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PAPER Special Section on Foundations of Computer Science

Computational Complexity of Yajisan-Kazusan and Stained Glass*

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SUMMARY Yajisan-Kazusan and Stained Glass are Nikoli's pencil puzzles. We study the computational complexity of Yajisan-Kazusan and Stained Glass puzzles. It is shown that deciding whether a given instance of each puzzle has a solution is NP-complete.

key words: *Yajisan-Kazusan, Stained Glass, pencil puzzle, computational complexity, NP-complete*

1. Introduction

Yajisan-Kazusan is played on a rectangular grid of cells (see Fig. 1(a)). Some of the cells contain numbers with an arrow indicating an orthogonal direction. The purpose of the puzzle to paint every cell in black or white (see Fig. 1(i)) according to the following rules [1]: (1) The number in a white cell indicates the number of black cells in the direction the associated arrow points to. (A black cell may or may not contain a valid number.) (2) Black cells must not be orthogonally adjacent. (3) All white cells must be connected as part of a single contiguous region.

Figure 1(a) is an initial configuration of a Yajisan-Kazusan puzzle. From Figs. 1(b)–(i), the reader can understand basic techniques for finding a solution. (b) Since the gray cell contains an invalid number, it must be colored black (see (c)). The three red cells adjacent to the gray cell must be colored white. If the blue cell is colored black, then the white cell at the top right corner is isolated. Thus, the blue cell is also colored white. (c) The gray cell and four red cells must be colored black and white, respectively. (d) Two gray cells and five red cells must be colored black and white, respectively. The blue cell must be colored white so that the bottom left cell is not isolated. (e) The two gray cells in the first column must be colored black, and thus the three red cells and two blue cells must be colored white. Then, the gray cell in the sixth column is colored black. (f) If the gray cell and two red cells are colored black and white respectively, then the green cell must be colored black (see (g)). (g) is an invalid painting of cells, since the red number 1 points to two black cells. Hence, in (f), the gray cell and two red cells must be colored white and black, respectively

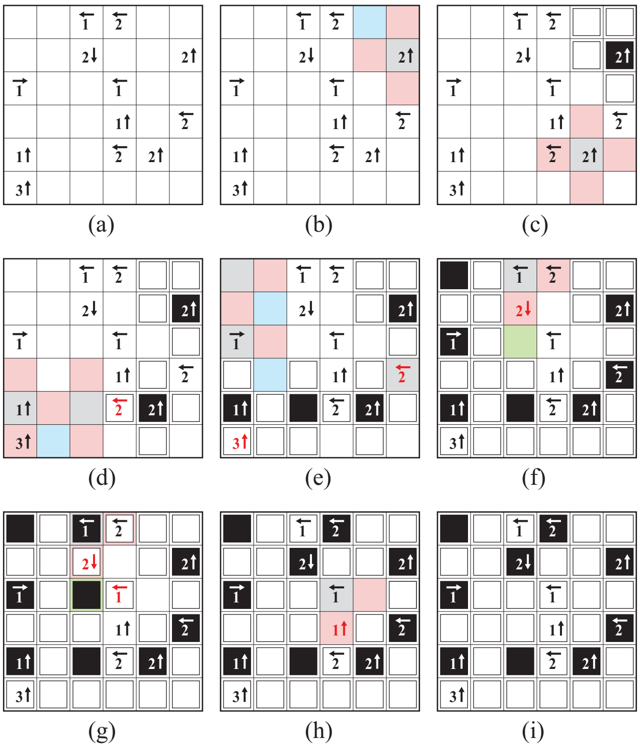

Fig. 1 (a) Initial configuration of a Yajisan-Kazusan puzzle. (b)–(i) are the progress from the initial configuration to a solution. (g) is an invalid painting of cells.

(see (h)). Also, the green cell in (f) is colored white. (h) If the gray cell and two red cells are colored black and white respectively, then the red number 1 becomes invalid. (i) is one of the multiple solutions. (There is another solution such that the red cell with number 1 in (h) is colored black.)

Stained Glass is played on a rectangular field (see Fig. 2(a)). The field is partitioned into *pieces* (see nine pieces $a, b, \ldots, i$ in Fig. 1(b)). Each piece is separated by vertical, horizontal, and diagonal line segments, and small colored circles are placed on some of line segments. (In the figure, there are two blue circles, two red circles, and three yellow circles.)

The purpose of the puzzle is to paint every piece in blue or red (see Fig. 2(f)) according to the following rules [2]: (1) A blue circle denotes that there are more blue pieces touching that circle than red pieces. (2) A red circle denotes that there are more red pieces touching that circle than blue pieces. (3) A yellow circle denotes that there are an equal

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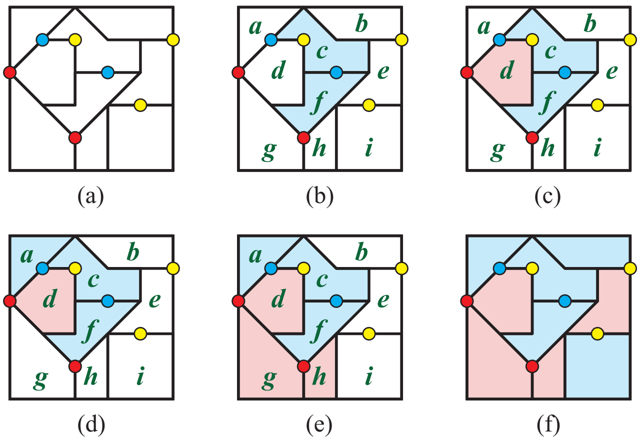

Fig. 2 (a) Initial configuration of a Stained Glass puzzle. (b)–(f) are the progress from the initial configuration to a solution.

number of blue and red pieces touching that circle. (In [10], the original Stained Glass puzzle uses colors black, white, and gray. In this paper, we use colors blue, red, and yellow for better visibility.)

Figure 2(a) is an initial configuration of a Stained Glass puzzle. (b) There is a blue circle which touches two pieces c and f. Thus, c and f must be colored blue. (c) Piece d must be colored red, since there is a yellow circle touching two pieces c and d. (d) Piece a must be colored blue, since there is a blue circle touching pieces a, c, and d. (e) Pieces g and h must be colored red. (f) is one of the multiple solutions. (There is another solution such that pieces b, e, and i are colored red, blue, and red, respectively.)

In this paper, we study the computational complexity of the decision version of the Yajisan-Kazusan and Stained Glass puzzles. The instance of the *Yajisan-Kazusan puzzle problem* is a rectangular grid of cells, where some of the cells contain a number with an arrow. The instance of the *Stained Glass puzzle problem* is a rectangular field partitioned into pieces, where a set of 3-colored circles are placed on the boundaries of pieces. Each problem is to decide whether there is a solution to the instance.

In Sects. 2 and 3, we will show that the Yajisan-Kazusan and Stained Glass puzzle problems are NP-complete. It is clear that the Yajisan-Kazusan puzzle problem belongs to NP, since the puzzle ends when every cell is painted in black or white. The Stained Glass puzzle problem also belongs to NP, since the puzzle ends when every piece is painted in blue or red.

There have been a lot of papers, which prove the NP-completeness of Nikoli's pencil puzzles. In the past few years, Choco Banana [8], Five Cells and Tilepaint [5], Moon-or-Sun, Nagareru, and Nurimeizu [6], Nondango [15], and Toichika [14] were shown to be NP-complete. Those NP-complete puzzles can be categorized into groups depending on which representative NP-complete problem they are reduced from. The categorization reflects fundamental differences in the inherent difficulty of puzzles within each group. (i) Choco Banana, Five Cells, Nondango, and Toichika are reduced from the 3SAT problem, (ii) Moon-or-Sun, Nagareru, and Nurimeizu are reduced from the Hamiltonian cycle problem, and (iii) Tilepaint is reduced from the 3-dimensional matching problem. The current paper demonstrates that Yajisan-Kazusan and Stained Glass fall into the first and third groups, respectively. One of the interesting related works is on zero-knowledge proof (ZKP) protocols. Recently, ZKP protocols were constructed for Nikoli's pencil puzzles: Five Cells [13], Moon-or-Sun [4], Nonogram [12], Nurimisaki and Kurodoko [11].

2. NP-completeness of Yajisan-Kazusan

In this section, we will show the following theorem.

Theorem 1: The Yajisan-Kazusan puzzle problem is NP-complete.

We give the definition of the 3-dimensional matching problem in Sect. 2.1. Then, we prove the NP-completeness of the Yajisan-Kazusan puzzle in Sect. 2.2.

2.1 3-Dimensional Matching Problem

An instance of the *3-Dimensional Matching problem* (3DM) is a set $M \subseteq X \times Y \times Z$, where X, Y, and Z are disjoint sets having the same number q of elements. The 3DM problem asks whether M contains a matching, i.e., a subset $M' \subseteq M$ such that $|M'| = q$ and no two elements of M' agree in any coordinate. This problem is known to be NP-complete [3]. For example, $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, $Z = \{z_1, z_2, z_3\}$, $M = \{e_1, e_2, \ldots, e_7\}$, and

$$\begin{aligned}
&e_1 = (x_1, y_1, z_1),\ e_2 = (x_1, y_2, z_3),\ e_3 = (x_2, y_2, z_3),\\
&e_4 = (x_2, y_3, z_1),\ e_5 = (x_3, y_1, z_2),\ e_6 = (x_3, y_2, z_2),\\
&e_7 = (x_3, y_3, z_2)
\end{aligned}$$

provide an instance of 3DM. For this instance, the answer is "yes," since there is a matching $M' = \{e_2, e_4, e_5\} \subseteq M$.

2.2 Transformation from an Instance of 3DM to a Yajisan-Kazusan Puzzle

We present a polynomial-time transformation from an arbitrary instance M of 3DM to a Yajisan-Kazusan puzzle K such that M contains a matching if and only if K has a solution.

Consider Fig. 3(a). This figure is composed of a *top gadget* of the first and second rows and an *e_i-gadget* of the third and fourth rows. The e_i-gadget corresponds to $e_i = (x_j, y_k, z_l) \in M$.

In the first row of Fig. 3(a), number 1 with an up-pointing arrow appears at every other position. In the second row, number 0 with a down-pointing arrow appears at every other position from the third through $(6q+4)$th columns. In the third row, (i) two red cells contain numbers $s = 3q - 2$ and $t = 3q$ with right-pointing arrows, and (ii) number 3 with an up-pointing arrow appears at the positions indicated by label $u \in \{x_1, x_2, \ldots, x_q, y_1, y_2, \ldots, y_q, z_1, z_2, \ldots, z_q\} -$

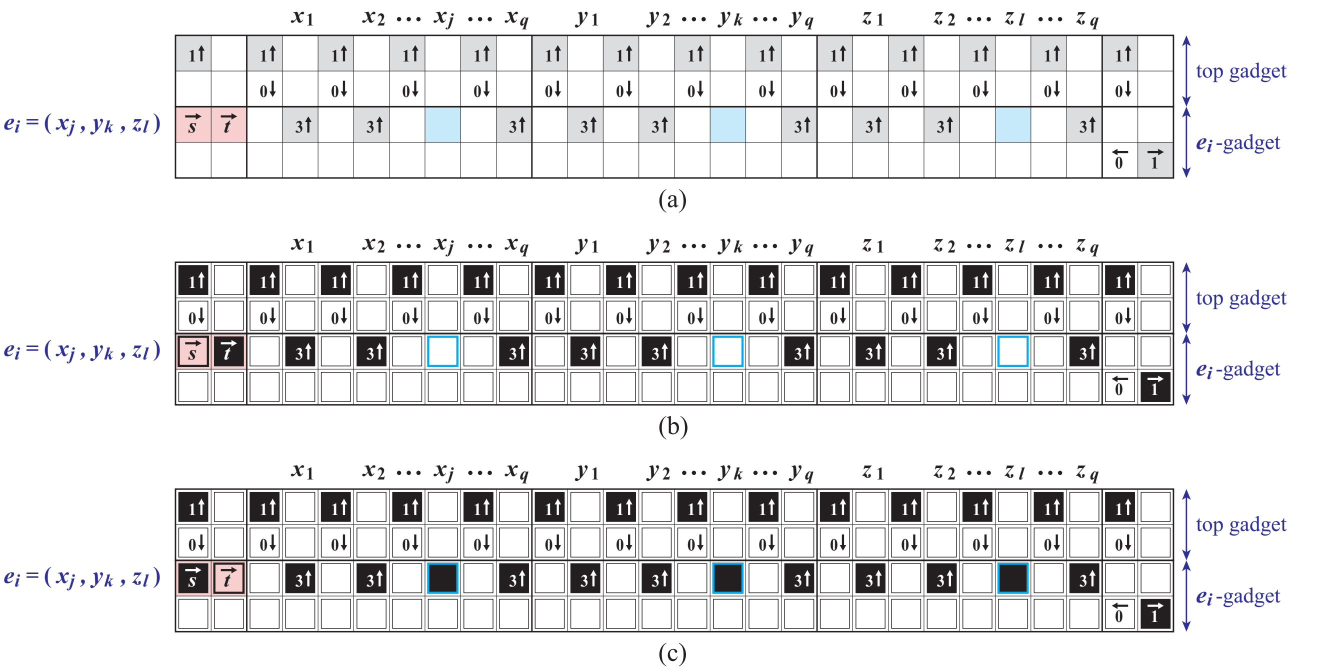

Fig. 3 In this figure, $s = 3q - 2$ and $t = 3q$. (a) The set of the first two rows is a top gadget. The set of the next two rows is an e_i-gadget transformed from $e_i = (x_j, y_k, z_l) \in M$. (b) and (c) are two possible solutions to (a).

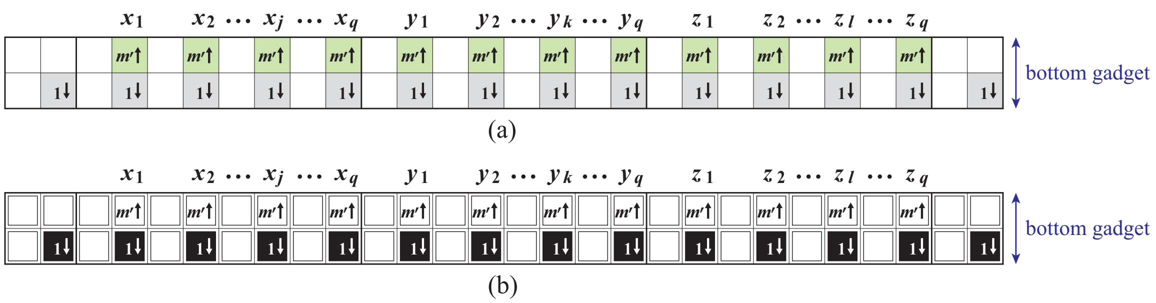

Fig. 4 (a) Bottom gadget. (b) Solution to this gadget when $m' = 0$.

$\{x_j, y_k, z_l\}$. In the fourth row, numbers 0 and 1 appear at the last two cells with left-pointing and right-pointing arrows, respectively.

In the first and third rows of Fig. 3(a), numbers 1 and 3 with up-pointing arrows are invalid. In the fourth row, number 1 with a right-pointing arrow is also invalid. Therefore, they must be colored black (see Figs. 3(b) and 3(c)). Since the two red cells contains numbers $s = 3q-2$ and $t = 3q$ with right-pointing arrows, exactly one of them must be colored black and the other must be colored white. Hence, Figs. 3(b) and 3(c) are two possible solutions to this gadget (see also Figs. 5 and 6).

Figure 4(a) is a *bottom gadget*. Fig. 4(b) is the unique solution to this gadget when $m' = 0$. (The value m' will be fixed to $m' = |M| - 1 = 6$ in Figs. 5 and 6.)

Figure 5 is the Yajisan-Kazusan puzzle K transformed from the instance $M = \{e_1, e_2, \ldots, e_7\}$, where $e_1 = (x_1, y_1, z_1)$, $e_2 = (x_1, y_2, z_3)$, $e_3 = (x_2, y_2, z_3)$, $e_4 = (x_2, y_3, z_1)$, $e_5 = (x_3, y_1, z_2)$, $e_6 = (x_3, y_2, z_2)$, and $e_7 = (x_3, y_3, z_2)$. This figure is composed of the top gadget, followed by e_i-gadgets for all $e_i \in M$, further followed by the bottom gadget. The green cells of the bottom gadget contain numbers $m' = |M| - 1 = 6$. Each e_i-gadget is placed at the $(2i+1)$th row, in which gray cells contain numbers $2i+1$ with up-pointing arrows. Thus, those numbers $2i+1$ are invalid (see Fig. 6).

Figure 6 is a solution to the puzzle K of Fig. 5. In Lemma 1, we will show that the instance M of 3DM has a matching if and only if the Yajisan-Kazusan puzzle K has a solution. From Fig. 6, one can see that there is a matching $M' = \{e_2, e_4, e_5\} \subseteq M$.

Lemma 1: *The instance M of 3DM has a matching if and only if there exists a solution to the instance K of the Yajisan-Kazusan puzzle.*

Proof. ($\Rightarrow$) Suppose that M has a matching $M' \subseteq M$,

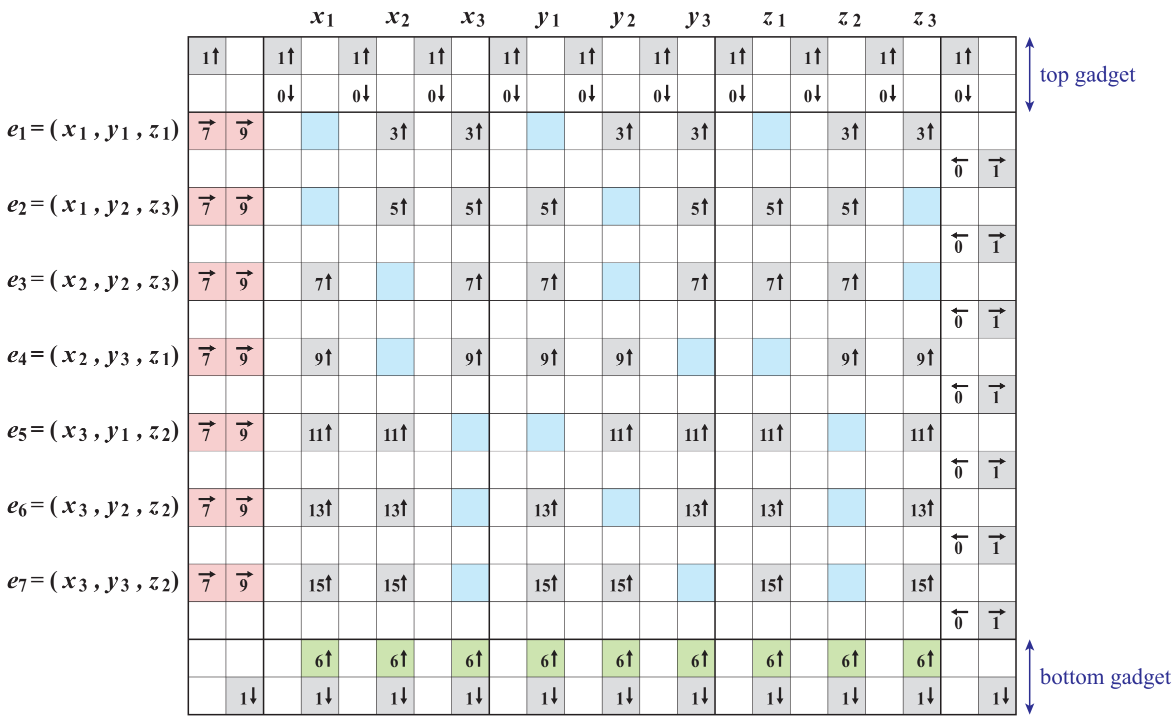

Fig. 5 Yajisan-Kazusan puzzle K transformed from 3DM $M = \{e_1, e_2, \ldots, e_7\}$, where $e_1 = (x_1, y_1, z_1)$, $e_2 = (x_1, y_2, z_3)$, $e_3 = (x_2, y_2, z_3)$, $e_4 = (x_2, y_3, z_1)$, $e_5 = (x_3, y_1, z_2)$, $e_6 = (x_3, y_2, z_2)$, and $e_7 = (x_3, y_3, z_2)$. The green cells of the bottom gadget contain numbers $m' = |M| - 1 = 6$.

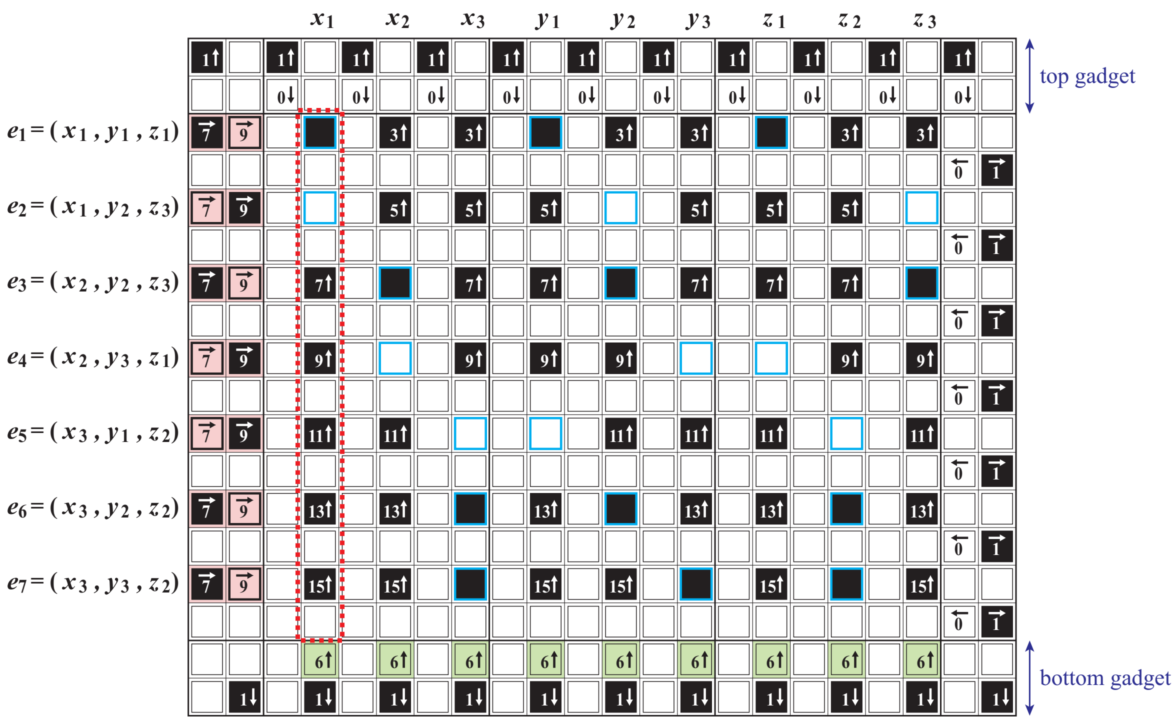

Fig. 6 Solution to the Yajisan-Kazusan puzzle K of Fig. 5. From this figure, one can see that there is a matching $M' = \{e_2, e_4, e_5\} \subseteq M$.

where $M \subseteq X \times Y \times Z$. (For example, consider instance $M = \{e_1, e_2, \ldots, e_7\}$ and its matching $M' = \{e_2, e_4, e_5\}$ in the captions of Figs. 5 and 6, respectively.) For every $u \in X \cup Y \cup Z$, there exists exactly one edge $e_{i'} \in M'$ such that

$e_{i'}$ has u as one of the three coordinates. (For example, for the coordinate $x_1 \in X \cup Y \cup Z$, there exists exactly one edge $e_2 = (x_1, y_2.z_3) \in M'$ such that e_2 has x_1 as one of its coordinates.) When $e_{i'} \in M'$, cells $\vec{s}$ and $\vec{t}$ can be colored white and black respectively in the $e_{i'}$-row (see cells $\vec{7}$ and $\vec{9}$ in the e_2-row of Fig. 6). Thus, the cell corresponding to coordinate u and edge $e_{i'}$ is colored white, and the remaining $|M| - 1$ cells corresponding to coordinate u and edges $e_i \in M - \{e_{i'}\}$ are colored black (see $|M| - 1 = 6$ black cells in the x_1-column from the 3rd through 16th rows). Therefore, for each $u \in X \cup Y \cup Z$, there are $m' = |M| - 1$ black cells in the u-column from the 3rd through $(2|M| + 2)$th rows (see the red dotted line frame in Fig. 6). Hence, the number m' $(= 6)$ in each green cell indicates the number of black cells in the direction the associated arrow points to. Thus, there is a solution to the instance K of the Yajisan-Kazusan puzzle.

($\Leftarrow$) Let K be a Yajisan-Kazusan puzzle transformed from an instance M of 3DM (see Fig. 5). Suppose that there is a solution to K (see Fig. 6). Consider an arbitrary coordinate $u \in X \cup Y \cup Z$ (for example, consider x_1 in Fig. 6). Since the green cell has number m' $(= 6)$ with an up-pointing arrow, there exists exactly one edge $e_{i'}$ such that the cell in the $e_{i'}$-row and the u-column is white (see the white cell in the e_2-row and the x_1-column). This implies that cells $\vec{s}$ and $\vec{t}$ are colored white and black respectively in the $e_{i'}$-row (see $\vec{7}$ and $\vec{9}$ in the e_2-row). Therefore, the exists a matching M' such that $e_{i'} \in M'$ if and only if cells $\vec{s}$ and $\vec{t}$ are colored white and black respectively in the $e_{i'}$-row (see three rows corresponding to edges in $M' = \{e_2, e_4, e_5\}$). □

3. NP-completeness of Stained Glass

In this section, we will show the following theorem.

Theorem 2: The Stained Glass puzzle problem is NP-complete.

We give the definition of the positive planar 1-in-3-SAT problem in Sect. 3.1. Then, we prove the NP-completeness of the Stained Glass puzzle in Sect. 3.2.

3.1 Positive Planar 1-in-3-SAT

Let $U = \{x_1, x_2, \ldots, x_n\}$ be a set of Boolean *variables*. Boolean variables take on values 0 (false) and 1 (true). A *clause* over U is a set of variables over U, such as $\{x_1, x_2, x_3\}$. A clause is *satisfied* by a truth assignment if and only if *exactly one* of its members is true under that assignment.

An instance of *positive planar 1-in-3-SAT* is a collection $C = \{c_1, c_2, \ldots, c_m\}$ of clauses over U such that (i) $|c_j| = 3$ for each $c_j \in C$ and (ii) the graph $G = (V, E)$, defined by $V = U \cup C$ and $E = \{ (x_i, c_j) \mid x_i \in c_j \in C \}$, is planar. The positive planar 1-in-3-SAT problem asks whether there exists some truth assignment for U that simultaneously satisfies all the clauses in C. This problem is known to be NP-complete [9].

For example, $U = \{x_1, x_2, x_3, x_4, x_5\}$, $C = \{c_1, c_2, c_3\}$, and $c_1 = \{x_1, x_2, x_5\}$, $c_2 = \{x_1, x_3, x_4\}$, and $c_3 = \{x_2, x_3, x_5\}$ provide an instance of positive planar 1-in-3-SAT. For this instance, the answer is "yes," since there is a truth assignment $(x_1, x_2, x_3, x_4, x_5) = (0, 1, 0, 1, 0)$ satisfying all clauses.

3.2 Transformation from an Instance of Positive Planar 1-in-3-SAT to a Stained Glass Puzzle

We present a polynomial-time transformation from an arbitrary instance C of positive planar 1-in-3-SAT to a Stained Glass puzzle S such that C is satisfiable if and only if S has a solution.

Variable x_i is transformed into a variable gadget as shown in Fig. 7(a). This gadget is a sequence of an even number of square pieces connected via yellow circles. In Fig. 7(b), if pieces a and b are colored red and blue respectively, then z will be colored blue. This corresponds to the assignment $x_i = 1$. In Fig. 7(c), if a and b are colored blue and red respectively, then z is colored red. This corresponds to the assignment $x_i = 0$.

Figure 8(a) is a clause gadget for $c_j = \{x_{i_1}, x_{i_2}, x_{i_3}\}$. In this gadget, both of pieces p and q must be colored blue (see Fig. 8(b,c,d)), since there is a blue circle touching them. In Fig. 8(a), there is a yellow circle touching four pieces z_1, z_2, z_3, and p; this yellow circle plays a key role in the 1-in-3 property. Note that each of the three pieces z_1, z_2, and z_3 corresponds to the rightmost piece z in Fig. 7. One can see that the gadget of Fig. 8(a) has a solution if and only if exactly one of x_{i_1}, x_{i_2}, and x_{i_3} is 1 (see Fig. 8(b,c,d)). (Figure 9 will be explained later.)

Figure 10 is a Stained Glass puzzle S transformed from positive planar 1-in-3-SAT $C = \{c_1, c_2, c_3\}$, where $c_1 = \{x_1, x_2, x_5\}$, $c_2 = \{x_1, x_3, x_4\}$, and $c_3 = \{x_2, x_3, x_5\}$. In Fig. 10, when we connect a variable gadget with a clause gadget, we sometimes need a connection gadget of odd length. In such a case, we use a gadget given in Fig. 9. In this gadget, two sets of pieces $\{s, t\}$ and $\{u, v\}$ are always colored blue and red, respectively (see Fig. 9(b,c)). If piece a is colored red (resp. blue), then piece z will be colored blue (resp. red). (In Fig. 10, we used three such connection gadgets between x_3 and c_3; x_4 and c_2; and x_5 and c_1.)

In Lemma 2, we will show that positive planar 1-in-3-SAT C is satisfiable if and only if there exists a solution to Stained Glass puzzle S. From the solution of puzzle S, one can see that the assignment $(x_1, x_2, x_3, x_4, x_5) = (0, 1, 0, 1, 0)$ satisfies all clauses of C. In the figure, there are three large pieces f_1, f_2, and f_3. Those pieces can be colored either blue or red with no restriction.

Lemma 2: *The instance C of positive planar 1-in-3-SAT is satisfiable if and only if there exists a solution to the instance S of the Stained Glass puzzle.*

Proof. ($\Rightarrow$) Suppose that instance C is satisfiable, i.e., there is a truth assignment to variables $x_1, x_2, \ldots, x_n$ satisfying all clauses of C. For such an assignment, exactly one of the three variables x_{i_1}, x_{i_2}, and x_{i_3} has value 1 for every clause $c_j = \{x_{i_1}, x_{i_2}, x_{i_3}\} \in C$. This implies that every clause gadget has a solution (see Fig. 8), since (i) one of the three

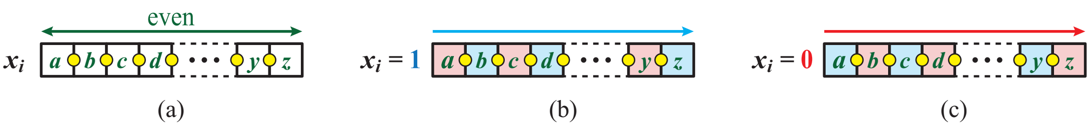

Fig. 7 (a) Variable gadget for x_i. (b) Solution corresponding to the assignment $x_i = 1$. (c) Solution corresponding to $x_i = 0$.

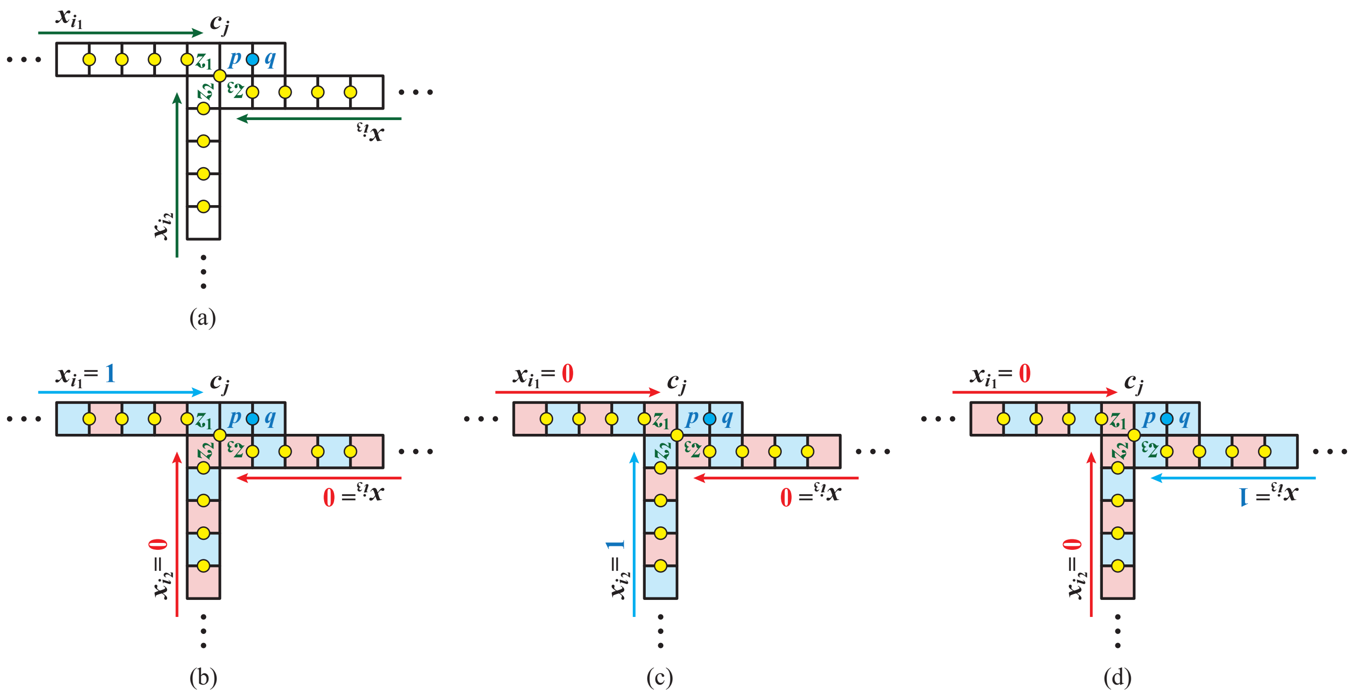

Fig. 8 (a) Clause gadget for $c_j = \{x_{i_1}, x_{i_2}, x_{i_3}\}$. (b,c,d) The gadget has a solution when exactly one of x_{i_1}, x_{i_2}, and x_{i_3} is 1.

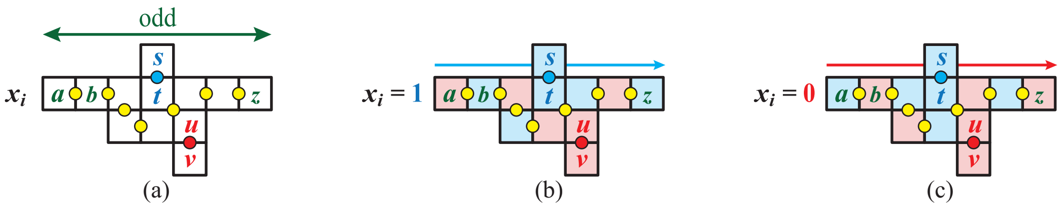

Fig. 9 (a) Connection gadget of odd length. (b) and (c) are two possible solutions corresponding to assignments $x_i = 1$ and $x_i = 0$, respectively.

pieces z_1, z_2, and z_3 is colored blue and (ii) two of them are colored red. As shown in Figs. 7 and 9, variable gadgets and connection gadgets have always solutions. Therefore, if C is satisfiable, then there is a solution to the instance S of the Stained Glass puzzle.

($\Leftarrow$) Let S be a Stained Glass puzzle transformed from an instance C of positive planar 1-in-3-SAT. Suppose that there is a solution to S (see Fig. 10). In each clause gadget $c_j = \{x_{i_1}, x_{i_2}, x_{i_3}\} \in C$, (i) one of the three pieces z_1, z_2, and z_3 is colored blue and (ii) two of them are colored red. This implies that there is a truth assignment to $(x_1, x_2, \ldots, x_n)$ satisfying every clause $c_j \in C$ such that $x_i = 0$ if and only if pieces a and b of the variable gadget x_i are colored blue and red respectively for all $i \in \{1, 2, \ldots, n\}$. □

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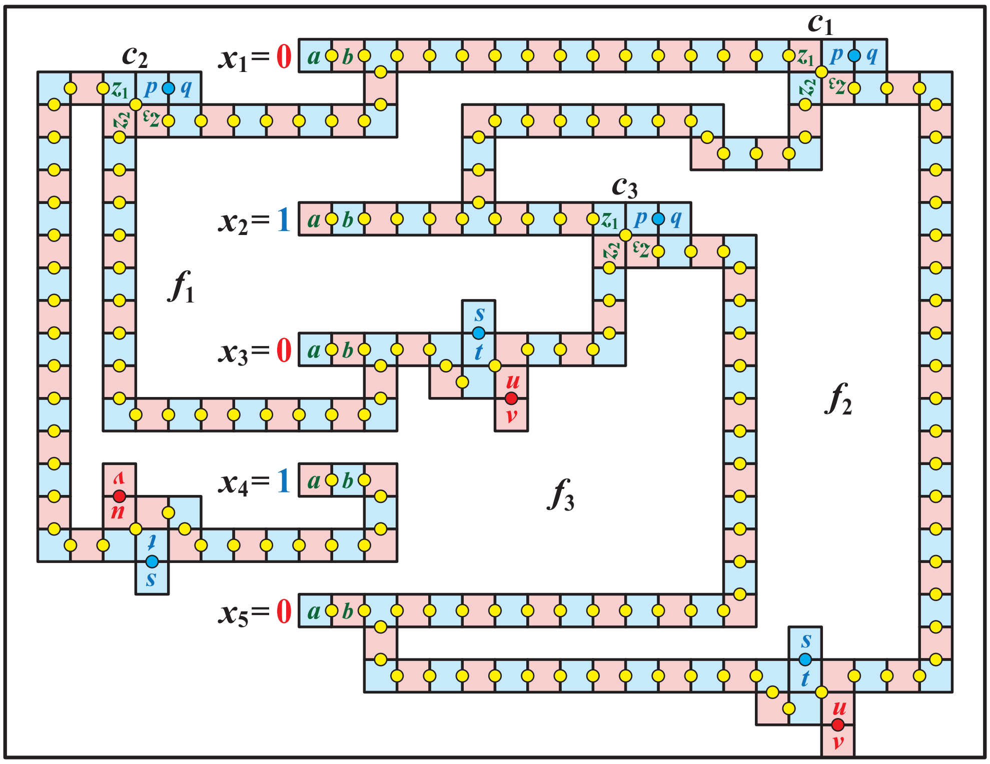

Fig. 10 Stained Glass puzzle S transformed from positive planar 1-in-3-SAT $C = \{c_1, c_2, c_3\}$, where $c_1 = \{x_1, x_2, x_5\}$, $c_2 = \{x_1, x_3, x_4\}$, and $c_3 = \{x_2, x_3, x_5\}$. From the solution of puzzle S, one can see that the assignment $(x_1, x_2, x_3, x_4, x_5) = (0, 1, 0, 1, 0)$ satisfies all clauses of C. (Each of the three large pieces f_1, f_2, and f_3 can be colored either blue or red with no restriction.)

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