PAPER

A Closed-Form of 2-D Maximally Flat Diamond-Shaped Half-Band FIR Digital Filters with Arbitrary Difference of the Filter Orders

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SUMMARY Two-dimensional (2-D) maximally flat finite impulse response (FIR) digital filters have flat characteristics in both passband and stopband. 2-D maximally flat diamond-shaped half-band FIR digital filter can be designed very efficiently as a special case of 2-D half-band FIR filters. In some cases, this filter would require the reduction of the filter lengths for one of the axes while keeping the other axis unchanged. However, the conventional methods can realize such filters only if difference between each order is 2, 4 and 6. In this paper, we propose a closed-form frequency response of 2-D low-pass maximally flat diamond-shaped half-band FIR digital filters with arbitrary filter orders. The constraints to treat arbitrary filter orders are firstly proposed. Then, a closed-form transfer function is achieved by using Bernstein polynomial.

key words: *2-D diamond-shaped filter, maximally flat, half-band filter, arbitrary different filter orders, closed-form expression*

1. Introduction

One of the basic operations in multirate digital signal processing is sampling rate conversion by upsampling and downsampling [1]–[4]. Especially, the 2 : 1 sampling rate conversion from the orthogonal to quincuncial sampling pattern is very important in the field of two dimensional (2-D) signal processing [5], [6]. 2-D low-pass diamond-shaped half-band FIR digital filters (2-D DH filters) are used to avoid aliasing after decimation. It is very easy to implement these filters because impulse response of them has a quincuncial sampling pattern and approximately half of the filter coefficients are zero.

Many design methods for 2-D DH filter have been proposed [7]–[18]. An optimization based design method [10], [16]–[18] are well-known. These methods realize a steep cut-off characteristic, however, it causes passband ripples that may distort input signals. On the other hand, many design methods for 2-D low-pass maximally flat diamond-shaped half-band FIR digital filter (2-D MFDH filter) have been proposed [11]–[15]. This filter can achieve high accurate extraction of input signal. This filter is often preferred for image signal processing because it usually has less ringing in the step response compared with filters having passband ripple. A design method of this filter was proposed by solving the linear simultaneous equations obtained from constraints about the magnitude flatness at $(\omega_1, \omega_2) = (0, 0)$ [11]. However, in this method, the linear simultaneous equations need to be formulated and solved each time when design specification is changed. To solve this problem, design methods of this filter was proposed by formulating the closed-form transfer function based on Bernstein polynomial [13]–[15], [19]. There is no need to solve a set of linear simultaneous equations.

In recent years, 2-D MFDH filters is required to have the different filter order for each axis [12]. 2-D MFDH filters with different orders are required in some applications, for example, interlace-to-noninterlace scanning converter in TV signal processing [12], [16], sampling rate conversion for different aspect ratio images, and sampling structure conversion for array systems. The design method of this filter by using linear equations was proposed [12]. In this method, the degree of freedom for frequency response of the filter increases according to difference of filter orders. Therefore, this method employs additional constraints at $(\omega_1, \omega_2) = (0, 0)$, in the $\omega_1 = \omega_2$ direction and for equi-amplitude line. However, the difference of filter orders of this method only can be set among 2, 4 and 6. Furthermore, as shown in Fig. 1(a) and 1(b), this method realize the magnitude response having peaks which may distort an input signal.

In this paper, we propose a design method for 2-D linear phase MFDH filters with arbitrary difference of filter orders and monotonically decreasing magnitude response. First, we propose novel constraints which are imposed only at $(\omega_1, \omega_2) = (0, 0)$. Next, the proposed magnitude response is achieved as a closed-form solution by using Bernstein polynomial. The parameter of the proposed method is only the flatness degree for magnitude response at $(\omega_1, \omega_2) = (0, 0)$. Then, we show design examples to confirm that the proposed method can design the filters regardless the filter order difference and realize the flat magnitude response. Furthermore, we show that all the impulse responses of the proposed filter has quincuncial sampling patterns.

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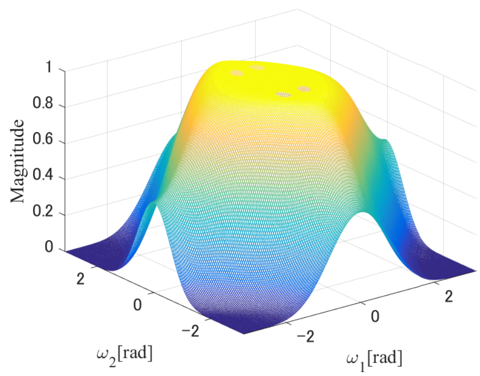
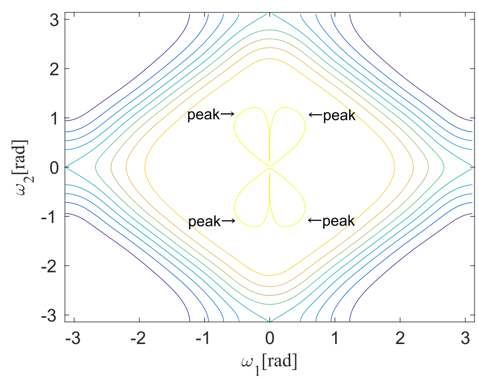


(a) The perspective plot



(b) The contour plot

Fig. 1 The magnitude response of (8×14) order 2-D MFDH filters designed by the conventional method [12].

2. Design Method

2.1 Definition of 2-D Half-Band FIR Digital Filters

In general, the frequency response of a $(2N_1 \times 2(N_1 + d))$ order liner phase 2-D FIR digital filter is given as

$$\begin{aligned} H(\omega_1, \omega_2) &= \sum_{n_1=0}^{2N_1} \sum_{n_2=0}^{2(N_1+d)} h(n_1, n_2) \mathrm{e}^{-j\omega_1 n_1} \mathrm{e}^{-j\omega_2 n_2} \\ &= H_0(\omega_1, \omega_2) \mathrm{e}^{-j\omega_1 N_1} \mathrm{e}^{-j\omega_2 (N_1+d)}, \end{aligned} \tag{1}$$

where $h(n_1, n_2)$, d, and $H_0(\omega_1, \omega_2)$ are the filter coefficients, an integer equal to or greater than 0, and the zero-phase frequency response. Here, the frequency response of the filter given by equation (1) is symmetric with respect to $(\omega_1, \omega_2) = (0, 0)$ in the frequency plane. Furthermore, the frequency response of equation (1) is symmetric with respect to the ω_1 and ω_2 axes because the coefficients of the linear phase 2-D FIR digital filter hold

$$\begin{aligned} h(n_1, n_2) &= h(2N_1 - n_1, n_2) \\ &= h(n_1, 2(N_1 + d) - n_2) \\ &= h(2N_1 - n_1, 2(N_1 + d) - n_2). \end{aligned} \tag{2}$$

Consequently, the frequency response of 2-D zero-phase FIR digital filters is given as

$$H_0(\omega_1, \omega_2) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_1+d} \tilde{h}(n_1, n_2) \cos n_1\omega_1 \cos n_2\omega_2, \tag{3}$$

where $\tilde{h}(n_1, n_2)$ is a coefficient led by $h(n_1, n_2)$ as shown in [13]. In this paper, we will introduce the design method for 2-D zero-phase DH filter because the filter coefficients of the linear phase filter are derived as time-shifted coefficients of the zero-phase filter.

A 2-D zero-phase FIR digital filter is said to be 2-D zero-phase DH filter if

$$H_0(\omega_1, \omega_2) + H_0(\pi - \omega_1, \pi - \omega_2) = 1 \tag{4}$$

is satisfied for arbitrary ω_1 and ω_2. Equation (1) indicates that the frequency response is symmetric about $(\omega_1, \omega_2, H_0) = (\pi/2, \pi/2, 0.5)$ in the space $\{(\omega_1, \omega_2, H_0) | 0 \leq \omega_1, \omega_2 \leq \pi\}$. Then, the coefficients of 2-D zero-phase DH FIR digital filters must satisfy [12]

$$\begin{cases} \tilde{h}(n_1, n_2) = \dfrac{1}{2} & (n_1 = n_2 = 0) \\ \tilde{h}(n_1, n_2) = 0 & (n_1 + n_2 = \text{even}) \end{cases}. \tag{5}$$

2.2 The Proposed Method

To design the $(2N_1 \times 2(N_1 + d))$ order 2-D zero-phase MFDH filter, it is necessary to match the number of the coefficients and the number of constraints. That is, the frequency response of this filter must satisfy the following constraints:

$$H_0(\omega_1, \omega_2)\big|_{\substack{\omega_1=0 \\ \omega_2=0}} = 1 \tag{6a}$$

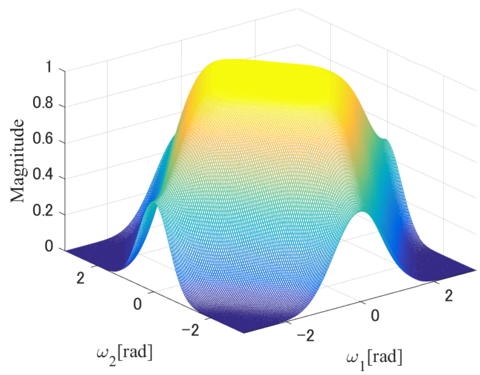
$$H_0(\omega_1, \omega_2)\big|_{\substack{\omega_1=0 \\ \omega_2=\pi}} = 0.5 \tag{6b}$$

$$\left.\frac{\partial^i H_0}{\partial\omega_1^j \partial\omega_2^{i-j}}\right|_{\substack{\omega_1=0 \\ \omega_2=0}} = 0 \quad \left(\begin{array}{l} i = 2, 4, \cdots, 2(N_1 + d - 1) \\ j = 0, 2, \cdots, 2(s(i) - 1) \end{array} \right). \tag{6c}$$

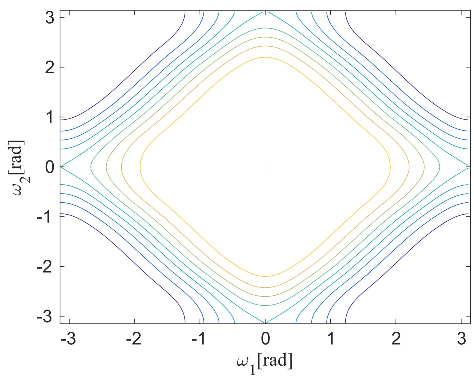
Note that the constraints at $(\omega_1, \omega_2) = (\pi, \pi)$ and $(\omega_1, \omega_2) = (\pi, 0)$ are derived to satisfy (4) from (6). In (6c), $s(i)$ is the flatness degree for the magnitude response at $(\omega_1, \omega_2) = (0, 0)$ and $(\omega_1, \omega_2) = (\pi, \pi)$ and given by

$$s(i) = \begin{cases} \dfrac{i}{2} + 1 & (i \leq 2(N_1 - 1)) \\ m(i) & \left(2N_1 \leq i \leq 2\left(N_1 + \dfrac{d}{2}\right)\right), \\ N_1 - m(4N_1 + 2d - i) + 1 & \left(i > 2\left(N_1 + \dfrac{d}{2}\right)\right) \end{cases} \tag{7}$$

where $m(i)$ is an integer parameter to control the shape of equi-amplitude line, and it is determined as follows:



(a) The perspective plot



(b) The contour plot

Fig. 2 The magnitude response of (8×14) order 2-D MFDH filter designed by the proposed method.

1. $m(2N_1) = N_1$
2. if d is an even integer, $m\left(2\left(N_1 + \frac{d}{2}\right)\right) = \lfloor \frac{N_1+1}{2} \rfloor$
3. For $2N_1 < i < 2(N_1 + d/2)$, $m(i)$ is set to satisfy $m(i-n) \geq m(i) > \lfloor \frac{N_1+1}{2} \rfloor$ with an even positive integer n

In the above $m(i)$ decision rules, $\lfloor x \rfloor$ denotes the maximum integer not exceeding x. From the last rule, the number of $m(i)$ to be set is $\lfloor (d-1)/2 \rfloor$. Therefore, there are $(N_1 - \lfloor N_1/2 \rfloor - 1)^{\lfloor (d-1)/2 \rfloor}$ combinations of constraints (6c) by changing parameter $m(i)$.

In this paper, a closed-form frequency response satisfying (6) is proposed using Bernstein polynomial as

$$H_0(\omega_1, \omega_2) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_1+d} f(n_1, n_2)\ b_{n_1,N_1}(x)\ b_{n_2,N_1+d}(y), \tag{8a}$$

where

$$b_{i,N}(x) = \binom{N}{i} x^i (1-x)^{N-i}, (0 \leq i \leq N) \tag{8b}$$

$$x = \frac{1 - \cos\omega_1}{2}$$

$$y = \frac{1 - \cos\omega_2}{2}.$$

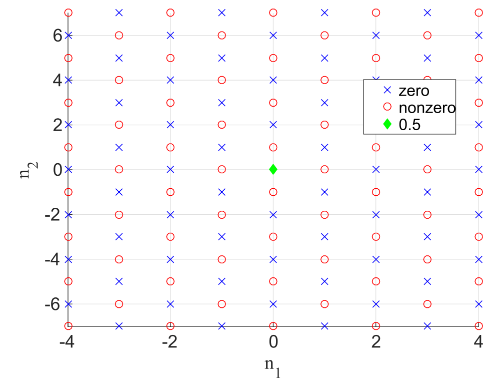
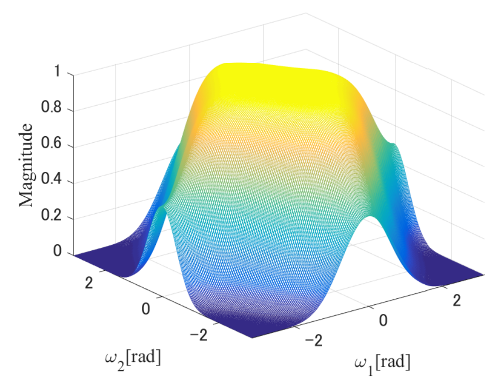
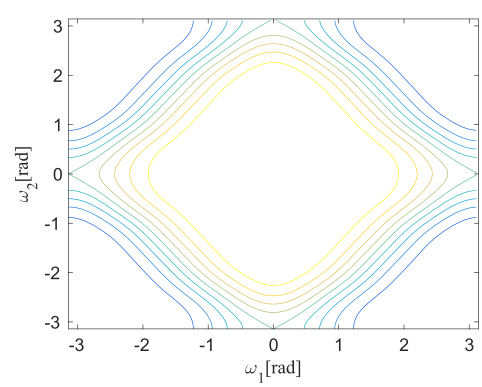


Fig. 3 The coefficients of (8×14) order zero-phase 2-D MFDH filter designed by the proposed method.



(a) The perspective plot



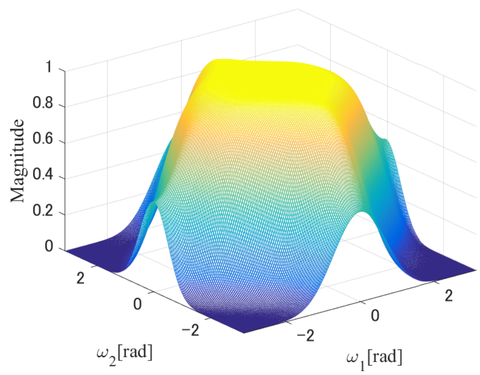
(b) The contour plot

Fig. 4 The magnitude response of (8×16) order 2-D MFDH filter with $m(10) = 3$.

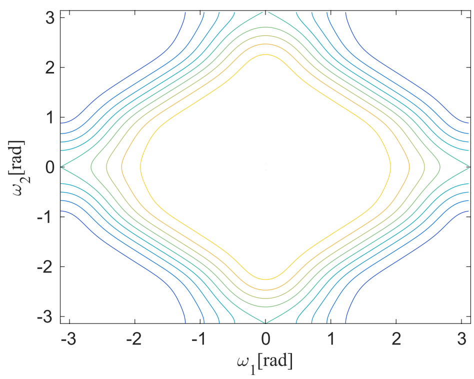
In above equation, $\binom{a}{n}$ is a binomial coefficient given as

$$\binom{a}{n} = \begin{cases} \dfrac{a(a-1)(a-2)\cdots(a-n+1)}{n!} & (0 < n \leq a) \\ 1 & (n = 0) \end{cases},$$

where a and n are an integer. In (8a), $f(\cdot)$ is the Bernstein coefficient derived as:

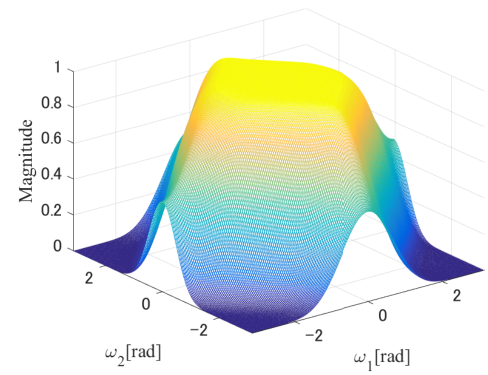


(a) The perspective plot

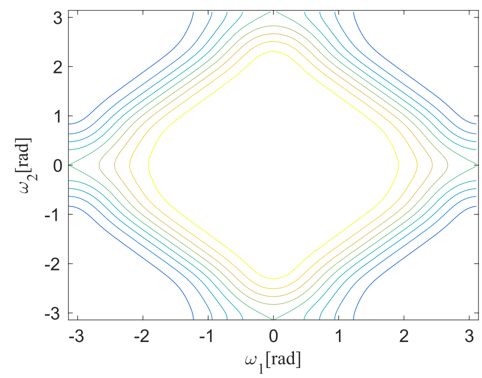


(b) The contour plot

Fig. 5 The magnitude response of (8×16) order 2-D MFDH filter with $m(10) = 4$.



(a) The perspective plot



(b) The contour plot

Fig. 6 The magnitude response of (8×18) order 2-D MFDH filter.

$$f(n_1, n_2) = \begin{cases} 1 & , (n_1, n_2) \in \mathcal{N} \\ 0 & , (N_1 - n_1, N_1 + d - n_2) \in \mathcal{N} \\ 0.5 & , \text{otherwise} \end{cases} \quad (8c)$$

$$\mathcal{N} = \{(n_1, n_2) \in \mathbb{N} \mid 0 \le n_1 + n_2 \le N_1 + d - 1, \\ 0 \le n_1 \le s(2n_1 + 2n_2) - 1\}. \quad (8d)$$

It is shown in the appendix that derivation of $f(\cdot)$, and (8a) satisfies (4).

By substituting $x = (2 - z_1 - z_1^{-1})/4$ and $y = (2 - z_2 - z_2^{-1})/4$ in (8a) and (1) , we can obtain the coefficients of the proposed filter.

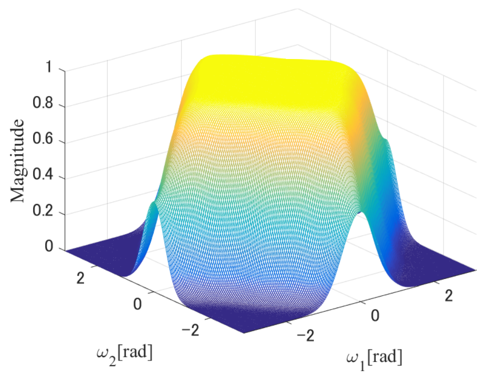
3. Design Examples

In this section, we will illustrate some magnitude responses of $(2N_1 \times 2(N_1 + d))$ order 2-D MFDH filters designed by the proposed method.

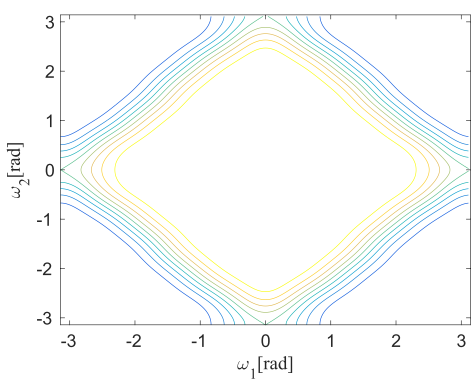
Example 1: In this example, we compare the proposed method and the conventional method for $N_1 = 4$ and $d = 3$ [12]. The magnitude response of the conventional method are already shown in Fig. 1(a) and 1(b). In this case, the parameter $m(i)$ to be set is only $m(10)$. From the decision rules of $m(i)$, $m(10)$ is set as 3 or 4. Figures 2(a) and 2(b) show the magnitude response of the proposed filter with $N_1 = 4$, $d = 3$ and $m(10) = 3$. It is clear from Fig. 1(b) that the magnitude response of conventional filter is not monotonically decreasing at $(\omega_1, \omega_2) = (0, 0)$. On the other hand, it is clear from Fig. 2(b) that the magnitude response of the proposed filter is monotonically decreasing over the whole frequency. Figure 3 illustrates the coefficients of the proposed filter. From Fig. 3, it is confirmed that the proposed method can achieve the quincuncial sampling pattern as same as the conventional method.

Example 2: In this example, we illustrate the 2-D MFDH filter with $d \ge 4$ design by the proposed method. Such filter can be designed only by the proposed method. In the case of design of filter with $N_1 = 4$ and $d = 4$, the parameter $m(i)$ to be set is only $m(10)$. From the decision rules of $m(i)$, the value of $m(10)$ is 3 or 4. Figures 4 and 5 show magnitude response of the proposed filter with $N_1 = 4$, $d = 4$ and $m(10) = \{3, 4\}$, respectively. From Figs. 4(a), 4(b), 5(a) and 5(b), it is confirmed that $m(10)$ controls the shape of the equi-amplitude lines.

Moreover, Figs. 6(a) and 6(b) show magnitude response of the proposed filter with $N_1 = 4$, $d = 5$, $m(10) = 4$ and $m(12) = 3$ and Figs. 7(a) and 7(b) show magnitude response of the proposed filter with $N_1 = 9$, $d = 5$, $m(20) = 7$ and $m(22) = 5$. From these figures, it is confirmed that 2-D MFDH filter with arbitrary d can be designed by the proposed method. Note that all of these filters are half-band



(a) The perspective plot



(b) The contour plot

Fig. 7 The magnitude response of (18×28) order 2-D MFDH filters.

characteristics and the filter coefficients of each filter has a quincuncial sampling pattern.

4. Conclusion

In this paper, we introduced a design method for 2-D MFDH filters with arbitrary different filter orders. To solve this problem, we proposed the new flatness constraints. Then, a closed-form frequency response with arbitrary filter orders were derived. The parameters of the proposed method are $m(i)$ which determines the shape of the equi-amplitude line, and N_1. Through design examples, it is confirmed that the proposed method can realize monotonically decreasing magnitude response for arbitrary d. Furthermore, it is also confirmed that the proposed method can realize various equi-amplitude line by setting $m(i)$, and the line approaches to the straight with appropriate $m(i)$.

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Appendix A: Derivation of Bernstein Coefficients (8)

From (4) and (8a), $f(\cdot)$ must satisfy the following relationship:

$$f(n_1, n_2) + f(N_1 - n_1, N_1 + d - n_2) = 1. \quad (\mathrm{A}\cdot 1)$$

From [14], the $2k$th partial derivative of (8a) is given as

$$\frac{\partial^{2k} H_0}{\partial \omega_1^{2r} \partial \omega_2^{2(k-r)}} = \frac{(2N_1)!(2(N_1+d))!}{(2(N_1-r))!(2(N_1+d-k+r))!}$$

$$\cdot \sum_{n_1=0}^{N_1-r} \sum_{n_2=0}^{N_1+d-k+r} \sum_{l_1=0}^{r} \sum_{l_2=0}^{k-r} \left\{ \frac{\binom{2(N_1-r)}{2n_1}\binom{2(N_1+d-k+r)}{2n_2}\binom{2r}{2l_1}\binom{2(k-r)}{2l_2}}{\binom{N_1-r}{n_1}\binom{N_1+d-k+r}{n_2}} \right.$$
$$\cdot (-1)^{2N_1+d-n_1-n_2-l_1-l_2} f(n_1+l_1, n_2+l_2)$$
$$\left. \cdot\, b_{n_1,N_1-r}(x)\, b_{n_2,N_1+d-k+r}(y) \right\}. \qquad (A\cdot 2)$$

From (8b), we have $b_{0,N}(0) = 1$, so that we obtain

$$\left.\frac{\partial^{2k} H_0}{\partial\omega_1^{2r}\partial\omega_2^{2(k-r)}}\right|_{\substack{\omega_1=0\\ \omega_2=0}} = \sum_{l_1=0}^{r}\sum_{l_2=0}^{k-r}\binom{r}{l_1}\binom{k-r}{l_2} f(l_1, l_2). \qquad (A\cdot 3)$$

From (6a), (6c), (8a), (A· 1) and (A· 3), we obtain

$$f(n_1, n_2) = \begin{cases} 1 & ,(n_1, n_2) \in \mathscr{N} \\ 0 & ,(N_1 - n_1, N_1 + d - n_2) \in \mathscr{N} \end{cases}. \qquad (A\cdot 4)$$

On the other hand, there are only three combinations of n_1 and n_2 which satisfies $(n_1, n_2) \notin \mathscr{N}$ and $(N_1 - n_1, N_1 + d - n_2) \notin \mathscr{N}$, i.e. $(n_1, n_2) = (N_1, 0), (0, N_1 + d)$ for any N_1 and d, and $(n_1, n_2) = (N_1/2, (N_1 + d)/2)$ for even N_1 and even d. Hence, from (A· 1), we have (A· 5),

$$f(N_1, 0) = f(0, N_1 + d) = 0.5 \text{ (for any } N_1 \text{ and } d)$$
$$f\left(\frac{N_1}{2}, \frac{N_1+d}{2}\right) = 0.5 \text{ (for even } N_1 \text{ and even } d). \qquad (A\cdot 5)$$

Appendix B: The Proof that (8a) Satisfies (4)

We obtain from (8a)

$$H_0(\omega_1, \omega_2)$$
$$= \sum_{(n_1,n_2)\in\mathscr{N}} b_{n_1,N_1}(x) b_{n_2,N_1+d}(y)$$
$$+\frac{1}{2} b_{N_1,N_1}(x) b_{0,N_1+d}(y) + \frac{1}{2} b_{0,N_1}(x) b_{N_1+d,N_1+d}(y)$$
$$+\frac{1}{2} b_{N_1/2,N_1}(x) b_{(N_1+d)/2,N_1+d}(y) \qquad (A\cdot 6)$$
$$H_0(\pi - \omega_1, \pi - \omega_2)$$
$$= \sum_{(N_1-n_1,N_1+d-n_2)\in\mathscr{N}} b_{n_1,N_1}(x) b_{n_2,N_1+d}(y)$$
$$+\frac{1}{2} b_{N_1,N_1}(x) b_{0,N_1+d}(y) + \frac{1}{2} b_{0,N_1}(x) b_{N_1+d,N_1+d}(y)$$
$$+\frac{1}{2} b_{N_1/2,N_1}(x) b_{(N_1+d)/2,N_1+d}(y). \qquad (A\cdot 7)$$

Then, all these summations exhaust all possible values for n_1 and n_2.

$$H_0(\omega_1, \omega_2) + H_0(\pi - \omega_1, \pi - \omega_2)$$
$$= \sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_1+d} b_{n_1,N_1}(x) b_{n_2,N_1+d}(y) = 1 \qquad (A\cdot 8)$$

From (A· 8), this equation is always equal to 1 [13].

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