PAPER

Conflict Management Method Based on a New Belief Divergence in Evidence Theory

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SUMMARY Highly conflicting evidence that may lead to the counter-intuitive results is one of the challenges for information fusion in Dempster-Shafer evidence theory. To deal with this issue, evidence conflict is investigated based on belief divergence measuring the discrepancy between evidence. In this paper, the pignistic probability transform belief χ^2 divergence, named as $BB\chi^2$ divergence, is proposed. By introducing the pignistic probability transform, the proposed $BB\chi^2$ divergence can accurately quantify the difference between evidence with the consideration of multi-element sets. Compared with a few belief divergences, the novel divergence has more precision. Based on this advantageous divergence, a new multi-source information fusion method is devised. The proposed method considers both credibility weights and information volume weights to determine the overall weight of each evidence. Eventually, the proposed method is applied in target recognition and fault diagnosis, in which comparative analysis indicates that the proposed method can realize the highest accuracy for managing evidence conflict.

key words: *Dempster-Shafer evidence theory, multi-source information fusion, evidence conflict, divergence, target recognition, fault diagnosis*

1. Introduction

Multi-source information fusion technology can integrate data from multiple sensors to make a unified decision [1]–[3]. As a distinguished multi-source information fusion method to resolve uncertainty problems, Dempster-Shafer evidence theory (D-S evidence theory) [4], [5] utilizes basic probability assignment (BPA) to depict incomplete and uncertain information, and the Dempster's combination rule can fuse uncertain information from different sources to improve decision level. In addition, D-S evidence theory has been extensively applied in plentiful fields, including image processing [6]–[8], supplier selection [9], risk analysis [10], [11], fault diagnosis [12]–[14], and so on. However, attributed to the complexity of targets and quantity of sensors, the information detected from different sensors may have significant conflict. When faced with the above situation, D-S evidence theory may generate the counter-intuitive result [15]. Therefore, how to manage highly conflicting information is still a challenge in D-S evidence theory.

To solve the challenge, the mainstream methods are primarily conducted by modifying the combination rule or pre-processing evidence before combination [16]–[23]. This paper concentrates on the latter. It is noted that the weighted average methods are commonly served as an effective approach to adjust the body of evidence. For example, Xiao modified evidence with a generalized evidential Jensen–Shannon (GEJS) divergence measure, the evidence weight is decided by the GEJS divergence among multiple sources of evidence [24]; Based on a new evidential correlation coefficient (ECC), a multi-source information fusion algorithm for conflict management was devised, where the evidence weight is calculated by the ECC between two pieces of evidence [25]; A new weighted average algorithm model based on DEMATEL was proposed to solve the conflicting evidence problem, where the total-relation matrix is determined by the similarity among evidence, then prominence and importance are considered to modify the conflicting evidence [26]. In particular, Xiao presented a modified evidence method based on the belief Jenson-Shannon (BJS) divergence to fuse conflicting evidence, but the BJS divergence does not take the influence of multi-element sets into account, treating evidence as probability distribution [27]. Furthermore, Gao and Xiao proposed a belief χ^2 ($B\chi^2$) divergence, but also neglected the impact of multi-element subsets, and thus introduced a reinforced belief χ^2 ($RB\chi^2$) divergence [28]. Although the above-mentioned weighted averaging methods can to some extent resolve conflict, there are still certain limitations that need to be overcome. As a consequence, a better generalized divergence based on χ^2 divergence is necessary to be explored.

The main motivation of this study lies in the following points:

- The BJS and $B\chi^2$ divergences overlook the uncertainty of evidence. Therefore, a new generalized divergence based on χ^2 divergence is worth exploring for more accurate dissimilarity measurement between evidence.
- It is significant to enhance the performance of the fusion system for achieving precise decision-making. Therefore, a novel algorithm needs to be designed to improve the accuracy of fusion.

In this article, a pignistic probability transform (BetP) belief χ^2 divergence, named as $BB\chi^2$ divergence, is proposed to measure the discrepancy between evidence. The $BB\chi^2$ divergence satisfies the properties of boundedness, nondegeneracy, and symmetry. Based on the $BB\chi^2$ divergence, a new multi-source information fusion method is devised. The method considers both credibility weights

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derived from the $BB\chi^2$ divergence and information volume weights generated by evidence uncertainty to produce the final weights. The proposed method is illustrated in target recognition and fault diagnosis to demonstrate its feasibility and superiority for conflict management in terms of higher accuracy.

The main contributions of this work are summarized as follows:

- Based on the pignistic probability transform (BetP) and χ^2 divergence, a new belief divergence, called $BB\chi^2$ divergence, is proposed. The $BB\chi^2$ divergence can reflect the interaction between singletons and multielement sets.
- Compared with other divergences, $BB\chi^2$ divergence can measure the discrepancy between evidence more accurately.
- Based on the $BB\chi^2$ divergence, a new multi-source information fusion method is designed. The effectiveness and superiority of the proposed method for handling conflict evidence are demonstrated in two applications of target recognition and fault diagnosis.

The remaining contents of this paper are arranged as follows: Sect. 2 briefly introduces a trace of preliminaries about Dempster-Shafer evidence theory, pignistic probability transform, Deng entropy and some divergence measures. In Sect. 3, a new pignistic probability transform (BetP) belief χ^2 divergence is proposed. Based on the $BB\chi^2$ divergence, a new multi-source information fusion method is devised in Sect. 4. In Sect. 5, two application cases in target recognition and fault diagnosis are implemented. Eventually, the conclusion is drawn in Sect. 6.

2. Preliminaries

In this section, some basic concepts about Dempster-Shafer evidence theory, pignistic probability transform, Deng entropy and divergence measure are introduced.

2.1 Dempster-Shafer Evidence Theory

Dempster-Shafer evidence theory is primitively presented by Dempster and perfected by Shafer, which can be learned as the generalization of probability theory. It extends basic events in probability theory to its power set space and introduces the basic probability assignment function. The concise knowledge about Dempster-Shafer evidence theory is introduced as follows.

Definition 1 (Frame of discernment): Let Θ be a finite and complete set, which is composed of N mutually exclusive elements, Θ is called a frame of discernment denoted as [4]

$$\Theta = \{\theta_1, \theta_2, \cdots, \theta_N\} \tag{1}$$

The power set of Θ consisting of 2^N elements is defined as

$$2^\Theta = \{\emptyset, \theta_1, \theta_2, \cdots, \theta_N, \{\theta_1, \theta_2\}, \cdots, \{\theta_1, \theta_2, \theta_3\}, \cdots, \Theta\} \tag{2}$$

The subsets of a frame of discernment Θ correspond to the propositions. For any $A \subseteq \Theta$, if $|A| = 1$, A is called a singleton; if $|A| > 1$, A is called a multi-element set, where $|A|$ indicates the cardinality of A.

Definition 2 (Basic probability assignment): Let Θ be a frame of discernment, $\forall A \subseteq \Theta$, if a function $m : 2^\Theta \to [0, 1]$ satisfies following two conditions:

$$\begin{cases} m(\emptyset) = 0 \\ \sum\limits_{A \subseteq \Theta} m(A) = 1 \end{cases} \tag{3}$$

m is called a basic probability assignment (BPA) or mass function on Θ [4], where $\emptyset$ is an empty set. $m(A)$ represents the exact belief assigned to A. If $m(A) \neq 0$, A is called a focal element.

Definition 3 (Belief function): Let m be a basic probability assignment on a frame of discernment Θ, if a function $Bel : 2^\Theta \to [0, 1]$ satisfies

$$Bel(A) = \sum_{B \subseteq A} m(B), \ A \in 2^\Theta \tag{4}$$

Bel is called a belief function on Θ [4]. where belief function meets

$$Bel(\emptyset) = 0, \ \ Bel(\Theta) = 1 \tag{5}$$

For a singleton A, it is clear that $Bel(A) = m(A)$.

Definition 4 (Plausibility function): Let m be a basic probability assignment on a frame of discernment Θ, if a function $Pl : 2^\Theta \to [0, 1]$ satisfies

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{A \cap B \neq \emptyset} m(B), \ \forall A \in 2^\Theta \tag{6}$$

Pl is called a plausibility function on Θ [4].

Definition 5 (Dempster's combination rule): Let m_1 and m_2 be two independent BPAs on a frame of discernment Θ, $m = m_1 \oplus m_2$ indicates new evidence after combination between m_1 and m_2, Dempster's combination rule is defined as [4]

$$\begin{cases} m(\emptyset) = 0 \\ m(A) = \dfrac{1}{1-k} \sum\limits_{A = B \cap C} m_1(B)\, m_2(C) \end{cases} \tag{7}$$

where $B, C \subseteq \Theta$, $k = \sum_{B \bigcap C = \emptyset} m_1(B)\, m_2(C)$ is called conflict coefficient, k satisfies $0 \leq k < 1$.

2.2 Pignistic Probability Transform

Pignistic probability transform can evenly assign belief of multi-element sets to singletons and transform evidence into probability distribution.

Definition 6 (Pignistic probability transform): Let m be a basic probability assignment on a frame of discernment Θ, pignisitic transform function $BetP_m : \Theta \rightarrow [0,1]$ is defined as [29]

$$BetP_m(\theta_i) = \sum_{\substack{A \subseteq \Theta \\ \theta_i \in A}} \frac{m(A)}{|A|} \tag{8}$$

where θ_i is an element of Θ, $A \subseteq \Theta$, $|A|$ is the cardinality of A.

2.3 Deng Entropy

In order to quantify the uncertainty of evidence, Deng developed a new belief entropy, called Deng entropy. It is defined as [30]

$$E_d = -\sum_{A \subseteq \Theta} m(A) \log_2 \frac{m(A)}{2^{|A|} - 1} \tag{9}$$

where m is a BPA defined on Θ, A is a focal element, $|A|$ is the cardinality of A.

2.4 Divergence Measure

Divergence measure is used to quantify the discrepancy between two probability distributions in information system. As a classical divergence, χ^2 divergence was proposed by Pearson [31] and defined as follows.

Definition 7 (χ^2 divergence): Given two probability distributions $P = (p_1, \ldots, p_n)$ and $Q = (q_1, \ldots, q_n)$ with $\sum_i p_i = \sum_i q_i = 1$, χ^2 divergence is denoted by

$$\chi^2(P,Q) = \sum_{i=1}^{n} \frac{(p_i - q_i)^2}{q_i} \tag{10}$$

In D-S evidence theory, how to measure the discrepancy between evidence is still in solving. In order to settle this problem, Xiao proposed Belief Jensen-Shannon divergence [27].

Definition 8 (Belief Jensen-Shannon divergence): Given two BPAs m_1 and m_2 defined on a frame of discernment Θ, composed of n mutually exclusive and collectively exhaustive elements, belief Jensen-Shannon divergence between m_1 and m_2 is defined as

$$\begin{aligned} BJS(m_1,m_2) = \tfrac{1}{2}\Big[&\textstyle\sum_i m_1(A_i) \log \frac{2m_1(A_i)}{m_1(A_i)+m_2(A_i)} \\ &+ \textstyle\sum_i m_2(A_i) \log \frac{2m_2(A_i)}{m_1(A_i)+m_2(A_i)}\Big] \end{aligned} \tag{11}$$

where $\sum_i m_j(A_i) = 1, (i = 1, \ldots, n; j = 1, 2)$.

Nevertheless, BJS divergence ignores the uncertainty of multi-element sets. It cannot sufficiently reflect the effect of different subsets of Θ. The restriction of BJS divergence is compendiously explained by Example 1.

Example 1: Suppose m_1, m_2 and m_3 are three BPAs defined on $\Theta = \{A, B\}$.

$$\begin{aligned} &m_1 : m_1(A) = 0.90 \quad m_1(B) = 0.05 \quad m_1(\Theta) = 0.05 \\ &m_2 : m_2(A) = 0.05 \quad m_2(B) = 0.90 \quad m_2(\Theta) = 0.05 \\ &m_3 : m_3(A) = 0.05 \quad m_3(B) = 0.05 \quad m_3(\Theta) = 0.90 \end{aligned}$$

In Example 1, m_1, m_2 and m_3 are mutually contradictory and respectively support A, B and Θ with belief value 0.90. Clearly, the conflict between m_1 and m_3 is similar to that between m_2 and m_3. Especially, the conflict between m_1 and m_2 is the most remarkable. Therefore, BJS divergence satisfies $BJS(m_1,m_2) > BJS(m_1,m_3) = BJS(m_2,m_3)$. However, by Eq. (11), we have

$$\begin{aligned} &BJS(m_1,m_2) = 0.6674 \quad BJS(m_1,m_3) = 0.6674 \\ &BJS(m_2,m_3) = 0.6674 \end{aligned}$$

From the result, it is discovered that $BJS(m_1,m_2) = BJS(m_1,m_3) = BJS(m_2,m_3)$, which doesn't conform to the intuition. Therefore, a proper belief divergence for getting more accurate inconsistency measurement is needed to be explored.

3. The Proposed Divergence Measure

A new pignistic probability transform (BetP) belief χ^2 divergence, named as $BB\chi^2$ divergence, is proposed to measure the evidence difference.

3.1 Definition of $BB\chi^2$ Divergence Measure

The Betp evenly distributes belief of multi-element sets to singletons and converts evidence into probability distribution. By this virtue, Betp can not only embody the difference between multi-element sets and singletons, but also reduce the uncertainty of the evidence. Considering this, χ^2 divergence is associated with the Betp to construct a new $BB\chi^2$ divergence. The definition of $BB\chi^2$ divergence is as follows.

Definition 9 ($BB\chi^2$ divergence): Given two BPAs m_1 and m_2 defined on Θ, consisting of n mutually exclusive and collectively exhaustive elements, $BB\chi^2$ divergence between m_1 and m_2 is defined as

$$\begin{aligned} BB\chi^2(m_1,m_2) = \tfrac{1}{2}\Big[&\chi^2\Big(BetP_{m_1}, \tfrac{BetP_{m_1}+BetP_{m_2}}{2}\Big) \\ &+\chi^2\Big(BetP_{m_2}, \tfrac{BetP_{m_1}+BetP_{m_2}}{2}\Big)\Big] \end{aligned} \tag{12}$$

where $BetP_m(\theta_i) = \sum_{\substack{A \subseteq \Theta \\ \theta_i \in A}} \frac{m(A)}{|A|}$, $\theta_i \in \Theta (i = 1, \ldots, n)$. The formula of $BB\chi^2$ divergence measure can be simplified as

$$BB\chi^2(m_1,m_2) = \frac{1}{2} \sum_{\theta_i \in \Theta} \frac{\left(BetP_{m_1}(\theta_i) - BetP_{m_2}(\theta_i)\right)^2}{BetP_{m_1}(\theta_i) + BetP_{m_2}(\theta_i)} \tag{13}$$

3.2 Properties of $BB\chi^2$ Divergence Measure

Let m_1 and m_2 be two BPAs defined on the frame of discernment Θ, $BB\chi^2$ divergence satisfies three properties as follows.

1. Boundedness: $0 \le BB\chi^2(m_1, m_2) \le 1$
2. Nondegeneracy: $BB\chi^2(m_1, m_2) = 0$ if and only if $m_1 = m_2$
3. Symmetry: $BB\chi^2(m_1, m_2) = BB\chi^2(m_2, m_1)$

Proof 1: (1) Suppose m_1 and m_2 are two BPAs defined on Θ. $BetP_m$ is the pignistic probability transform from m. Actually, it can be treated as a probability distribution. $BetP_{m_1}$, $BetP_{m_2}$ and $\frac{BetP_{m_1}+BetP_{m_2}}{2}$ are probability distributions, so we have

$$\chi^2\left(BetP_{m_1}, \frac{BetP_{m_1}+BetP_{m_2}}{2}\right) = \frac{1}{2}\sum_i \frac{\left(BetP_{m_1} - BetP_{m_2}\right)^2}{BetP_{m_1}+BetP_{m_2}} \ge 0 \tag{14}$$

$$\chi^2\left(BetP_{m_2}, \frac{BetP_{m_1}+BetP_{m_2}}{2}\right) = \frac{1}{2}\sum_i \frac{\left(BetP_{m_2} - BetP_{m_1}\right)^2}{BetP_{m_1}+BetP_{m_2}} \ge 0 \tag{15}$$

Therefore,

$$BB\chi^2(m_1, m_2) = \frac{1}{2}\left[\chi^2\left(BetP_{m_1}, \frac{BetP_{m_1}+BetP_{m_2}}{2}\right) + \chi^2\left(BetP_{m_2}, \frac{BetP_{m_1}+BetP_{m_2}}{2}\right)\right] \ge 0$$

According to Eq. (13), we have

$$\begin{aligned} BB\chi^2(m_1, m_2) &= \frac{1}{2}\sum_{\theta_i\in\Theta} \frac{\left(BetP_{m_1}(\theta_i) - BetP_{m_2}(\theta_i)\right)^2}{BetP_{m_1}(\theta_i)+BetP_{m_2}(\theta_i)} \\ &\le \frac{1}{2}\sum_{\theta_i\in\Theta} \frac{\left(BetP_{m_1}(\theta_i) + BetP_{m_2}(\theta_i)\right)^2}{BetP_{m_1}(\theta_i)+BetP_{m_2}(\theta_i)} \\ &= \frac{1}{2}\sum_{\theta_i\in\Theta}\left(BetP_{m_1}(\theta_i) + BetP_{m_2}(\theta_i)\right) \\ &= 1 \end{aligned}$$

Consequently, $0 \le BB\chi^2(m_1, m_2) \le 1$. The boundedness of $BB\chi^2$ divergence is proved. □

Proof 2: (2) Given two BPAs m_1 and m_2 defined on Θ. If $m_1 = m_2$, then $BetP_{m_1}$ transformed from m_1 equals to $BetP_{m_2}$ transformed from m_2 by the Eq. (8). Therefore, $BetP_{m_1} = BetP_{m_2} = \frac{BetP_{m_1}+BetP_{m_2}}{2}$, then we have

$$\begin{aligned} & BB\chi^2(m_1, m_2) = 0 \\ \Leftrightarrow\ & \frac{1}{2}\left[\chi^2\left(BetP_{m_1}, \frac{BetP_{m_1}+BetP_{m_2}}{2}\right) + \chi^2\left(BetP_{m_2}, \frac{BetP_{m_1}+BetP_{m_2}}{2}\right)\right] = 0 \\ \Leftrightarrow\ & \chi^2\left(BetP_{m_1}, \frac{BetP_{m_1}+BetP_{m_2}}{2}\right) = 0 \\ \Leftrightarrow\ & BetP_{m_1} = \frac{BetP_{m_1}+BetP_{m_2}}{2} \\ \Leftrightarrow\ & BetP_{m_1} = BetP_{m_2} \\ \Leftrightarrow\ & m_1 = m_2 \end{aligned}$$

Therefore, the nondegeneracy of $BB\chi^2$ divergence is proved. □

Proof 3: (3) Given two BPAs m_1 and m_2 defined on Θ, we have

$$BB\chi^2(m_1, m_2) = \frac{1}{2}\left[\chi^2\left(BetP_{m_1}, \frac{BetP_{m_1}+BetP_{m_2}}{2}\right) + \chi^2\left(BetP_{m_2}, \frac{BetP_{m_1}+BetP_{m_2}}{2}\right)\right]$$

$$BB\chi^2(m_2, m_1) = \frac{1}{2}\left[\chi^2\left(BetP_{m_2}, \frac{BetP_{m_2}+BetP_{m_1}}{2}\right) + \chi^2\left(BetP_{m_1}, \frac{BetP_{m_2}+BetP_{m_1}}{2}\right)\right]$$

It is obvious that $BB\chi^2(m_1, m_2) = BB\chi^2(m_2, m_1)$. As a result, the symmetry of $BB\chi^2$ divergence is proved. □

3.3 Performance of $BB\chi^2$ Divergence Measure

Recalling Example 1, the calculation for $BB\chi^2$ divergence is showed as follows.

Firstly, the BetPs of m_1, m_2 and m_3 are calculated as

$$\begin{aligned} BetP_{m_1}(A) &= 0.9250 \quad BetP_{m_1}(B) = 0.0750 \\ BetP_{m_2}(A) &= 0.0750 \quad BetP_{m_2}(B) = 0.9250 \\ BetP_{m_3}(A) &= 0.5000 \quad BetP_{m_3}(B) = 0.5000 \end{aligned}$$

for simplicity, the Betps of m_1, m_2 and m_3 are denoted as

$$m_1 : (0.9250,\ 0.0750) \quad m_2 : (0.0750,\ 0.9250)$$
$$m_3 : (0.5000,\ 0.5000)$$

Finally, the $BB\chi^2$ divergence measures are obtained as

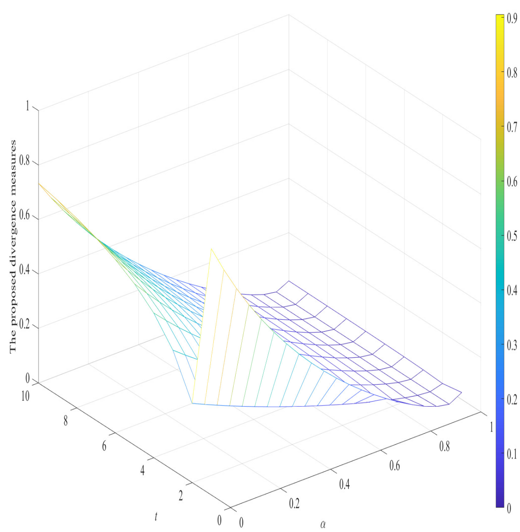
$$BB\chi^2(m_1, m_2) = 0.7225 \quad BB\chi^2(m_1.m_3) = 0.2204$$
$$BB\chi^2(m_2.m_3) = 0.2204$$

The result indicates that $BB\chi^2(m_1, m_2) > BB\chi^2(m_1.m_3) = BB\chi^2(m_2.m_3)$, it is in line with the previous analysis about discrepancy among evidence. As a consequence, it is verified that $BB\chi^2$ divergence overcomes the deficiency of BJS divergence and is more valid to measure the discrepancy.

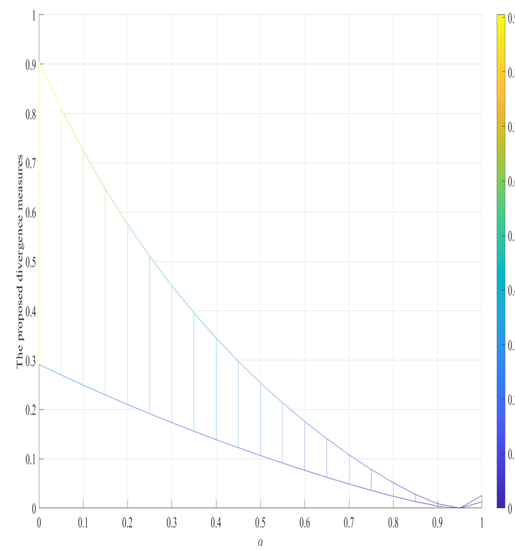
Example 2: Suppose m_1 and m_2 are two BPAs defined on Θ, A_t is a variable set defined as Table 1, α varies from 0 to 1.

Table 1 The variation of set A_t

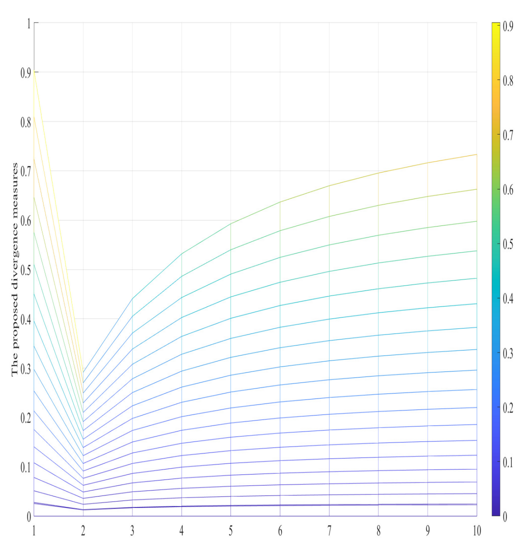
t	A_t
1	$\{A\}$
2	$\{A, B\}$
3	$\{A, B, C\}$
4	$\{A, B, C, D\}$
5	$\{A, B, C, D, E\}$
6	$\{A, B, C, D, E, F\}$
7	$\{A, B, C, D, E, F, G\}$
8	$\{A, B, C, D, E, F, G, H\}$
9	$\{A, B, C, D, E, F, G, H, I\}$
10	$\{A, B, C, D, E, F, G, H, I, J\}$



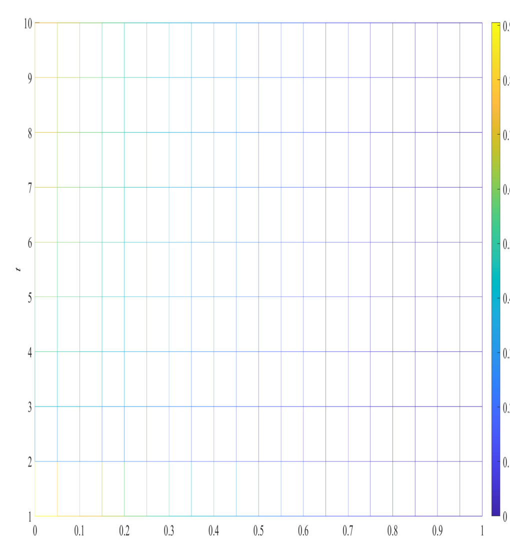
(a) The proposed divergence measures with α and t



(b) The proposed divergence measures with α



(c) The proposed divergence measures with t



(d) The variation of α and t

Fig. 1 The behavior of $BB\chi^2$ divergence measure in Example 2

$$m_1: \ m_1(B) = \alpha \quad m_1(A_t) = 1 - \alpha$$
$$m_2: \ m_2(B) = 0.95 \quad m_2(A_t) = 0.05$$

In this example, m_1 and m_2 have same focal elements, i.e., B and A_t, the $BB\chi^2$ divergence measures between them are depicted as Fig. 1. The ranges of t and α are appeared in

Fig. 1 (d).

As shown in Fig. 1 (a), it is clear that the proposed divergence measure is greater than zero and smaller than one, which verifies the boundedness of the proposed divergence.

As shown in Fig. 1 (b), when α equals to zero, the conflict degree between m_1 and m_2 is the largest. With the value of α increasing, the conflict degree between m_1 and m_2 becomes smaller and smaller. When α equals to 0.95, m_1 and m_2 are completely identical. Thus, the proposed divergence measure decreases as zero.

As Fig. 1 (c) shows, as t is one, the proposed divergence measure is the largest. It is the reason that there is no intersection between the propositions B and A. As t is two, the value of the proposed divergence measure is the lowest. With t increasing, the uncertainty about A_t is enlarging due to the addition of members different from A and B, the inconsistency between the evidence is growing.

3.4 Comparative Analysis

For the purpose of explaining the superiority of $BB\chi^2$ divergence further, a numerical example is exploited to make comparison with the BJS divergence, $B\chi^2$ and $RB\chi^2$ divergence in [28], and analyze the convergence of divergence.

Example 3: Suppose m_1 and m_2 are two BPAs defined on Θ, A_t is a variable set defined as Table 1.

$$m_1 : m_1(B) = 0.05 \quad m_1(\{A_t\}) = 0.95$$
$$m_2 : m_2(B) = 0.95 \quad m_2(\{A_t\}) = 0.05$$

When t is one, A is highly conflicting with B, the proposed divergence measure is the largest. As t increases to two, the value of the proposed divergence measure is the lowest. As the uncertainty of A_t enlarges, the proposed divergence measure between the evidence is increasing. As depicted in Fig. 2, with the variation of t, it is found that the BJS and $B\chi^2$ divergence measure keep unchanged, it is unable to appear correct evidence difference tendency. By contrast, $RB\chi^2$ and $BB\chi^2$ divergence measures are more accurate and consistent with the changing situation.

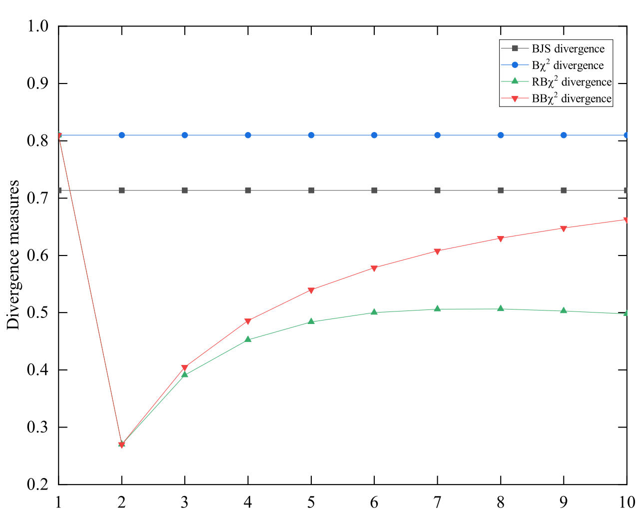


Fig. 2 The comparison of BJS, $B\chi^2$, $RB\chi^2$ and $BB\chi^2$ divergence

Actually, the range of t can be expanded to the infinity. Consequently, for exploring the strength of $BB\chi^2$ divergence further, the convergence of divergence measure will be discussed. When m_1 and m_2 include multi-element sets consisting of more than two elements ($t > 2$), the general formulas of $BetP_m s$ of m_1 and m_2 are presented as follows.

$$BetP_{m_1}(B) = m_1(B) + \frac{m_1(\{A_t\})}{t},$$
$$BetP_{m_1}(A) = \cdots = BetP_{m_1}(X_t) = \frac{m_1(\{A_t\})}{t}$$
$$BetP_{m_2}(B) = m_2(B) + \frac{m_2(\{A_t\})}{t},$$
$$BetP_{m_2}(A) = \cdots = BetP_{m_2}(X_t) = \frac{m_2(\{A_t\})}{t}$$

Where X_t is the last member of set A_t in Table 1, $X_t \in \Theta$. When the specific belief values corresponding to the propositions in m_1 and m_2 are substituted to $BetP_{m_1}$ and $BetP_{m_1}$, $BB\chi^2$ divergence measure is calculated as

$$BB\chi^2(m_1, m_2) = \frac{0.81(t-1)}{t+1}$$

In addition, $RB\chi^2$ divergence measure is calculated as

$$RB\chi^2(m_1, m_2) = \frac{1}{2}\frac{\left(\frac{1.05}{0.1+0.95t} - \frac{1.95}{1.9+0.05t}\right)^2}{\frac{1.05}{0.1+0.95t} + \frac{1.95}{1.9+0.05t}} + \frac{1}{2}\frac{\left(\frac{0.95}{0.1+0.95t} - \frac{0.05}{1.9+0.05t}\right)^2 * (t-1)}{\frac{0.95}{0.1+0.95t} + \frac{0.05}{1.9+0.05t}}$$

If t is close to the infinity, the convergence of $BB\chi^2(m_1, m_2)$ and $RB\chi^2(m_1, m_2)$ is showed in Table 2.

Easy to know, as t becomes larger and larger, the discrepancy degree between m_1 and m_2 will increase. Correspondingly, the values of divergence measure should increase. However, from Table 2, the limit of $RB\chi^2$ divergence is zero. Dissimilarily, the limit of $BB\chi^2$ divergence is 0.81, the reason why it doesn't reach to 1 is that the two pieces of evidence can't be completely conflicting as t is increasing. Thus, $BB\chi^2$ divergence is more reasonable and effective for evidence conflict measurement.

For confirming that the $BB\chi^2$ divergence performs more better than $RB\chi^2$ divergence, a numerical example is employed, where the disturbance is added to the evidence.

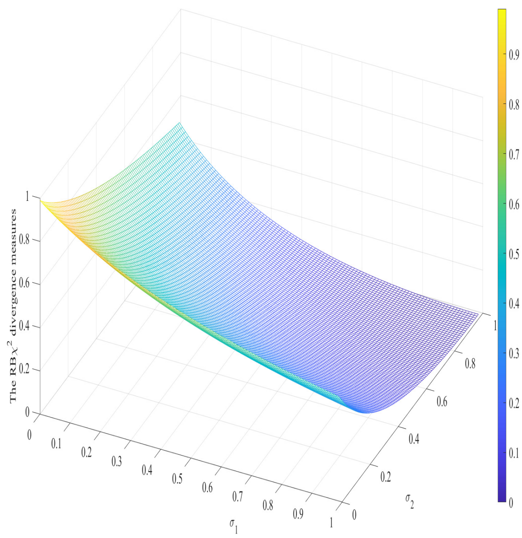
Example 4: Suppose m_1 and m_2 are two BPAs defined on $\Theta = \{A, B, C\}$, σ_1 and σ_2 are the disturbance added to m_1 and m_2 respectively.

$$m_1 : m_1(A) = 0.99 - \sigma_1 \quad m_1(B) = 0.01 \quad m_1(\Theta) = \sigma_1$$
$$m_2 : m_2(B) = 0.01 \quad m_2(C) = 0.99 - \sigma_2 \quad m_2(\Theta) = \sigma_2$$

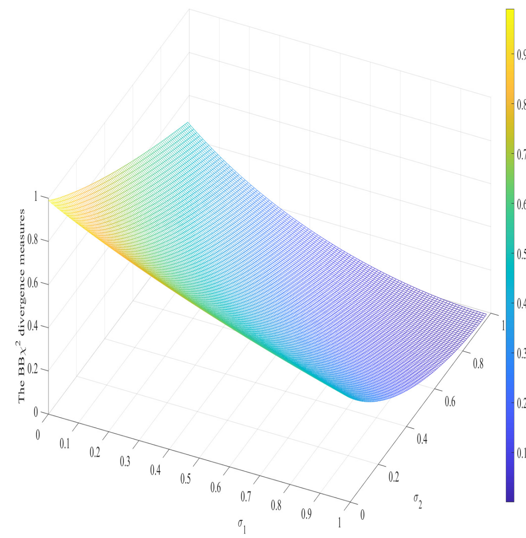
As shown in Fig. 3 (a) and 3 (b), the $RB\chi^2$ and $BB\chi^2$ divergence measure decrease as the disturbance σ_1 and σ_2

Table 2 The convergence of divergence measure

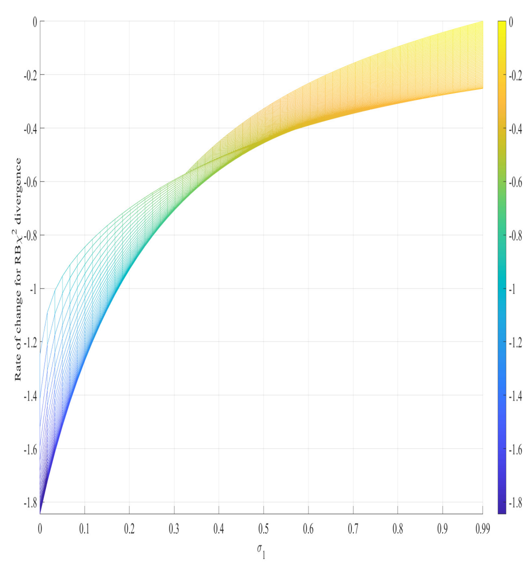
Divergence	Limit of divergence measure
$RB\chi^2$	$RB\chi^2\,(m_1, m_2) \to 0, (t \to \infty)$
$BB\chi^2$	$BB\chi^2\,(m_1, m_2) \to 0.81, (t \to \infty)$



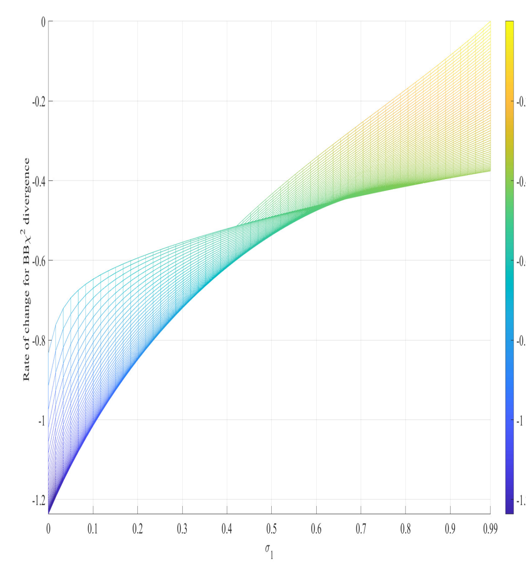
(a) The $RB\chi^2$ divergence measures with σ_1 and σ_2



(b) The $BB\chi^2$ divergence measures with σ_1 and σ_2



(c) Rate of change for the $RB\chi^2$ divergence with σ_1



(d) Rate of change for the $BB\chi^2$ divergence with σ_1

Fig. 3 The comparison and rate of change of $RB\chi^2$ and $BB\chi^2$ divergence measure

enlarge. It is because that belief assigned to A and C is decreasing and belief endowed to Θ is increasing, which leads to the two pieces of evidence more approaching.

Although the changing trend of $BB\chi^2$ divergence measure in Fig. 3 (b) seems similar to $RB\chi^2$ divergence measure in Fig. 3 (a), there is difference on the extent of variation. As we can see in Fig. 3 (a), it can be found that the larger values of $RB\chi^2$ divergence measure extremely aggregate on a narrow area. While in Fig. 3 (b), the larger values of $BB\chi^2$ divergence measure assemble on a relatively large area in general. Analyzing such difference, firstly, $BB\chi^2$ divergence is calculated as

$$BB\chi^2\,(m_1, m_2) = \frac{1}{2}\left[\frac{\left(0.99-\frac{2\sigma_1}{3}-\frac{\sigma_2}{3}\right)^2}{0.99-\frac{2\sigma_1}{3}+\frac{\sigma_2}{3}} + \frac{\left(\frac{\sigma_1}{3}-\frac{\sigma_2}{3}\right)^2}{0.02+\frac{\sigma_1}{3}+\frac{\sigma_2}{3}}\right.$$

$$+\frac{\left(\frac{\sigma_1}{3}-0.99+\frac{2\sigma_2}{3}\right)^2}{\frac{\sigma_1}{3}+0.99-\frac{2\sigma_2}{3}}\Bigg]$$

Then, the rate of change refers to the partial derivative with respect to σ_1, indicating the speed at which the divergences change as σ_1 varies. The rate of change for $RB\chi^2$ and $BB\chi^2$ divergence varying with σ_1 are respectively depicted in Fig. 3 (c) and 3 (d). As presented in Fig. 3 (c), it is discovered that the rate of change for $RB\chi^2$ divergence ranges from −1.8 to 0, the rate of change for $BB\chi^2$ divergence varies from −1.2 to 0 in Fig. 3 (d). With the wider variation range of the partial derivative, the $RB\chi^2$ divergence decreases more quickly than $BB\chi^2$ divergence. This is the reason why large values of $RB\chi^2$ divergence measure exceedingly aggregate on a smaller area. This also concludes that the $RB\chi^2$ divergence is more susceptible to the disturbance than $BB\chi^2$.

4. The Proposed Multi-Source Information Fusion Method Based on $BB\chi^2$ Divergence Measure

Based on $BB\chi^2$ divergence, a new multi-source information fusion method is proposed to handle conflicting bodies of evidence before combination. The smaller $BB\chi^2$ divergence is, the more similar evidence is. For this functional character of $BB\chi^2$ divergence, bodies of evidence can be endowed with different credibility weights to indicate different significance. In other words, $BB\chi^2$ divergence can embody difference and connection between evidence. Furthermore, when evidence has high uncertainty, information entropy of evidence becomes large. It represents that evidence will be given higher information volume weights. Therefore, in the proposed fusion method, $BB\chi^2$ divergence is utilized to construct a divergence matrix to obtain credibility weights. Deng entropy is used to acquire the information volume weights of evidence. Eventually, the comprehensive weights by integrating $BB\chi^2$ divergence and Deng entropy are able to fully reflect relationship between evidence. The flowchart of this new multi-source information fusion method is showed as Fig. 4.

Assume that the frame of discernment is $\Theta = \{\theta_1, \theta_2, \ldots, \theta_m\}$. There are n sensors, from which n pieces of evidence are denoted as $m_1, m_2, \ldots, m_n$. The detailed calculation steps of the proposed fusion algorithm are given as follows.

Step 1: Construct divergence measure matrix
According to $BB\chi^2$ divergence Eq. (13), we calculate the divergence between evidence $m_i\,(i = 1, 2, \ldots, n)$ and $m_j\,(j = 1, 2, \ldots, n)$ as d_{ij} and construct the divergence measure matrix $DMM = (d_{ij})_{n\times n}$ as follows:

$$DMM = \begin{bmatrix} 0 & \cdots & d_{1i} & \cdots & d_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{i1} & \cdots & 0 & \cdots & d_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ d_{n1} & \cdots & d_{ni} & \cdots & 0 \end{bmatrix} \quad (16)$$

Step 2: Generating the credibility weights W_c
The average divergence $\bar{d}(m_i)$ of evidence m_i can be calculated by Eq. (17). Furthermore, the support degree $Sup(m_i)$ of evidence m_i is inversely proportional to $\bar{d}(m_i)$, which is defined and normalized by Eqs. (18) and (19):

$$\bar{d}(m_i) = \frac{\sum_{j=1}^{n} d_{ij}}{n-1}, i = 1, \ldots, n \quad (17)$$

$$Sup(m_i) = \frac{1}{\bar{d}(m_i)}, i = 1, \ldots, n \quad (18)$$

$$W_c(m_i) = \frac{Sup(m_i)}{\sum_{j=1}^{n} Sup(m_j)}, i = 1, \ldots, n \quad (19)$$

Step 3: Forming the information volume weights W_{IV}
Deng entropy $E_d(m_i)$ of evidence m_i can be calculated by Eq. (9). Then, the information volume $IV(m_i)$ of evidence m_i is denoted and normalized by Eqs. (20) and (21):

$$IV(m_i) = e^{E_d(m_i)},\ i = 1, \ldots, n \quad (20)$$

$$W_{IV}(m_i) = \frac{IV(m_i)}{\sum_{j=1}^{n} IV(m_j)},\ i = 1, \ldots, n \quad (21)$$

Step 4: Producing the final weights W
The final weights $W(m_i)$ of evidence m_i is presented as:

$$W(m_i) = \frac{W_c(m_i) \times W_{IV}(m_i)}{\sum_{j=1}^{n} W_c(m_j) \times W_{IV}(m_j)}, i = 1, \ldots, n \quad (22)$$

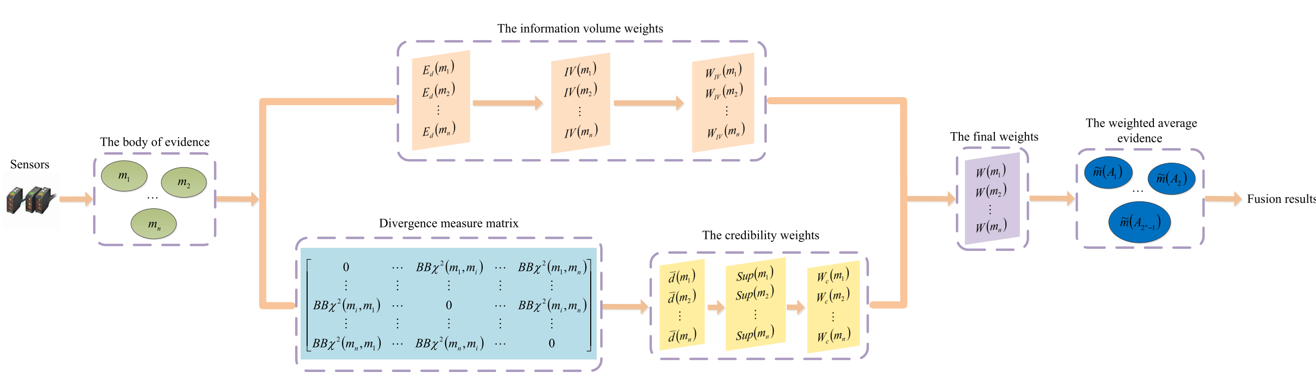


Fig. 4 The flowchart of the proposed multi-source information fusion method

Step 5: Weighting the body of evidence
The weighted average evidence is calculated as:

$$\widetilde{m}(A) = \sum_{i=1}^{n} W(m_i) \times m_i(A), A \subseteq \Theta \tag{23}$$

Step 6: The weighted average evidence needs to be fused by $n-1$ times with the Dempster's combination rule, the eventual fused result can be obtained as

$$m = \underbrace{\widetilde{m} \oplus \widetilde{m} \oplus \cdots \oplus \widetilde{m}}_{n-1\ times} \tag{24}$$

5. Application

To verify the feasibility and superiority of the proposed multi-source information fusion method, two applications of target recognition and fault diagnosis are implemented among the proposed method and some other methods such as Dempster [4]'s method, Murphy [32]'s method, Deng [33]'s method, Xiao [27]'s method and Xiao [34]'s method.

5.1 Application in Target Recognition

In a multisensor-based target recognition system, five installed sensors, S_1, S_2, S_3, S_4 and S_5, are located at different positions to monitor the objectives. The frame of discernment, consisting of three types of targets, is represented as $\Theta = \{A, B, C\}$. On this frame, the target information collected from S_1, S_2, S_3, S_4 and S_5 is modeled as five BPAs, which are m_1, m_2, m_3, m_4 and m_5 listed in Table 3. This example is cited from Xiao [34].

As can be found from Table 3, the five pieces of evidence are conflicting. Among them, m_2 strongly supports the target B, it deviates from the mainstream perception that m_1, m_3, m_4 and m_5 stand for the target A. According to the proposed algorithm in Sect. 4, evidence fusion process is showed as follows.

Step 1: the divergence measure matrix DMM is calculated as:

$$DMM = \begin{bmatrix} 0 & 0.4105 & 0.1526 & 0.1225 & 0.1225 \\ 0.4105 & 0 & 0.7470 & 0.6959 & 0.6959 \\ 0.1526 & 0.7470 & 0 & 0.0033 & 0.0033 \\ 0.1225 & 0.6959 & 0.0033 & 0 & 0 \\ 0.1225 & 0.6959 & 0.0033 & 0 & 0 \end{bmatrix}$$

Step 2-Step 4: the calculation results contained in each step are presented in Table 4.

Step 5: the weighted average evidence is obtained as:

$$\widetilde{m}(A) = 0.5446$$
$$\widetilde{m}(B) = 0.1484$$
$$\widetilde{m}(C) = 0.0807$$
$$\widetilde{m}(\{A, C\}) = 0.2263$$

Step 6: the weighted average evidence needs to be fused by 4 times with the Dempster's combination rule.

In Table 5, the eventual fusion results of m_1, m_2, m_3, m_4 and m_5, generated by the proposed fusion algorithm and another five methods for comparison, are presented. As seen in Table 5, all methods can correctly recognize the target A. Amongst these methods, the Dempster [4]'s method identifies the real target with the lowest belief. Same as Deng [33]'s method, Xiao [27]'s method attains a larger belief value than the Dempster [4]'s method and Murphy [32]'s method, but it is based on the BJS divergence measure which ignores the influence of multi-element sets. Furthermore, Xiao [34]'s

Table 3 The BPAs modeled from five sensors in target recognition

	A	B	C	$\{A, C\}$
m_1	0.40	0.28	0.30	0.02
m_2	0.01	0.90	0.08	0.01
m_3	0.63	0.06	0.01	0.30
m_4	0.60	0.09	0.01	0.30
m_5	0.60	0.09	0.01	0.30

Abbreviation: BPA, basic probability assignment.

Table 4 The results obtained by Step 2-Step 4 of the proposed method in target recognition

	m_1	m_2	m_3	m_4	m_5
$\bar{d}$	0.2021	0.6374	0.2266	0.2054	0.2054
Sup	4.9490	1.5690	4.4137	4.8675	4.8675
W_c	0.2395	0.0759	0.2136	0.2355	0.2355
E_d	1.7087	0.5770	1.7265	1.8179	1.8179
IV	5.5215	1.7808	5.6209	6.1586	6.1586
W_{IV}	0.2188	0.0706	0.2227	0.2440	0.2440
W	0.2379	0.0243	0.2160	0.2609	0.2609

Table 5 The combination results using different methods in target recognition

Method	m(A)	m(B)	m(C)	m(A, C)	Target
Dempster [4]	0.8657	0.0168	0.1167	0.0007	A
Murphy [32]	0.9694	0.0175	0.0110	0.0021	A
Deng [33]	0.9885	0.0013	0.0079	0.0023	A
Xiao [27]	0.9885	0.0015	0.0077	0.0023	A
Xiao [34]	0.9888	0.0015	0.0073	0.0024	A
Proposed method	0.9898	0.0003	0.0078	0.0021	A

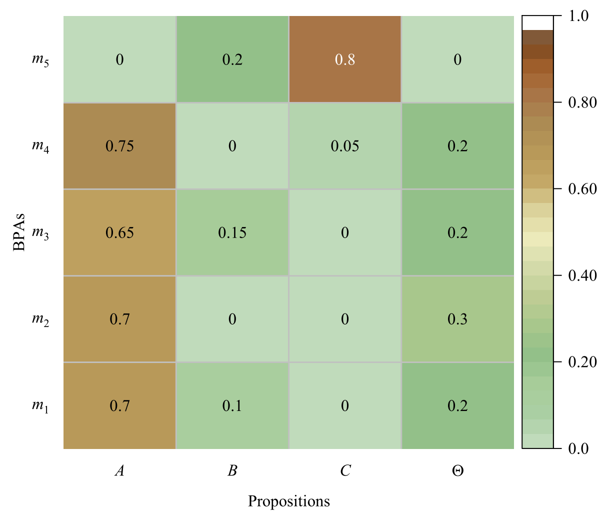


Fig. 5 The BPAs modeled from five sensors in fault diagnosis

method assigns a higher belief than the Xiao [27]'s, however, the information volume of evidence hasn't been considered into its algorithm. Differently, in the proposed method, $BB\chi^2$ divergence with the defect of BJS divergence overcome can achieve better discrepancy measure performance. In addition, the information volume weight is considered to fully decide the credibility of each evidence. Therefore, the proposed method gives the highest belief to A.

5.2 Application in Fault Diagnosis

In the automobile system, three kinds of faults, low oil pressure, air leakage in the intake system, and solenoid valve jam, may happen. For determining which type of fault occurs in the system, sensors S_1, S_2, S_3, S_4 and S_5 are used to collect the diagnosis data. The frame of discernment, containing the three types of faults, can be represented as $\Theta = \{A, B, C\}$. The collected data is modeled as five pieces of evidence showed in Fig. 5. The sensor S_5 breaks down due to speed overload, so m_5 is served as noisy data. This example is cited from [28].

From Fig. 5, m_1, m_2, m_3, m_4 supports that A is the real fault type. Contrarily, m_5 gives the majority of belief to C. m_5 is conflicting with m_1, m_2, m_3, m_4. The fusion results of the proposed method and the comparative methods are showed in Fig. 6.

As displayed in Fig. 6 (a), Dempster [4]'s method treats C as the true fault type, because the Dempster rule

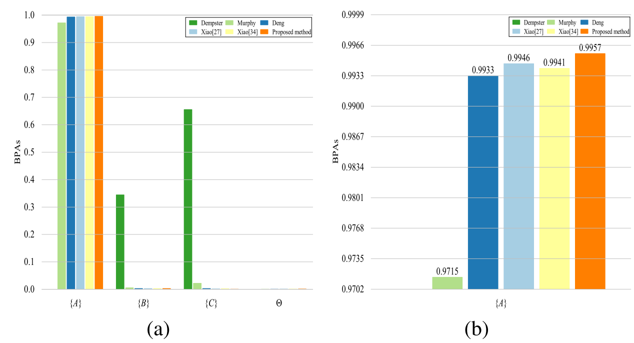


Fig. 6 The combination results using different methods in fault diagnosis

fail to handle the highly conflicting evidence. Except the Dempster [4]'s method, the remaining methods, Murphy [32]'s method, Deng [33]'s method, Xiao [27]'s method, Xiao [34]'s method and the proposed method, can effectively deal with evidence conflict and recognize the correct fault type A.

As seen in Fig. 6 (b), compared with Xiao [27]'s method, the proposed method gets a more higher accuracy. The reason is that the $BB\chi^2$ divergence can reflect the interaction between singletons and multi-element subsets. Meanwhile, the proposed method makes use of not only the $BB\chi^2$ divergence to obtain the credibility weight but also the uncertainty of the evidence to obtain the information volume weight. Based on the two kinds of weights, the final weight is comprehensively determined to achieve better decision level.

6. Conclusion

In this paper, a new belief divergence, called $BB\chi^2$ divergence, is presented to characterize the discrepancy between evidence. The advantages of the $BB\chi^2$ divergence are:

1. By introducing the pignistic probability transform, the proposed $BB\chi^2$ divergence takes the uncertainty of multi-element sets into account and embodies the relationship between singletons and multi-element sets.
2. Compared with other divergences, $BB\chi^2$ divergence is more reasonable and accurate.

Based on the $BB\chi^2$ divergence, a new multi-source information fusion method is designed. In the applications of target recognition and fault diagnosis, the proposed fusion method outperforms other related methods with the highest accuracy. Therefore, the proposed method provides a promising solution for dealing with conflict evidence.

However, it is found that the limitations of the $BB\chi^2$ divergence are:

1. When BetP in $BB\chi^2$ divergence converts evidence into probabilities, some information may be lost.
2. If BPA is in the form of interval values, calculating $BB\chi^2$ divergence divergence becomes challenging.

In our future work, we intend to further explore a divergence directly from BPA, and the fusion method when the BPA of evidence is an interval value.

Acknowledgments

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