PAPER

Multi-Tree-Based Peer-to-Peer Video Streaming with a Guaranteed Latency*

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SUMMARY This paper considers Peer-to-Peer (P2P) video streaming systems, in which a given video stream is divided into b stripes and those stripes are delivered to n peers through b spanning trees under the constraint such that each peer including the source can forward at most b stripes. The delivery of a stripe to n peers is said to be a **k-hop delivery** if *all peers* receive the stripe through a path of length at most k. Let $B_k = \sum_{i=0}^{k-1} b^i$. It is known that under the above constraint, k-hop delivery of b stripes to n peers is possible only if $n \leq B_k$. This paper proves that $(k + 1)$-hop delivery of b stripes to n peers is possible *for any* $n \leq B_k$; namely, we can realize the delivery of stripes with a *guaranteed latency* while it is slightly larger than the minimum latency. In addition, we derive a necessary and sufficient condition on n to enable a k-hop delivery of b stripes for $B_k - b + 2 \leq n \leq B_k - 1$; namely for n's close to B_k.

key words: *P2P video streaming system, guaranteed latency, tree-structured overlay*

1. Introduction

Video streaming over Peer-to-Peer (P2P) networks has attracted considerable attention in the past two decades [4]–[6], [13], [17], [19]–[21], [23], [24], [26], [27]. In **P2P video streaming**, peers participating in the network contribute their upload capacity to help a stable dissemination of the streaming data. For example, in tree-based systems [5], [6], [20], [28], peers are organized in a tree-structured overlay and the streaming data which is "pushed" by the media server located at the root of the tree, is delivered to the downstream peers by repeating store-and-relay operation. It is known that the efficiency of a tree-structured streaming could be effectively improved by adopting *multiple trees* instead of a single tree [4], [19]; namely by dividing the given stream into b stripes $s_1, s_2, \ldots, s_b$ and by delivering those stripes through different spanning trees. Such a division of a video stream is generally realized in such a way that the j^{th} stripe, for $1 \leq j \leq b$, consists of the $(bi + j)$th chunks in the given stream for $i \geq 0$ [1].

Let us consider a P2P system consisting of $n + 1$ peers. We assume that any two peers in the system are connected by bi-directional communication links (e.g., through UDP or TCP connections) so that they can directly communicate with each other. A video stream is issued by a designated peer called **source**, and is subscribed by the other n peers. Each peer including the source has an upload capacity of amount b so that it can simultaneously upload at most b stripes to other peers, while the capacity of each link and the download capacity of each peer are assumed to be sufficiently large [1]. In other words, each peer can *forward* received stripes to other peers as long as the amount of simultaneous uploads does not exceed b.

Delivery of a stripe to n peers is said to be a **k-hop delivery** if *all peers* receive the stripe through a path of length at most k [1]. In P2P video streaming systems, it is naturally requested to realize a k-hop delivery of *all stripes* for small k's such as two or three [4], [25]. When the capacity of each peer is b, the number of peers which enable a k-hop delivery of a stripe is at most $B_k = \sum_{i=0}^{k-1} b^i$ (see Sect. 3 for the details), but as will be described later, 2-hop delivery to $n \leq b + 1$ peers is *not always possible* as long as the capacity of peers is bounded by b. To overcome such a situation [1], we examined cases in which the capacity of every peer increases to $\tilde{b}$ $(\geq b)$, and derived a necessary and sufficient condition on $\tilde{b}$ to enable 2-hop delivery of b stripes to n peers. In addition, we clarified that 2-hop delivery of b stripes to n peers is possible if the capacity of the source is augmented by n/b by an external server [10]. In the current paper, we slightly relax the requirement on the number of hops and prove that $(k + 1)$-hop delivery of b stripes to n peers is possible for any $n \leq B_k$; namely, we can always realize the delivery of b stripes with a *guaranteed latency* while it is slightly greater than the minimum latency. In addition, concerned with the possibility of k-hop delivery of b stripes to $B_k - \delta_k$ peers, we derive a necessary and sufficient condition for $1 \leq \delta_k \leq b$ and several sufficient conditions for $\delta_k \geq b + 1$. Such a performance guarantee is crucial for delay-sensitive applications such as patient monitoring and real-time manufacturing.

The remainder of this paper is organized as follows. Section 2 overviews related work. Section 3 gives preliminaries. Section 4 proves a theorem concerned with the $(k + 1)$-hop delivery of b stripes to $n \leq B_k$ peers. Sections 5 and 6 are concerned with the k-hop delivery of b stripes to $n = B_k - \delta_k$ peers for $1 \leq \delta_k \leq b$ and $\delta_k \geq b + 1$, respectively. Finally, Sect. 7 concludes the paper with future work.

2. Related Work

Video streaming systems based on the P2P technology have

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been widely used in recent years. Those systems can be classified into several types by the way of delivering video contents to the subscribers, e.g., mesh type such as Bullet [12], PRIME [16], CoolStreaming/DONet [24] and a hybrid of mesh and tree such as mTreebone [22]. In particular, the demonstration experiments conducted by NHK science & technology research laboratories during London Olympics in 2012 [18] shows that mesh-based P2P realizes the delivery of live contents (e.g., the final of the men's singles of tennis tournament) to more than 1600 subscribers in 1.5 Mbps in a stable manner. Among those systems, in this paper, we focus on multiple trees as the underlying topology of the overlay network.

The idea of using multiple trees for the delivery of video streams was firstly adopted in SplitStream [4]. SplitStream divides a given video stream into several sub-streams (i.e., stripes) and delivers those sub-streams through different spanning trees to have disjoint sets of internal nodes. In other words, in SplitStream, each peer joins at most one spanning tree as an internal node and joins all of the other spanning trees as a leaf node. Such a construction of the set of spanning trees enables peers to contribute their upload capacity with low cost, which balances the load of all peers participating in the streaming system [2], [4], [7].

Theoretical aspects concerned with multiple-tree-based P2P video streaming have also been studied in recent years. Liu [14] considered the problem of minimizing the broadcast time of each chunk contained in the given stream under the constraint such that each peer can upload at most one chunk at a time, and proposed an algorithm which broadcasts every chunk to n subscribers in $\lceil \log_2 n \rceil$ hops. In this algorithm, any two consecutive chunks are delivered through different binomial trees since in order to enable the delivery of chunks in $\lceil \log_2 n \rceil$ hops, every peer receiving a chunk at time t must continuously upload the chunk until the chunk is received by all subscribers (i.e., note that to complete the broadcast of a chunk to n subscribers in $\lceil \log_2 n \rceil$ steps, the number of subscribers receiving the chunk must *double* in each step). A generalization of the Liu's result to the cases in which each peer can upload at most k chunks at a time, was done in [3] and an extension to the cases in which each peer has constant number of neighbors in the overlay and the upload capacity of peers is not uniform was given in [8] and [9], respectively. In addition to the above results, the upper bound on the network capacity of multiple-tree-based P2P was discussed in different contexts; e.g., [15] discussed the network capacity of peer-assisted live streaming systems and [11] considered the problem of constructing multiple trees which maximize the network capacity by considering the topology of the underlying physical network.

Zhao *et al.* analyzed the network capacity of multiple-tree-structured P2Ps in the context of one-view multiparty video conferencing (MPVC, for short) [25]. In one-view MPVC, each user participating in the video conference can watch the stream published by a specific peer selected beforehand. The delivery of each stream is conducted with the support of helper peers in addition to the publisher and subscribers, and to provide theoretical bounds on the network capacity, they assume that each stream can be divided into sub-streams with *arbitrary fractions*; e.g., a stream with bit-rate r can be divided into two sub-streams of bit-rate ϵr and $(1-\epsilon)r$ for arbitrary $\epsilon > 0$, while the length of each delivery path is bounded by two.

In [1], Ando and Fujita introduced the notion of k-hop delivery for multiple-tree-structured P2P video streaming. They considered P2P systems consisting of homogeneous peers and focused on the upload capacity of peers which enables the 2-hop delivery of b stripes to n subscribing peers. They derived tight lower bounds on the upload capacity for *any* combination of b and n [1]. Although tight lower bounds *equals to b or slightly greater than b* for $n \le b+1$, it linearly increases as n increases for $n > b+1$, and requests each peer to have an ability of forwarding c video streams when the number of peers becomes $c \times b$.

3. Preliminaries

3.1 Model of P2P Video Streaming

This paper considers the problem of delivering b stripes from the source to n subscribers with as short latency as possible. Each peer including the source has upload capacity b so that it can forward b stripes to other peers at the same time, where b stripes and their receivers might be different. Since the capacity of each peer is b and there are b stripes to be delivered to n subscribers, when $n > 1$, at least one peer must receive a stripe through *intermediate* peers; namely it needs two or more hops. We say that the delivery of a stripe is **k-hop delivery** [1] if all of n peers receive the stripe within k hops from the source. By definition, there are at most

$$B_k = 1 + b + b^2 + \cdots + b^{k-1}$$

peers which can receive a stripe within k hops from the source (recall that the upload capacity of the source is bounded by b). In fact, one-hop delivery of a stripe is possible *if and only if* $n = 1$. On the other hand, 2-hop delivery of a stripe is possible *only if* $n \le 1 + b$.

3.2 Elementary Bound for $k = 2$

At first, let us consider the case of $k = 2$. When $n = 1 + b$ $(= B_2)$, 2-hop delivery of b stripes to n peers can be done in the following manner: 1) in the first hop, the source sends b stripes to b $(\le n)$ peers so that each peer receives exactly one stripe; and 2) in the second hop, each peer receiving a stripe forwards it to the other $n-1$ $(\le b)$ peers. Although such a simple scheme correctly works even for $n = b$, it collapses for smaller n's, since it forces at least one peer to forward $b' \ge \lceil b/n \rceil \ge 2$ stripes to the other peers in the second hop. Such a forwarding of b' stripes to $n-1$ peers is possible *only if* $b' \times (n-1) \le b$. Conversely, if $b' \times (n-1) \le b$, 2-hop delivery of b stripes to n peers can be

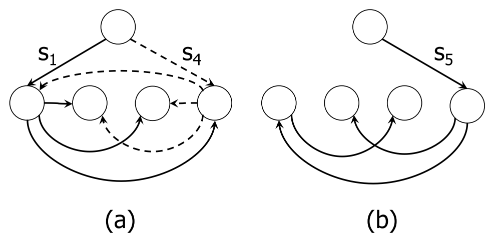

Fig. 1 3-hop delivery of five stripes to four peers, where (a) represents the delivery of stripes s_1 and s_4, and (b) represents the delivery of stripe s_5.

realized in the following manner: 1) in the first hop, the source sends b strips to n peers so that each peer receives at most $\lceil b/n \rceil$ stripes; and 2) in the second hop, each peer forwards received stripes to the other $n-1$ peers. Hence the following elementary claim follows.

Theorem 1: 2-hop delivery of b stripes to n peers is possible iff

$$\left\lceil \frac{b}{n} \right\rceil \times (n-1) \leq b.$$

This theorem indicates that 2-hop delivery of 5 stripes to 4 peers is *impossible*, while it is possible for $n = 3$ or 5 (i.e., it is not monotonic). However, it is also true that the delivery of 5 stripes is possible for any $n \leq 6$ $(= B_2)$ if we allow one more hop for the delivery of stripes; e.g., 3-hop delivery of 5 $(= b)$ stripes to 4 $(= n)$ peers can be realized as follows (see Fig. 1 for illustration):

1. in the first hop, the source sends stripe s_i to peer p_i for each $1 \leq i \leq 4$, and stripe s_5 to peer p_4;
2. in the second hop, p_i forwards s_i to the other three peers for each $1 \leq i \leq 4$, and simultaneously, p_4 forwards stripe s_5 to peers p_1 and p_2 (note that p_4 uses its capacity of amount 3 for stripe s_4 and capacity of amount 2 for stripe s_5); and
3. in the third hop, p_1 forwards stripe s_5 to p_3.

The reader should note that in this extended scheme, several stripes *share* the capacity of peers p_1 and p_4 in realizing their 3-hop delivery, while such a sharing is not possible if we need to deliver all stripes within 2 hops.

3.3 Equivalent Decision Problem for $k \geq 3$

In this paper, we will extend the above argument to the case of $k \geq 3$. At first, assuming $n = B_k$, we introduce two values X_i and Y_i defined as follows:

$$X_i \stackrel{\text{def}}{=} b^{i-1} \text{ and}$$

$$Y_i \stackrel{\text{def}}{=} 1 + b + \cdots + b^{i-2} = \frac{b^{i-1} - 1}{b - 1}.$$

X_i is the maximum number of peers which can receive a stripe in exactly i hops from the source, and Y_i denotes the number of peers which must *exclusively* use its upload capacity for the delivery of a stripe provided that b^{i-1} $(= X_i)$ peers receive the stripe in the i^{th} hop. Here, word "exclusive" means that the peer fully uses its upload capacity of amount b for the delivery of the stripe (if the capacity of amount $b' < b$ is used for the stripe, it is not considered to be an exclusively used peer even if the remaining capacity is not used for the other stripe). For example, $Y_2 = 1$ holds, since to deliver a stripe to b $(= X_2)$ peers in the second hop, exactly one peer must exclusively use its upload capacity of amount b for the stripe. The reader should note that those Y_i peers cannot be *shared* with other stripes, as long as $n = B_k$. With the above notions, value B_k can be represented as $B_k = Y_{k+1} = bY_k + 1 = X_k + Y_k$.

A k-hop delivery of b stripes to n peers is possible if $B_k - 1 \leq n \leq B_k$, and it is trivially impossible if $n \geq B_k + 1$. Thus in the following, without loss of generality, we assume $n = B_k - \delta_k$ for some $\delta_k \geq 2$. According to the reduction of n by δ_k, the total capacity used for the delivery of a stripe also reduces by δ_k, since n equals to the number of nodes in the delivery tree, and the total capacity equals to the number of edges in the delivery tree. In other words, according to the reduction of n, at most δ_k peers become "sharable" with other stripes, and the total amount of capacity used for a stripe reduces by exactly δ_k. Let

$$S(s) = \{u_1, u_2, \ldots, u_{\delta_k}\}$$

be the set of such peers (the reader should note that we do not need to mind the location of those peers in the delivery tree, and should just mind that such a set $S(s)$ exists for any δ_k and s).

In this paper, we are interested in the assignment of *reductions of the upload capacity* to peers in set $S(s)$ so that: 1) the total amount of reductions for a stripe equals to δ_k, and 2) it enables as much sharing of peers with other stripes as possible. The reader should note that since $Y_k - \delta_k$ peers have already been exclusively used for each stripe, we can exclude them from the candidate for sharing. Thus, the number of peers m which are not exclusively used for any stripe, i.e., peers which can be shared with other stripe under an appropriate assignment of reductions, is given as

$$\begin{aligned} m &= (B_k - \delta_k) - b(Y_k - \delta_k) \\ &= B_k - bY_k + (b-1)\delta_k \\ &= (b-1)\delta_k + 1 \\ &= b\delta_k - \delta_k + 1, \end{aligned}$$

where the third equality uses equality $B_k = bY_k + 1$. To simplify the exposition, in the following, we merely consider those m peers and will neglect the other peers. For example, we often use an argument such that: *If no peer in $S(s)$ is shared with other stripe for any s, then at least $b\delta_k$ peers must be used for the k-hop delivery of b stripes, but it is impossible since it exceeds the number of available peers $m = b\delta_k - \delta_k + 1 \leq b\delta_k - 1$, where the last inequality is due to $\delta_k \geq 2$.*

For the same reason, k-hop delivery of b stripes to $B_k - \delta_k$ peers is possible *only if* the capacity of peers can

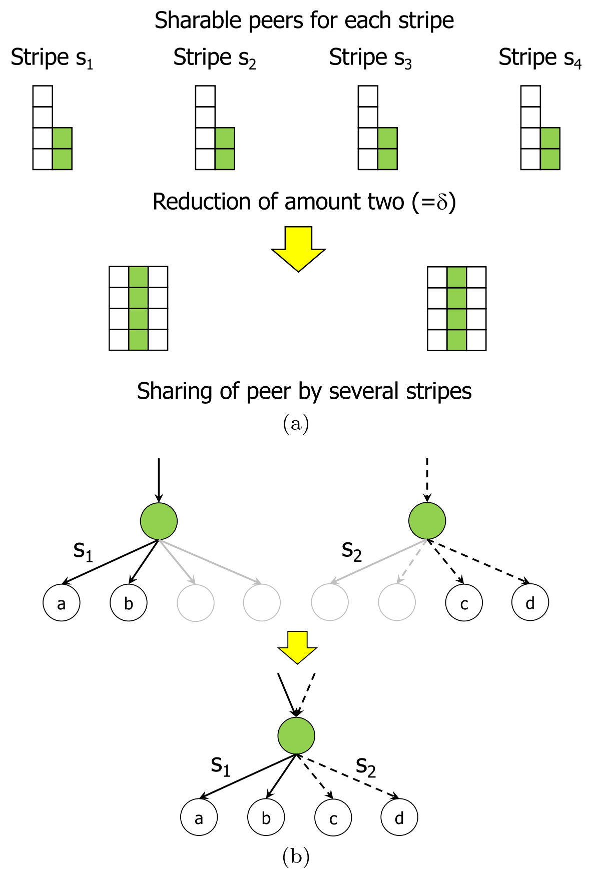

Fig. 2 Set $S(s_i)$ for four (= b) stripes. (a) it first reduces the upload capacity of peers in $S(s_i)$ by $\delta_k = 2$ and then conducts the sharing of peers by several stripes. (b) shows a part of resulting overlay according to the sharing of upload capacity.

be shared so that the total number of peers used for the delivery of b stripes does not exceed m. Conversely, if such a sharing of capacity is possible and if $\delta_k \leq b^{k-2}$, k-hop delivery of b stripes to $B_k - \delta_k$ peers is also possible. In fact, given such a sharing of capacity, k-hop delivery of b stripes can be realized by simply reducing the number of peers receiving the stripe in the k^{th} hop by δ_k. More concretely, we may construct an overlay so that: 1) $S(s)$ consists of δ_k peers which receive stripe s in the $(k-1)$st hop and 2) the upload capacity of peers in $S(s)$ is shared with other stripe according to the given sharing of capacity. See Fig. 2 for illustration.

The above observation implies that if $\delta_k \leq b^{k-2}$, the k-hop delivery of b stripes to $B_k - \delta_k$ peers is possible if and only if the answer to the following simplified decision problem is "yes," where d corresponds to δ_k in the original problem.

Equivalent Decision Problem

Input: Prepare b boxes of width d and height b, and fill each box with d items of width one and height b. See Fig. 3 for illustration. Note that each box corresponds to a stripe s, the height of items corresponds to the upload capacity of peers, and the width of box corresponds to the size of set $S(s)$.

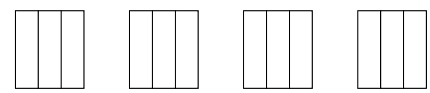

Fig. 3 Equivalent decision problem for $b = 4$ and $d = 3$. There are four (= b) boxes with three (= d) columns, where each column is filled with an item of height four (= b).

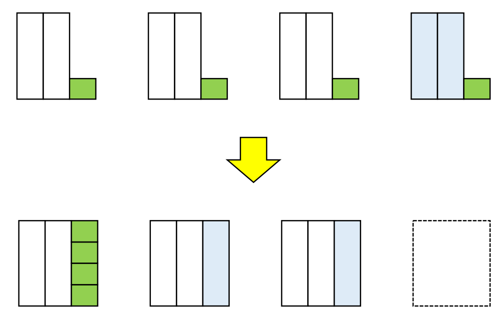

Fig. 4 Reduction of the size of items and the rearrangement of resulting items, which increases the number of unused items to three ($\geq d - 1$).

Operation: For each box, reduce the length of items in the box so that the total amount of reductions equals to d, and rearrange all items so that the total size of items packed into each column does not exceed b. After the rearrangement, a column is said to be used if it contains at least one item.

Question: Is it possible to realize reductions and rearrangement so that the number of unused columns is at least $d - 1$. Note that if the number of unused columns is $d-1$, the number of used columns becomes $b \times d - d + 1 = m$.

In Fig. 4, the size of items in a box reduces by three (= d), and after conducting rearrangement of resulting items, the number of unused columns becomes three ($\geq d - 1$). In the following, to clarify the exposition, we often prove claims by using this simplified decision problem. For example, since several items can be packed into one column only if $d \geq b/2$, we have the following claim concerned with the impossibility of k-hop delivery.

Lemma 1: k-hop delivery of b stripes to $B_k - \delta_k$ peers is impossible if $2 \leq \delta_k < b/2$.

4. Main Theorem on $(k + 1)$-Hop Delivery

This section proves the following theorem.

Theorem 2: $(k + 1)$-hop delivery of b stripes to n peers is possible for any $n \leq B_k$.

This theorem indicates that *by allowing one more hop*, we can always complete the delivery of b stripes to at most B_k peers with a guaranteed latency. Proof of the theorem relies on the following lemma.

Lemma 2: k-hop delivery of b stripes to $B_k - \delta_k$ peers is possible if $\delta_k = pb - q$ for some integers $p \geq 1$ and $0 \leq q \leq p$.

Proof. Consider the corresponding simplified problem. If d is a multiple of b with $d \geq b$, we can reduce the size of d/b (≥ 1) items in each box to zero, which generates $b \times d/b = d$ unused columns. If d is a multiple of $b-1$ with $d \geq b-1$, we can reduce the size of $\frac{d}{b-1}$ (≥ 1) items in each box to one and rearrange them so that each column contains exactly b such items, which generates $b \times \frac{d}{b-1} - \frac{d}{b-1} = d$ unused columns.

If $d = pb + p'(b-1)$ for some p and p' with $p + p' \geq 1$, by letting $d_1 = pb$ and $d_2 = p'(b-1)$, we can apply the first and the second reductions to d_1 and d_2, respectively, which generates $d = d_1 + d_2$ unused columns. Finally, since the above equality can be restated as

$$d = pb + p'(b-1) = (p + p')b - p',$$

the claim follows. Q.E.D.

If $\delta_k \geq (b-1)^2$, we can be restate δ_k as $\delta_k = pb + p'(b-1)$ by using two integers $p \geq 0$ and $p' \geq 1$. Thus by Lemma 2, k-hop delivery of b stripes to $B_k - \delta_k$ peers is possible if $\delta_k \geq (b-1)^2$. Since $B_{k+1} - B_k = b^k \geq (b-1)^2$ for any $b \geq 2$ and $k \geq 2$, the theorem follows.

5. Bound on k-Hop Delivery for Small δ_k

Unlike $(k+1)$-hop delivery, k-hop delivery is *not always possible* for any $n \leq B_k$. The following Lemma holds.

Lemma 3: k-hop delivery of b stripes to $B_k - \delta_k$ peers is possible if $\delta_k = b - \lfloor b/\sigma \rfloor$ for some integer $\sigma \geq 2$.

Proof. We prove the claim on the simplified problem. If $d = b - \lfloor b/\sigma \rfloor$ for some $\sigma \geq 2$, then we can reduce the size of an item in each box to $\lfloor b/\sigma \rfloor$, and pack resulting σ items into a column since $\lfloor b/\sigma \rfloor \times \sigma \leq b$ holds, where equality holds iff b is a multiple of σ. Since it generates $\lceil b/\sigma \rceil$ columns containing item of size $\lfloor b/\sigma \rfloor$, such a rearrangement reduces the number of used columns to

$$b(d-1) + \lceil b/\sigma \rceil = bd - (b - \lceil b/\sigma \rceil) \leq bd - d + 1,$$

where the last inequality is due to $d = b - \lfloor b/\sigma \rfloor$. Hence the lemma follows. Q.E.D.

The next theorem gives a *necessary and sufficient condition* to enable k-hop delivery of b stripes to $B_k - \delta_k$ peers for $\delta_k \leq b$.

Theorem 3: When $\delta_k < b-1$, k-hop delivery of b stripes to $B_k - \delta_k$ peers is possible iff $\delta_k = b - \lfloor b/\sigma \rfloor$ for some integer $\sigma \geq 2$ or $\delta_k = b - b/\sigma + 1$ for some integer $\sigma \geq 1$ dividing b.

Proof. By Lemma 3, it is enough to prove that: If $\delta_k \neq b - \lfloor b/\sigma \rfloor$ for any integer $\sigma \geq 2$, then k-hop delivery is possible iff $\delta_k = b - b/\sigma + 1$ for some integer $\sigma \geq 1$ dividing b. Assume that δ_k satisfies

$$b - \lfloor b/\sigma \rfloor < \delta_k < b - \lfloor b/(\sigma+1) \rfloor \tag{1}$$

for some $\sigma \geq 1$. Since $\delta_k < b-1$, this condition is equivalent to $\delta_k \neq b - \lfloor b/\sigma' \rfloor$ for any $\sigma' \geq 2$. Then, in the corresponding simplified problem, since we can pack at most σ items into a column, and the number of columns containing an item with a reduced size is at least $\lceil b/\sigma \rceil$, the number of used columns is at least

$$\begin{aligned} b(d-1) + \lceil b/\sigma \rceil &= bd - (b - \lceil b/\sigma \rceil) \\ &\geq bd - (b - \lfloor b/\sigma \rfloor) + f(b,\sigma) \\ &> bd - d + f(b,\sigma), \end{aligned}$$

where $f(x,y)$ is a function which returns 0 if $x \equiv 0 \pmod{y}$ and returns 1 otherwise, and the last inequality is due to $d > b - \lfloor b/\sigma \rfloor$. This implies that k-hop delivery is possible if and only if σ divides b and $d = b - b/\sigma + 1$. Hence the theorem follows. Q.E.D.

In the following, we explore the case of $\delta_k \geq b$, and derives several sufficient conditions to enable k-hop delivery of b stripes to $B_k - \delta_k$ peers. Recall that we have known that k-hop delivery to $B_k - \delta_k$ peers is possible if $\delta_k \geq (b-1)^2$ for any given $b \geq 2$.

6. Sufficient Conditions for Larger δ_k

This subsection derives several sufficient conditions to enable k-hop delivery of b stripes to $n = B_k - \delta_k$ peers for $\delta_k \geq b$. Let us consider the corresponding simplified problem. If $n = B_k - b$, i.e., if $d = b$, the size of an item in a box can be reduced from b to zero which generates b ($= d$) unused columns. Similarly, if $d = b + 1$, we can generate b ($= d - 1$) unused columns. Hence we can conclude that k-hop delivery of b stripes to $B_k - \delta_k$ peers is possible if $\delta_k = b$ or $b + 1$. As for $\delta_k = b + 2$, we have the following lemma.

Lemma 4: For any $b \geq 6$, k-hop delivery of b stripes to $B_k - (b+2)$ peers is possible.

Proof. If $b \neq 11$, we can solve the corresponding simplified problem in the following manner. Let $x = \lfloor b/3 \rfloor$, $y = b - 3x$ and $\eta = \lceil (x+y)/4 \rceil$. By definition, it holds $x \geq 2$ and $0 \leq y \leq 2$. Since $b = 3x + y$, we have

$$2\eta + x \geq (3x+y)/2 = b/2;$$

namely we can pack two items of size $b - (2\eta + x)$ into one column. Similarly, since $x \leq b/3$, we can pack three items of size x into one column.

- The size of several items is reduced as follows. For each $i \in \{1, 2, \ldots, \eta\}$, 1) select six boxes; 2) select two items from each box; and 3) reduce the size of selected items to $x + 2(i-1)$ and $b - (2i + x)$, respectively. Since

$$\begin{aligned} &b - \{x + 2(i-1)\} + b - \{b - (2i+x)\} \\ &= b + (2i + x) - \{x + 2(i-1)\} \\ &= b + 2, \end{aligned}$$

we can certainly generate such a pair of items by using

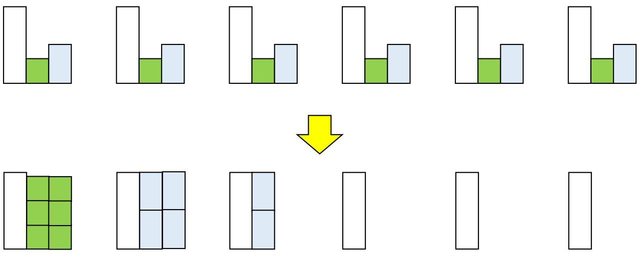

Fig. 5 Explanation of Lemma 4. At first, we select six boxes and select two items from each of selected boxes. We then reduce the size of the first item to $x = \lfloor b/3 \rfloor$ (painted green) and the size of the second item to $b-(2+x)$ (painted light blue). If $\eta = 1$, then we rearrange the resulting 12 items so that three items of size x are packed into a column and two items of size $b-(2+x)$ are into a column.

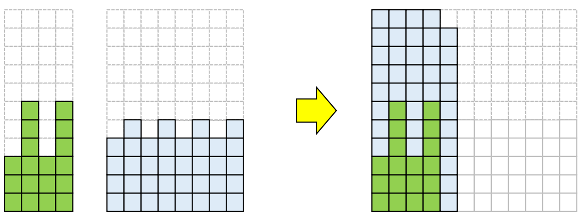

Fig. 6 Ad hoc method for $b = 11$ in the proof of Lemma 4.

the amount of reductions d $(= b+2)$. Note that it generates 12η items in 6η boxes. See Fig. 5 for illustration.

- Since there are six items of size x, pack them into two columns so that each column contains three items of size x. Next, for each $i \in \{1, 2, \ldots, \eta-1\}$, pack an item of size $b-(2i-x)$ and an item of size $2i+x$ into the same column, which generates six such columns. Finally, pack two items of size $b-(2\eta+x)$ into a column, which generates three such columns.

Since $\eta = \lceil (x+y)/4 \rceil$ and $b = 3x+y$, we have

$$\begin{aligned} b - 6\eta &\geq b - 6\left(\frac{x+y}{4} + \frac{3}{4}\right) \\ &= b - \frac{3}{2}\left(\frac{b-y}{3} + y\right) - \frac{9}{2} \\ &= \frac{b}{2} - \frac{9+2y}{2}, \end{aligned}$$

which implies that $b - 6\eta \geq 0$ holds if $6 \leq b \leq 10$ or $b \geq 12$. If $b - 6\eta \geq 1$, for each of those $b - 6\eta$ boxes, we may simply select one item and reduce its size to zero. Then, the number of unused columns becomes

$$\begin{aligned} &(b-6\eta) + \{12\eta - (2 + 6(\eta-1) + 3)\} \\ &= (b-6\eta) + (6\eta+1) \\ &= b+1 = d-1. \end{aligned}$$

On the other hand, if $b = 11$, we can use the following *ad hoc* method for the rearrangement of items. See Fig. 6 for illustration.

- Classify 11 boxes into three types so that two boxes are of Type A, four boxes are of Type B, and five boxes are of Type C. For Type A boxes, generate an item of size 3 and an item of size 6 (note that such a reduction is possible since $(b-3)+(b-6) = 2b-9 = 13 = b+2$); for Type B boxes, generate an item of size 4 and an item of size 5; and for Type C boxes, reduce the size of an item to 0.
- Rearrange the resulting items so that two columns contain three items of size 3, 4, and 4; two columns contain two items of size 5 and 6; and one column contains two items of size 5.

Then, the number of unused columns becomes $5+(12-5) = 12 = d-1$. Hence the lemma follows. Q.E.D.

The argument used in the proof of Lemma 4 can be extended as follows.

Theorem 4: For any $b \geq 18$, k-hop delivery of b stripes to $B_k - (b+c)$ peers is possible for any integer c satisfying $3 \leq c \leq \sqrt{b/2}$.

Proof. If $b \geq 2c^2 \geq 18$, we can solve the corresponding simplified problem in the following manner. Let $x = \lfloor b/2c \rfloor$, $y = b - 2cx$, and

$$\eta = \left\lceil \frac{(2c-2)x+y}{2c} \right\rceil = \left\lceil x + \frac{y-2x}{2c} \right\rceil.$$

Note that $x \geq 2c$ holds. Since

$$\begin{aligned} c\eta + x &\geq \frac{(2c-2)x+y}{2} + x \\ &= \frac{2cx+y}{2} = \frac{b}{2}, \end{aligned}$$

we can pack two items of size $b-(c\eta+x)$ into a column. Similarly, since $x \leq b/2c$, we can pack $2c$ items of size x into one column.

- The size of several items is reduced as follows. For each $i \in \{1, 2, \ldots, \eta\}$, 1) select $2c$ boxes; 2) select two items from each box; and 3) reduce the size of selected items to $x + c(i-1)$ and $b-(ci+x)$, respectively. Note that it generates $4c\eta$ items of reduced size in $2c\eta$ boxes.
- Since there are $2c$ items of size x, pack them into one column. Next, for each $i \in \{1, 2, \ldots, \eta-1\}$, pack an item of size $b-(ci+x)$ and an item of size $ci+x$ into one column, which generates $2c$ such columns. Finally, pack two items of size $b-(c\eta+x)$ into a column, which generates c such columns.

Now let us certify that we can select $2c\eta$ such boxes if $b \geq 2c^2$. If $b \geq 2c^2$, it holds $x \geq c$ since

$$x = \lfloor b/2c \rfloor \geq \lfloor 2c^2/2c \rfloor = c.$$

Since $y \leq 2c-1$, $x \geq c$ implies $y \leq 2x$, which further implies $x = \eta$ by the definition of η. Finally, if $x = \eta$ then $b \geq 2c\eta$ since $b = 2cx + y \geq 2cx$.

Table 1 The value of c covered by Corollary 1.

b	c
2	1
3	1
4	1, 2
5	1, 2
6	1, 2, 3
7	1, 3
8	1, 2, 3, 4
9	1, 2, 4
10	1, 2, 4, 5
11	1, 2, 5
12	1, 2, 3, 4, 5, 6
13	1, 2, 3, 6
14	1, 2, 3, 6, 7
15	1, 2, 3, 7
16	1, 2, 3, 4, 7, 8
17	1, 2, 4, 8
18	1, 2, 3, 4, 8, 9
19	1, 2, 3, 4, 9
20	1, 2, 3, 4, 5, 9, 10

If $b-2c\eta \geq 1$, for each of the remaining $b-2c\eta$ boxes, we may simply select one item and reduce its size to zero. Then, the number of unused columns becomes

$$\begin{aligned}&(b-2c\eta)+\{4c\eta-(1+2c(\eta-1)+c)\}\\&=b+c-1=d-1.\end{aligned}$$

Hence the claim follows. Q.E.D.

The reader should note that the argument used in the above proof correctly works as long as $x=\eta\geq 1$. This condition can be restated as $y\leq 2x$; i.e., $b\leq 2(c+1)x=2(c+1)\lfloor b/2c\rfloor$. Hence we have the following claim.

Corollary 1: k-hop delivery of b stripes to $B_k-(b+c)$ peers is possible for any integer c satisfying $2c\leq b\leq 2(c+1)\lfloor b/2c\rfloor$.

For each $2\leq b\leq 20$, the value of c satisfying the above inequality is summarized as follows: Theorem 4 indicates that k-hop delivery of b stripes to $B_k-(b+3)$ peers is possible for any $b\geq 18$, but it does not clarify whether it is possible for $b\leq 17$; which implies that even if it is possible, we need to use an ad hoc method for $b=9,10,11$, and 17. For example, the above construction can be extended as follows.

Lemma 5: For any even $b\geq 2$, k-hop delivery of b stripes to $B_k-\{3b/2-\lfloor b/2\sigma\rfloor\}$ peers is possible for any integer $\sigma\geq 2$.

Proof. Consider the corresponding simplified problem. By letting $x=\lfloor b/\sigma\rfloor$, we have $d=(3/2)b-\lfloor x/2\rfloor$. For each box, select two items, and reduce the size of an item to $b/2-\lceil x/2\rceil$ and the size of another item to x. Since $(b-x)+b/2+\lceil x/2\rceil=(3/2)b-\lfloor x/2\rfloor=d$, we can generate such a pair of items. We then rearrange $2b$ items so that: 1) $b/2$ columns contain two items of size $b/2-\lceil x/2\rceil$ and one item of size x; and 2) the remaining items of size x are packed into as small number of columns as possible. Note that the first packing is possible since $2\times\lceil x/2\rceil\geq x$ and the second packing uses $\lceil b/2\sigma\rceil$ columns since it should pack $b/2$ items of size x so that each column contains σ such items.

The number of unused columns becomes

$$\begin{aligned}2b-(b/2+\lceil b/2\sigma\rceil)&=(3/2)b-\lceil b/2\sigma\rceil\\&\geq(3/2)b-\lfloor b/2\sigma\rfloor-1=d-1.\end{aligned}$$

Hence the claim follows. Q.E.D.

The argument used in the above proof correctly works for any $x\geq 2$ if $b/2$ items of size x can be packed into $\lceil x/2\rceil$ columns, which is realized if $2x^2\leq b$; namely if $x\leq\sqrt{b/2}$. Hence the following claim follows.

Corollary 2: For any even $b\geq 2$, k-hop delivery of b stripes to $B_k-(3/2)b-x$ peers is possible for any integer $x\leq\sqrt{b/2}$.

7. Concluding Remarks

This paper considers the problem of delivering b stripes to n peers with a guaranteed latency. More concretely, we prove that $(k+1)$-hop delivery of b stripes to n peers is possible *for any* $n\leq\sum_{i=0}^{k-1}b^i$. A future work is to derive a necessary and sufficient condition on n to enable a k-hop delivery of b stripes for $n\leq B_k-b-2$.

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