disturbance, $t \in \{0, 1, 2, \dots\}$ is the discrete time. The scalar $\sigma(t) \in \mathcal{I} := \{1, 2, \dots M\}$ is the mode (switching signal). For each $i \in \mathcal{I}$, matrices $A_i \in \mathcal{R}^{n\times n}$ and $B_i \in \mathcal{R}^{n\times m}$ are assigned. The matrix $D \in \mathcal{R}^{n\times l}$ is given independently for the mode. The p-th element of $w(t)$ and the p-th column of the matrix D are denoted by $w_p(t)$ and D_p, respectively. Assume that $|w_p(t)| \le W_p$ is satisfied, where W_p is a given scalar. Assume also that the system (1) is stabilizable in the case of $W_p = 0$.

In this paper, we propose a control law that combines self-triggered sampling and event-triggered control. In self-triggered sampling, when the state is sampled at time t_k ($k = 0, 1, 2, \dots$), the next sampling time t_{k+1} is given by

$$t_{k+1} = t_k + S(x(t_k)), \tag{2}$$

where $S(x(t_k))$ is the sampling interval depending on the sampled state $x(t_k)$. The controller that determined both the control input and the mode is given by

$$u(t_k) = \begin{cases} u_{\text{new}} & \text{if a certain condition holds,} \\ u(t_k - 1) & \text{otherwise,} \end{cases} \tag{3}$$

$$\sigma(t_k) = \begin{cases} \sigma_{\text{new}} & \text{if a certain condition holds,} \\ \sigma(t_k - 1) & \text{otherwise,} \end{cases} \tag{4}$$

where u_{new} and σ_{new} are a new control input and a new mode calculated when a certain condition holds, respectively. In other words, updating both the control input and the mode may occur, only when an event-triggering condition is satisfied.

Next, uniformly ultimately boundedness (UUB) [16] is defined as follows.

Definition 1: The closed-loop system composed of the plant (1) and a certain controller is said to be uniformly ultimately bounded (UUB) in a convex and compact set $\mathcal{X}$ containing the origin in its interior, if for every initial condition $x(0) = x_0$, there exists $T(x_0)$ such that for $k \ge T(x_0)$ and $T(x_0) \in \{0, 1, 2, \dots\}$, the condition $x(k) \in \mathcal{X}$ holds.

Using the notion of UUB, it is expected that a longer sampling interval can be obtained at the neighborhood of the origin. Based on these preparations, we consider the following problem.

Problem 1: For the switched linear system (1), design $S(x(t_k))$ in (2), an event-triggering condition, u_{new} in (3), and σ_{new} in (4).

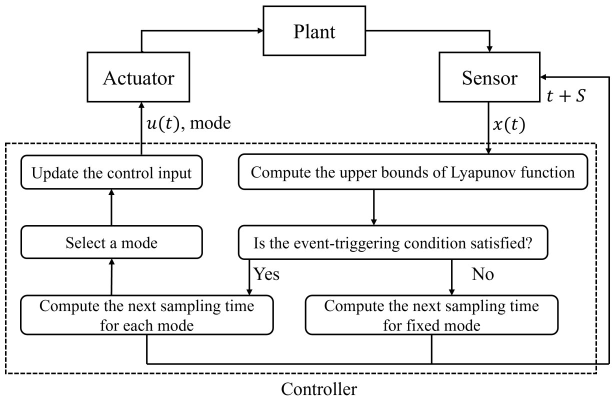

Fig. 1 Outline of the proposed procedure.

3. Proposed method

3.1 Outline

In this subsection, we explain the outline of the proposed procedure shown in Fig. 1. In the proposed procedure, an event-triggering condition is introduced using the upper bounds of the Lyapunov function. The sampling interval $S(x(t_k))$ is calculated in both the case where an event-triggering condition is satisfied and the case where an event-triggering condition is not satisfied. Hence, the function of $S(x(t_k))$ must be derived for each case. Using the proposed procedure shown in Fig. 1, we can achieve that the closed-loop system is UUB while the number of updates of the control input, the mode, and the sampled state is decreased.

3.2 Preparation

In this subsection, we consider the discrete-time switched linear system (1) with no disturbance ($W_p = 0$).

Using $A_i, B_i, i \in \mathcal{I}$, consider the following simultaneous Lyapunov equations:

$$(A_i + B_iK_i)^\top P(A_i + B_iK_i) - P = -Q_i, \ \ i \in \mathcal{I}. \tag{5}$$

Assume that there exist both the positive definite symmetric matrix $P \in \mathcal{R}^{n\times n}$ and the matrix $K_i \in \mathcal{R}^{m\times n}$. The matrix K_i represents the state feedback gain that stabilizes $A_i + B_iK_i$. The matrix $Q_i \in \mathcal{R}^{n\times n}$ is an arbitrary positive definite symmetric matrix. When there exist P and K_i, the closed-loop system with the state-feedback controller $u(t) = K_{\sigma(t)}x(t)$ is asymptotically stable under arbitrary switching [15]. In other words, for the discrete-time switched linear system (1) with no disturbance, the following common Lyapunov function $V(t)$ monotonically decreases:

$$V(t) = x^\top(t)Px(t) = \left\| P^{\frac{1}{2}}x(t) \right\|^2. \tag{6}$$

Next, we define

$$\bar{A} := \sum_{i=1}^{M} \alpha_i(A_i + B_iK_i), \ \ \sum_{i=1}^{M} \alpha_i = 1, \ \ \alpha_i \ge 0, \tag{7}$$

where α_i, $i \in \mathcal{I}$ are given. There exists the positive definite symmetric matrix $Q \in \mathcal{R}^{n\times n}$ satisfying the following Lyapunov equation:

$$\bar{A}^\top P\bar{A} - P = -Q. \tag{8}$$

See Appendix A for further details. From (8), $\bar{A}$ is stable. Then, we can obtain the following lemma.

Lemma 1: For the discrete-time switched linear system (1) with no disturbance, assume that P, K_i in (5) and Q in (8) are given. Consider $u(t) = K_{\sigma(t)}x(t)$ as a controller. Then, there exist $\sigma(t) \in \mathcal{I}$ (a mode at each time) such that the following inequality holds for any time t:

$$V(t) \leqq (1-\lambda)^t V(0),\ \ \lambda := \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}.$$

Proof : Consider the system given by $x'(t+1) = \bar{A}x'(t)$. From (8), for any initial state $x'(0)$, we can obtain

$$\frac{x'^\top(1)Px'(1)}{x'^\top(0)Px'(0)} = 1 - \frac{x'^\top(0)Qx'(0)}{x'^\top(0)Px'(0)} \leq 1-\lambda,$$

which implies that there exists $\bar{i} \in \mathcal{I}$ satisfying

$$x'^\top(0)(A_{\bar{i}} + B_{\bar{i}}K_{\bar{i}})^\top P(A_{\bar{i}} + B_{\bar{i}}K_{\bar{i}})x'(0) \leq (1-\lambda)x'^\top(0)Px'(0).$$

For the closed-loop system consisting of the system (1) with no disturbance and the controller $u(t) = K_{\sigma(t)}x(t)$, suppose that the initial state $x(0)$ is arbitrarily given. From the above result, there exists a mode $\sigma(0)$ satisfying

$$V(1) \leq (1-\lambda)V(0).$$

Using this result recursively, we can obtain Lemma 1. □

3.3 Upper Bound of the Common Lyapunov Function Based on the Current State

In this subsection and the next subsection, we derive two kinds of upper bounds of the common Lyapunov function for the discrete-time switched linear system (1) with disturbances. These upper bounds are used in design of the sampling interval $S(x(t_k))$ and an event-triggering condition.

Let $V_{\hat{u},i}(t+N|t)$ denote the predicted common Lyapunov function at $t+N$ for the system (1), where the current state $x(t) = x_t$ is given (t is the current time), and assume that the mode and the control input are given by $\sigma(t) = \sigma(t+1) = \cdots = \sigma(t+N-1) = i$ and $u(t) = u(t+1) = \cdots = u(t+N-1) = \hat{u}$, respectively. Let $\hat{V}_{\hat{u},i}(t+N|t)$ denote the upper bound of $V_{\hat{u},i}(t+N|t)$ satisfying

$$\max_{w(t+j),\ j=0,1,\ldots,N-1} V_{\hat{u},i}(t+N|t) \leq \hat{V}_{\hat{u},i}(t+N|t).$$

The upper bound $\hat{V}_{\hat{u},i}(t+N|t)$ is used for evaluating the performance when the control input and the mode are not updated in self-triggered sampling and event-triggered control.

From (1) and (6), $V_{\hat{u},i}(t+N|t)$ can be derived as

$$V_{\hat{u},i}(t+N|t) = \left\| P^{\frac{1}{2}}\left(A_i^N x_t + \sum_{j=0}^{N-1} A_i^j\left(B_i\hat{u} + Dw(t+j)\right)\right)\right\|^2,$$

where P is a solution for simultaneous Lyapunov equations (5). Considering $|w_p(t)| \leq W_p$, we can obtain

$$\begin{aligned}\max_{\substack{w(t+j),\\ j=0,1,\ldots,N-1}} V_{\hat{u},i}(t+N|t) \leq{}& \left\| P^{\frac{1}{2}}(A_i^N x_t + \sum_{j=0}^{N-1}(A_i^j B_i\hat{u}))\right\|^2 \\ &+ 2\left\| P^{\frac{1}{2}}(A_i^N x_t + \sum_{j=0}^{N-1}(A_i^j B_i\hat{u}))\right\| \\ &\cdot \sum_{p=1}^{l}\sum_{j=0}^{N-1}\left\|P^{\frac{1}{2}}A_i^j D_p\right\| W_p \\ &+ \left(\sum_{p=1}^{l}\sum_{j=0}^{N-1}\left\|P^{\frac{1}{2}}A_i^j D_p\right\| W_p\right)^2 \\ =:{}& \hat{V}_{\hat{u},i}(t+N|t).\end{aligned}$$

3.4 Upper Bound of the Common Lyapunov function Based on the Initial State

Next, we derive the upper bound of the common Lyapunov function $V(t)$ for the system (1) when the initial state $x(0)$ is given. In this paper, we consider the upper bound, which does not depend on the mode at each time.

Based on Lemma 1, consider the system given by

$$x'(t+1) = \bar{A}x'(t) + Dw(t) \tag{9}$$

and the Lyapunov function

$$\begin{aligned}V'(t) &= x'^\top(t)Px'(t) \\ &= \left\| P^{\frac{1}{2}}\bar{A}^t x'(0) + P^{\frac{1}{2}}\sum_{j=0}^{t-1}\bar{A}^j Dw(j)\right\|^2,\end{aligned}$$

where P is a solution for simultaneous Lyapunov equations (5). Let $\bar{V}(t)$ denote the upper bound of $V'(t)$ satisfying

$$\max_{w(j),\ j=0,1,\ldots,t-1} V'(t) \leq \bar{V}(t).$$

Considering the worst disturbance at each time, we can obtain

$$\max_{\substack{w(j),\\ j=0,1,\ldots,t-1}} V'(t) \leq (1-\lambda)^t V'(0)$$

$$+ 2\left\|P^{\frac{1}{2}}\bar{A}^t x'(0)\right\| \sum_{p=1}^{l}\sum_{j=0}^{t-1}\left\|P^{\frac{1}{2}}\bar{A}^j D_p\right\| W_p + \left(\sum_{p=1}^{l}\sum_{j=0}^{t-1}\left\|P^{\frac{1}{2}}\bar{A}^j D_p\right\| W_p\right)^2 =: \bar{V}(t)$$

See also [14] for further details. For $\bar{V}(t)$, the following lemma holds.

Lemma 2: Assume that P, K_i in (5) and Q in (8) are given. For two systems (1) and (9), assume that $x(0) = x'(0)$ holds (i.e., $V(0) = V'(0)$). For the discrete-time switched linear system (1), consider $u(t) = K_{\sigma(t)}x(t)$ as a controller. Then, there exists $\sigma(t) \in \mathcal{I}$ such that $V(t) \leq \bar{V}(t)$ holds for any time t.

Proof : Since the matrix D for disturbances is the same in two systems (1) and (9), this lemma is immediately derived from Lemma 1 and the definition of $\bar{V}(t)$. □

Remark 1: In this paper, we assume that the matrix D is not switched. There are two reasons. First, this assumption is needed to introduce the system given by (9). Next, when the dynamics are widely switched, there does not exist a common Lyapunov function in many cases. Hence, it is desirable that the dynamics are not widely switched and some physical parameters in A_i, B_i, $i \in \mathcal{I}$ are slightly switched.

3.5 Derivation of Event-Triggering Condition and Sampling Interval Function

Using $\hat{V}_{\hat{u},i}(t+N|t)$ and $\bar{V}(t)$, we derive the sampling interval $S(x(t_k))$ in (2) and an event-triggering condition in (3), and (4).

First, the set $\mathcal{X}$ in Problem 1 is given by

$$\mathcal{X} = \{x \mid x^\top Px \leq \beta\}, \tag{10}$$

where β is given scalar satisfying

$$\beta > \lim_{t\to\infty} \bar{V}(t).$$

Then, under an appropriate control law, there exists $T(x(0))$ such that $x(t) \in \mathcal{X}$ holds for any $t \geq T(x(0))$. Using β, we propose the following event-triggering condition:

$$\hat{V}_{u(t_k-1),\sigma(t_k-1)}(t_k+1|t_k) > \max(\bar{V}(t_k+1), \beta). \tag{11}$$

The sampling interval $S(x(t_k))$ in (2) and the controller (3), (4) are rewritten as

$$S(x(t_k)) = \begin{cases} S'(x(t_k)) & \text{if (11) is satisfied,} \\ S''(x(t_k)) & \text{otherwise,} \end{cases} \tag{12}$$

$$u(t_k) = \begin{cases} u_{\text{new}} & \text{if (11) is satisfied,} \\ u(t_k-1) & \text{otherwise,} \end{cases} \tag{13}$$

$$\sigma(t_k) = \begin{cases} \sigma_{\text{new}} & \text{if (11) is satisfied,} \\ \sigma(t_k-1) & \text{otherwise,} \end{cases} \tag{14}$$

respectively.

Next, consider the case where (11) is satisfied. In this case, we derive both the mode and the control input such that the sampling interval becomes longer, while the closed-loop system is UUB in the set $\mathcal{X}$. As a preparation, we define

$$S'_i(x(t_k)) := \min\left\{\tau \geq 1 \mid \hat{V}_{K_i x(t_k),i}(t_k+\tau+1|t_k) > \max(\bar{V}(t_k+\tau+1), \beta)\right\}. \tag{15}$$

Using $S'_i(x(t_k))$, the set $\mathcal{I}_{\text{pre}}$ is defined by

$$\mathcal{I}_{\text{pre}} := \arg\max_{i\in\mathcal{I}} S'_i(x(t_k)),$$

which represents the set of modes such that the sampling interval is the longest when both the mode and the control input are updated. Moreover, the set $\mathcal{I}_{\text{new}}$ is defined by

$$\mathcal{I}_{\text{new}} := \arg\min_{i'\in\mathcal{I}_{\text{pre}}} \hat{V}_{K_{i'}x(t_k),i'}(t_k + \max_{i\in\mathcal{I}} S'_i(x(t_k))|t_k).$$

A new mode σ_{new} in (14) can be derived as an arbitrary element of the set $\mathcal{I}_{\text{new}}$. By choosing σ_{new} from the $\mathcal{I}_{\text{new}}$, it is expected that the future Lyapunov function is as small as possible. Using the obtained new mode σ_{new}, a new control input u_{new} in (13) and $S'(t_k)$ in (12) can be derived as

$$u_{\text{new}} = K_{\sigma_{\text{new}}}x(t_k),$$
$$S'(x(t_k)) = \max_{i\in\mathcal{I}} S'_i(x(t_k)), \tag{16}$$

respectively.

Finally, consider the case where (11) is not satisfied. In this case, since the mode and the control are not updated (i.e., $u(t_k) = u(t_k-1)$ and $\sigma(t_k) = \sigma(t_k-1)$), we consider deriving only $S''(t_k)$ in (12). Then, $S''(t_k)$ can derived as

$$S''(x(t_k)) = \min\left\{\tau \geq 1 \mid \hat{V}_{u(t_k),\sigma(t_k)}(t_k+\tau+1|t_k) > \max(\bar{V}(t_k+\tau+1), \beta)\right\}.$$

3.6 Proposed Procedure

We present the proposed procedure of event-triggering control with self-triggered sampling.

Procedure of event-triggering control with self-triggered sampling:

Step 1: Set $t = 0$ and $x(0) = x_0$.

Step 2: Calculate $S'(x(t))$, σ_{new}, and u_{new}.

Step 3: Apply both the control input $u(t) = u_{\text{new}}$ and the mode $\sigma(t) = \sigma_{\text{new}}$ to the plant. Set $S(x(t)) = S'(x(t))$.

Step 4: Update $t := t + S(x(t))$.

Step 5: Wait until t, and measure $x(t)$.

Step 6: If the event-triggering condition (11) is satisfied, then go to Step 2, otherwise Step 7.

Step 7: Calculate $S''(x(t))$, set $S(x(t)) = S''(x(t))$, and go to Step 4.

See also Fig.1. For the closed-loop system applied the above procedure, we can obtain the following theorem.

Theorem 1: Assume that P, K_i in (5) and Q in (8) are given. For the discrete-time switched linear system (1), the closed-loop system driven by the above procedure is UUB in the set $\mathcal{X}$.

Proof : The control input and the mode are updated when (11) holds. From the definition of $\hat{V}_{\hat{u},i}(t+N|t)$ and Lemma 2, by these updates, the following condition holds:

$$V(t_k+1) \le \max(\bar{V}(t_k+1), \beta).$$

Hence, the closed-loop system is UUB in the set $\mathcal{X}$. □

In the above procedure, we set $S(x(t_k)) = 1$. The modified procedure gives an event-triggered control method. In this case, the number of updates of the control input and the mode may be decreased, but it is required that the state is measured at each time. From Theorem 1, it is guaranteed that the closed-loop system is UUB in the set $\mathcal{X}$.

On the other hand, in the above procedure, we modify Step 6 to "go to Step 2". In the modified procedure, the event-triggering condition is not used. Since the next sampling time is calculated, the modified procedure gives a self-triggered control method. In this case, from (16) (i.e., (15)), it is guaranteed that the closed-loop system is UUB in the set $\mathcal{X}$.

Thus, the proposed procedure includes both event-triggered control and self-triggered control as a special case.

4. Numerical Example

We present a numerical example to show the effectiveness of the proposed method. Consider the discrete-time switched linear system with three modes ($M = 3$). The matrices are given by

$$A_1 = \begin{bmatrix} 1.1 & 0.9 \\ 0 & 1.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.5 \\ 0.9 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1.15 & 0.8 \\ 0.1 & 1.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 1.1 & 0.9 \\ -0.1 & 1.1 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -0.4 \\ 0.9 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}.$$

Assume that $|w(t)| \le 0.5$. By solving simultaneous Lyapunov equations, we can obtain

$$P = \begin{bmatrix} 0.0138 & 0.0199 \\ 0.0199 & 0.0357 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -0.9805 & -2.5790 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.9761 & -2.0599 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -0.9838 & -2.7312 \end{bmatrix},$$

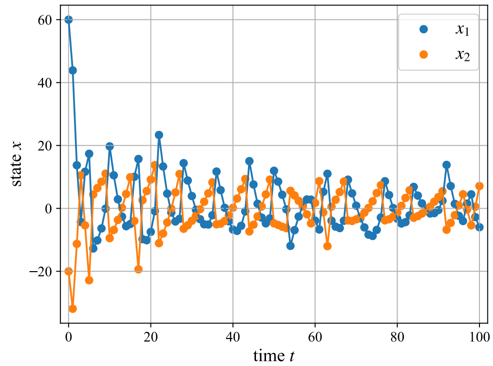

Fig. 2 Time response of the state.

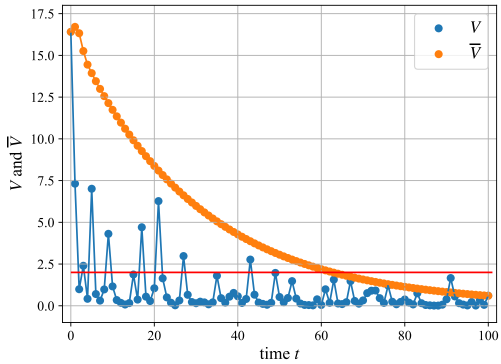

Fig. 3 Time response of V and $\bar{V}$.

$$Q_1 = \begin{bmatrix} 0.0068 & 0.0102 \\ 0.0102 & 0.0220 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 0.0064 & 0.0114 \\ 0.0114 & 0.0254 \end{bmatrix},$$
$$Q_3 = \begin{bmatrix} 0.0069 & 0.0103 \\ 0.0103 & 0.0225 \end{bmatrix}.$$

In addition, we set $\alpha_1 = 0.4$, $\alpha_2 = 0.3$, and $\alpha_3 = 0.3$.

We present a simulation result. The initial state and the parameter β in (10) are given by $x(0) = [60, -20]^\top$ and $\beta = 2$, respectively. The disturbance is generated by uniformly distributed random numbers in the interval $[-0.5, 0.5]$. Fig. 2 shows the time response of the state. From this figure, we see that the state convergences the neighborhood of the origin. Fig. 3 shows the time response of the Lyapunov function V and its upper bound $\bar{V}$. From this figure, we see that the closed-loop system is UUB. Fig. 4 shows the control input. From this figure, we see that the control input is sometimes not updated. Fig. 5 shows the mode sequence. From this figure, we see that the mode is not fixed, and is sometimes changed. Fig. 6 shows the event occurrence. From this figure, we see that even if the state is sampled, the control input and the mode are not necessarily updated. Thus, it is guaranteed that the closed-loop system is UUB, while the communication cost is reduced.

Finally, we discuss the effect of mode switches. We consider the following four cases: i) the mode is switched based on the proposed method, ii) the mode is fixed as 1, iii)

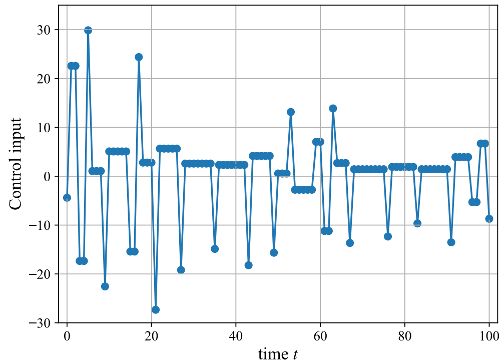

Fig. 4 Control input.

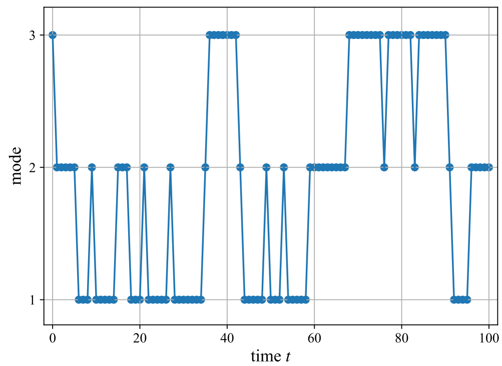

Fig. 5 Mode sequence.

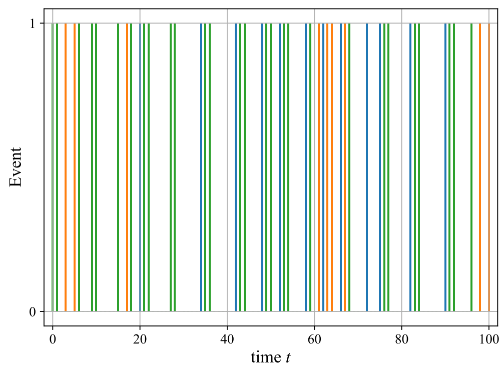

Fig. 6 Event occurrence. The blue line implies that the state is sampled, and both the control input and the mode are not updated. The orange line implies that the state is sampled, the control input is updated, but the mode is not updated. The green line implies that the state is sampled, and both the control input and the mode are updated.

the mode is fixed as 2, and iv) the mode is fixed as 3. Since we consider UUB as a control specification, it is not appropriate to discuss the convergence speed of the Lyapunov function. We focus on the number of times that the state is sampled and the number of times that the control input is updated. Table 1 shows the computation result, where the disturbance is the same for all cases. From this table, we see that these numbers can be reduced by mode switches.

Table 1 Effect of mode switches, where "#1" and "#2" are the number of state samplings and the number of control input updates, respectively.

	#1	#2
Case i)	49	37
Case ii)	52	39
Case iii)	63	54
Case iv)	66	63

5. Conclusion

In this paper, we proposed a new method of event-triggered control with self-triggered sampling for switched linear systems. In the proposed method, the control input, the mode, and the sampling interval are calculated using the upper bounds of the common Lyapunov function. Since the proposed method performs state measurements and control input updates only when necessary, the communication and energy costs can be reduced. It is guaranteed that the closed-loop system is UUB.

A control method using common Lyapunov functions is frequently conservative. One of the future efforts is to develop a method where a common Lyapunov function is not used. In addition, applying the proposed method to real and practical systems is also important.

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Appendix A: Derivation of the Lyapunov equation (8)

For the matrix M, let $M \succ 0$ denote that M is positive definite. Then, simultaneous Lyapunov equations (5) can be rewritten as

$$P - (A_i + B_i K_i)^\top P (A_i + B_i K_i) \succ 0, \ \ i \in \mathcal{I},$$

which can be equivalently transformed into

$$\begin{bmatrix} P & (A_i + B_i K_i)^\top P \\ P(A_i + B_i K_i) & P \end{bmatrix} \succ 0, \ \ i \in \mathcal{I}$$

by applying the Schur complement [17]. Using α_i, $i \in \mathcal{I}$ in (7), we can obtain

$$\begin{bmatrix} P & \bar{A}^\top P \\ P\bar{A} & P \end{bmatrix} \succ 0,$$

that is, $P - \bar{A}^\top P \bar{A} \succ 0$. This implies that there exists the positive definite matrix Q satisfying the Lyapunov equation (8).

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