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PAPER *Special Section on Foundations of Computer Science*

Strategies and Equilibria on Indistinguishability of Winning Objectives and Related Decision Problems*

Rindo NAKANISHI[†a)], ***Nonmember***, **Yoshiaki TAKATA**[††b)], ***Member***, ***and*** **Hiroyuki SEKI**[†c)], ***Fellow***

SUMMARY Game theory on graphs is a basic tool in computer science. In this paper, we propose a new game-theoretic framework for studying the privacy protection of a user who interactively uses a software service. Our framework is based on the idea that an objective of a user using software services should not be known to an adversary because the objective is often closely related to personal information of the user. We propose two new notions, $\mathcal{O}$-indistinguishable strategy ($\mathcal{O}$-IS) and objective-indistinguishability equilibrium (OIE). For a given game and a subset $\mathcal{O}$ of winning objectives (or objectives in short), a strategy of a player is $\mathcal{O}$-indistinguishable if an adversary cannot shrink $\mathcal{O}$ by excluding any objective from $\mathcal{O}$ as an impossible objective. A strategy profile, which is a tuple of strategies of all players, is an OIE if the profile is locally optimal in the sense that no player can expand her set of objectives indistinguishable from her real objective from the viewpoint of an adversary. We analyze the complexities of deciding the existence of $\mathcal{O}$-IS and prove the decidability of the existence of OIE under a weaker assumption on rationality.
key words: *graph game, Muller objective,* $\mathcal{O}$*-indistinguishable strategy, objective-indistinguishability equilibrium*

1. Introduction

Indistinguishability is a basic concept in security and privacy, meaning that anyone who does not have the access right to secret information cannot distinguish a target secret data from other data. For example, a cryptographic protocol may be considered secure if the answer from an adversary who tries to attack the protocol is indistinguishable from a random sequence [1]. A database is k-anonymous if we cannot distinguish a target record from at least $k-1$ records whose public attribute values are the same as those of the target record [2].

In this paper, we apply indistinguishability to defining and solving problems on privacy of a user who interacts with other users and/or software tools. The basic idea of this study is that we consider an objective of a user should not be public because the objective is often closely related to personal information of the user. For example, users of e-commerce websites may select products to purchase depending on their preference, income, health condition, etc., which are related to private information of the users; hence, they may not want the target products to become public unnecessarily. In this study, we try to formalize the above idea based on the indistinguishability of objectives from the viewpoint of an adversary who may observe the user's behavior.

We adopt a multiplayer non-zero-sum game played on a game arena, which is a finite directed graph with the initial vertex [3], [4], as our basic framework. A game has been used as the framework of reactive synthesis problem [5], [6]. It is natural to require that a reactive system acting as an agent of a human user should not unnecessarily make the user's privacy public, and this paper proposes a framework for capturing the suitability of the behavior of a player (which may be a synthesized reactive system) when we consider the user's objective is private information.

One of the main concerns in game theory is to decide whether there is a winning strategy for a given player p and if so, to construct a winning strategy for p. A strategy of player p is a function that selects a next move of p based on the current position or the history of a play. A strategy σ of a player p is called a winning strategy if the player p always wins by using σ, i.e., any play consistent with the strategy σ satisfies her winning objective regardless of the other players' strategies. From the viewpoint of reactive synthesis, a winning strategy for a designated player represents the behavior of the synthesized system that satisfies its objective regardless of the behaviors of the other environmental entities. Note that there may be more than one winning strategies for a player; she can choose any one among such winning strategies. In the literatures, a winning objective is *a priori* given as a component of a game. In this study, we regard that a winning objective of a player is her private information and hence she wants to choose a winning strategy that maximizes the indistinguishability of her winning objective from the viewpoint of an adversary who may observe the play and recognize which players win the game. For a subset $\mathcal{O}$ of winning objectives which a player p wants to be indistinguishable from one another, we say that a strategy of p is $\mathcal{O}$-indistinguishable if an adversary cannot make $\mathcal{O}$ smaller as the candidate set of winning objectives. The paper discusses the decidability and complexity of some problems related to $\mathcal{O}$-indistinguishability.

Another important problem in game theory is to find a good combination of strategies of all players, which provides a locally optimal play. A well-known criterion is Nash equilibrium. A combination of strategies (called a strategy profile) is a Nash equilibrium if any player losing the game in

[†]Graduate School of Informatics, Nagoya University, Furo-cho, Chikusa, Nagoya 464-8601, Japan
[††]School of Informatics, Kochi University of Technology, Tosayamada, Kami City, Kochi 782-8502, Japan
*This work was supported by JSPS KAKENHI Grant Number 22H03568.
a) E-mail: rindo@sqlab.jp
b) E-mail: takata.yoshiaki@kochi-tech.ac.jp
c) E-mail: seki@i.nagoya-u.ac.jp

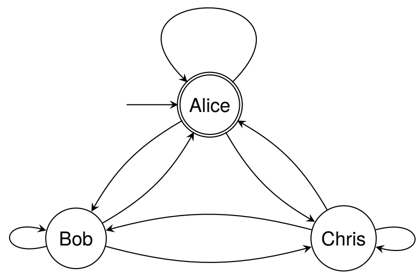

Fig. 1 1-player game arena with a Büchi objective

that strategy profile cannot make herself a winner by changing her strategy alone. This paper introduces objective-indistinguishability equilibrium (OIE) as a criterion of local optimality of a strategy profile; a strategy profile is OIE if and only if no player can extend the indistinguishable set of winning objectives by changing her strategy alone. An OIE shares good properties with other kind of equilibria. If a combination of strategies is an OIE, then every player can protect her privacy in a locally optimal way in the sense that any player cannot extend the candidate set of winning objectives (as her privacy) by unilaterally changing her strategy. The paper also provides the decidability results on OIE.

Example 1.1: Figure 1 shows a 1-player game. The player is a spy. Alice is her buddy. The player wants to communicate with Alice many times and she does not want an adversary to find out that Alice is her buddy. In this game, the objective of the player is to visit the accepting vertex Alice infinitely often. Visiting a vertex corresponds to communicating with the person written on that vertex.

We assume that an adversary knows the game arena, the play and whether the player wins. We also assume that an adversary knows the objective of the player is a Büchi objective†. We examine the following three strategies of the player, all of which result in the player's winning.

1. Always choose Alice as the next vertex, i.e., the play will be Alice Alice Alice $\cdots$. In this case, the player wins because she visits Alice infinitely often. An adversary knows that at least Alice is an accepting vertex because the player won and she visited only Alice infinitely often.
2. Choose Bob as the next vertex when the player is in Alice, and Alice when the player is in Bob, i.e., the play will be Alice Bob Alice Bob $\cdots$. In this case, the player wins and an adversary knows that at least one of Alice and Bob is an accepting vertex. Compared to the case 1, the vertex Bob is added to the candidate set of accepting vertices.
3. Choose Bob as the next vertex when the player is in Alice, Chris when the player is in Bob, and Alice when the player is in Chris, i.e., the play will be Alice Bob Chris Alice $\cdots$. In this case, the player wins and an adversary knows that at least one of Alice, Bob and Chris is an accepting vertex. Compared to the case 2, the vertex Chris is added to the candidate set of accepting vertices.

†A Büchi objective defined in Definition 2.2 in Section 2 is specified by a set of accepting vertices. If the play visits an accepting vertex infinitely often, then the player wins. Otherwise, the player loses.

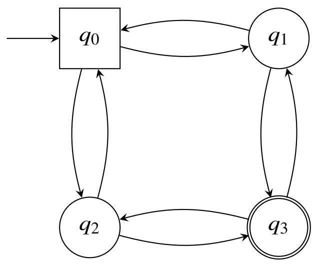

Fig. 2 1-player game arena with a Streett objective

Example 1.2: Figure 2 shows a 1-player game. The player is a web server. If the player receives a request from a user infinitely often, the player should send HTML files to the user infinitely often. Visiting the vertex q_0, shown as a rectangle, corresponds to receiving a request from a user, and visiting the vertex q_3, shown as a double circle, corresponds to sending HTML files to the user. Hence, the server satisfies her objective if (and only if) the play visits q_3 infinitely often whenever it visits q_0 infinitely often, which is a typical liveness property. An adversary guesses the objective of the player. As the same as Example 1.1, assume that an adversary knows the game arena, the play, whether the player wins, and the fact that the objective of the player is a Streett objective††. For example, when the player visits all vertices infinitely often, an adversary only knows that at least one of q_0, q_1, q_2 and q_3 is a send vertex. When the player visits only q_1 and q_3 infinitely often, an adversary knows that at least one of q_1 and q_3 is a send vertex or at least one of q_0 and q_2 is a request vertex.

Related work

There is a generalization of games where each player can only know partial information on the game, which is called an imperfect information game [7]–[11]. While the indistinguishability proposed in this paper shares such restricted observation with imperfect information games, the large difference is that we consider an adversary who is not a player but an individual who observes partial information on the game while players themselves may obtain only partial information in imperfect information games.

Among a variety of privacy notions, k-anonymity is well-known. A database D is *k-anonymous* [2], [12] if for any record r in D, there are at least $k-1$ records different from r such that the values of quasi-identifiers of r and

††A Streett objective defined in Definition 2.2 in Section 2 is specified by a set of pairs (F_k, G_k) of vertices F_k and G_k $(1 \leq k \leq n)$. If the play visits a vertex in G_k infinitely often or the play does not visit any vertices of F_k infinitely often for all pairs (F_k, G_k) $(1 \leq k \leq n)$, then the player wins. Otherwise, the player loses.

these records are the same. Here, a set of quasi-identifiers is a subset of attributes that can 'almost' identify the record such as {zip-code, birthday, income}. Hence, if D is k-anonymous, an adversary knowing the quasi-identifiers of some user u cannot identify the record of u in D among the k records with the same values of the quasi-identifiers. Methods for transforming a database to the one satisfying k-anonymity have been investigated [13], [14]. Refined notions have been proposed by considering the statistical distribution of the attribute values [15], [16].

However, these notions suffer from so called non-structured zero and mosaic effect. Actually, it is known that there is no way of protecting perfect privacy from an adversary who can use an arbitrary external information except the target privacy itself. The notion of ε-differential privacy where $\varepsilon > 0$ was proposed to overcome the weakness of the classical notions of privacy. A query Q to a database D is *ε-differentially private* (abbreviated as ε-DP) [17], [18] if for any person u, the probability that we can infer whether the information on u is contained in D or not by observing the result of $Q(D)$ is negligible (very small) in terms of ε. (Also see [19], [20].) As the privacy protection of individual information used in data mining and machine learning is becoming a serious social problem [21], methods of data publishing that guarantees ε-DP have been extensively studied [21]–[25].

Quantitative information flow (abbreviated as QIF) [26], [27] is another way of formalizing privacy protection or information leakage. QIF of a program P is the mutual information of the secret input X and the public output Y of the program P in the sense of Shannon theory where the channel between X and Y is a program which has logical semantics. Hence, QIF analysis uses not only the calculation of probabilities but also program analysis [28].

We have mentioned a few well-known approaches to formally modeling privacy protection in software systems; however, these privacy notions, even QIF that is based on the logical semantics of a program, share the assumption that private information is a static value or a distribution of values. In contrast, our approach assumes that privacy is a purpose of a user's behavior. The protection of this kind of privacy has not been studied to the best of our knowledge. In [29], the following synthesis problem of privacy preserving systems is discussed: For given multivalued linear-time temporal logic (LTL) formulas representing secrets as well as an LTL formula representing a specification, decide whether there is a reactive program that satisfies the specification while keeping the values of the formulas representing secrets unknown. The paper [29] treats the secrets as values as in the previous studies, and the approach is very different from ours.

This paper is an extended version of [30]. There are two major differences from [30]. First, we improved the definitions of the functions $\mathrm{Obj}^{p,O_p}_{\Omega,knw}$ defined in Section 3, which are the key parts in our framework. Second, under the above new definitions, we analyzed the complexities of the existences of $\mathcal{O}$-IS defined in Section 3 for which only decidability was proved in [30]. The complexities are summarized in Table 1 where row headers are classes of objectives defined in Section 2 and column headers are types of information that an adversary can use defined in Section 3. $\mathrm{Obj}^{p,O_p}_{\Omega,knw}$ intuitively represents the set of candidate objectives of player p an adversary can guess. Both in [30] and this paper, we regard that an adversary assumes the players behave rationally and use this assumption to guess the players' objectives. It is assumed in [30] that when a player is a loser, there is no winning strategy for the player. However, when a player is a winner, we asymmetrically made no assumption on the player's rationality in [30]. Hence, in [30], an adversary assumed a player behaved rationally when she lost while the adversary did not assume so when the player won, which is not a balanced definition.

Table 1 The complexities of Problem 4.1

	pw, gw or pgw	pg
Büchi or co-Büchi	coNP (b2) P-hard (a1)	P-complete (a1) (b1)
Streett	PSPACE (b3) coNP-hard (a2)	coNP-complete (a2) (b2)
Rabin	PSPACE-complete (a3) (b3)	PSPACE-complete (a3) (b3)
Muller	EXPTIME (b4) P-hard (a1)	P-complete (a1) (b1)

2. Preliminaries

Definition 2.1: A game arena is a tuple $\mathcal{G} = (P, V, (V_p)_{p\in P}, v_0, E)$, where P is a finite set of players, V is a finite set of vertices, $(V_p)_{p\in P}$ is a partition of V, namely, $V_i \cap V_j = \emptyset$ for all $i \neq j$ $(i, j \in P)$ and $\bigcup_{p\in P} V_p = V$, $v_0 \in V$ is the initial vertex, and $E \subseteq V \times V$ is a set of edges.

As defined later, a vertex in V_p is controlled by a player p, i.e., when a play is at a vertex in V_p, the next vertex is selected by player p. This type of games is called *turn-based*. There are other types of games, concurrent games [7] and stochastic game [31]–[33]. In this paper, we consider only deterministic turn-based games.

Play and history

An infinite sequence of vertices $v_0v_1v_2\cdots$ $(v_i \in V, i \geq 0)$ starting from the initial vertex v_0 is a *play* if $(v_i, v_{i+1}) \in E$ for all $i \geq 0$. A *history* is a non-empty (finite) prefix of a play. The set of all plays is denoted by *Play* and the set of all histories is denoted by *Hist*. We often write a history as hv where $h \in Hist \cup \{\varepsilon\}$ and $v \in V$. For a player $p \in P$, let $Hist_p = \{hv \in Hist \mid v \in V_p\}$. That is, $Hist_p$ is the set of histories ending with a vertex controlled by player p. For a play $\rho = v_0v_1v_2\cdots \in Play$, we define $Inf(\rho) = \{v \in V \mid \forall i \geq 0.\ \exists j \geq i.\ v_j = v\}$.

Strategy

For a player $p \in P$, a *strategy* of p is a function $\sigma_p : Hist_p \to V$ such that $(v, \sigma_p(hv)) \in E$ for all $hv \in Hist_p$. At a vertex $v \in V_p$, player p chooses $\sigma_p(hv)$ as the next vertex according to her strategy σ_p. Note that because the domain of σ_p is $Hist_p$, the next vertex may depend on the whole history in general. Let $\Sigma^p_{\mathcal{G}}$ denote the set of all strategies of p. A *strategy profile* is a tuple $\boldsymbol{\sigma} = (\sigma_p)_{p\in P}$ of strategies of all players, namely $\sigma_p \in \Sigma^p_{\mathcal{G}}$ for all $p \in P$. Let $\Sigma_{\mathcal{G}}$ denote the set of all strategy profiles. For a strategy profile $\boldsymbol{\sigma} \in \Sigma_{\mathcal{G}}$ and a strategy $\sigma'_p \in \Sigma^p_{\mathcal{G}}$ of a player $p \in P$, let $\boldsymbol{\sigma}[p \mapsto \sigma'_p]$ denote the strategy profile obtained from $\boldsymbol{\sigma}$ by replacing the strategy of p in $\boldsymbol{\sigma}$ with σ'_p. We define the function $\mathrm{out}_{\mathcal{G}} : \Sigma_{\mathcal{G}} \to Play$ as $\mathrm{out}_{\mathcal{G}}((\sigma_p)_{p\in P}) = v_0v_1v_2\cdots$ where $v_{i+1} = \sigma_p(v_0\cdots v_i)$ for all $i \geq 0$ and for $p \in P$ with $v_i \in V_p$. We call the play $\mathrm{out}_{\mathcal{G}}(\boldsymbol{\sigma})$ the *outcome* of $\boldsymbol{\sigma}$. We also define the function $\mathrm{out}^p_{\mathcal{G}} : \Sigma^p_{\mathcal{G}} \to 2^{Play}$ for each $p \in P$ as $\mathrm{out}^p_{\mathcal{G}}(\sigma_p) = \{v_0v_1v_2\cdots \in Play \mid v_i \in V_p \Rightarrow v_{i+1} = \sigma_p(v_0\cdots v_i)$ for all $i \geq 0\}$. A play $\rho \in \mathrm{out}^p_{\mathcal{G}}(\sigma_p)$ is called a play consistent with the strategy σ_p of player p. By definition, for a strategy profile $\boldsymbol{\sigma} = (\sigma_p)_{p\in P} \in \Sigma_{\mathcal{G}}$, it holds that $\bigcap_{p\in P} \mathrm{out}^p_{\mathcal{G}}(\sigma_p) = \{\mathrm{out}_{\mathcal{G}}(\boldsymbol{\sigma})\}$.

Objective

In this paper, we assume that the result that a player obtains from a play is either a winning or a losing. Each player has her own winning condition over plays, and we represent a winning condition by a subset $O \subseteq Play$ of plays; i.e., the player wins if the play belongs to the subset O. We call the subset O the *objective* of that player. In this paper, we focus on the following important classes of objectives.

Definition 2.2: Let $U \subseteq V$ be a subset of vertices, $(F_k, G_k)_{1\leq k\leq l}$ be pairs of sets $F_k, G_k \subseteq V$ and $\mathcal{F} \subseteq 2^V$ be a subset of subsets of vertices. We will use U, $(F_k, G_k)_{1\leq k\leq l}$ and $\mathcal{F}$ as finite representations for specifying an objective as follows.

- Büchi objective:
  $\text{Büchi}(U) = \{\rho \in Play \mid Inf(\rho) \cap U \neq \emptyset\}$.
- co-Büchi objective:
  $\text{co-Büchi}(U) = \{\rho \in Play \mid Inf(\rho) \cap U = \emptyset\}$.
- Rabin objective:
  $\text{Rabin}\big((F_k, G_k)_{1\leq k\leq l}\big) = \{\rho \in Play \mid 1 \leq \exists k \leq l.\ Inf(\rho) \cap F_k = \emptyset \wedge Inf(\rho) \cap G_k \neq \emptyset\}$.
- Streett objective:
  $\text{Streett}\big((F_k, G_k)_{1\leq k\leq l}\big) = \{\rho \in Play \mid 1 \leq \forall k \leq l.\ Inf(\rho) \cap F_k \neq \emptyset \vee Inf(\rho) \cap G_k = \emptyset\}$.
- Muller objective:
  $\text{Muller}(\mathcal{F}) = \{\rho \in Play \mid Inf(\rho) \in \mathcal{F}\}$.

Note that each objective defined in Definition 2.2 is also a Muller objective: For example, for any $U \subseteq V$, $\text{Büchi}(U) = \text{Muller}(\{I \subseteq V \mid I \cap U \neq \emptyset\})$, and for any $(F_k, G_k)_{1\leq k\leq l}$ where $F_k, G_k \subseteq V$, $\text{Rabin}\,((F_k, G_k)_{1\leq k\leq l}) = \text{Muller}(\mathcal{F})$ where $\mathcal{F} = \bigcup_{1\leq k\leq l}\{I \subseteq V \mid I \cap F_k = \emptyset \wedge I \cap G_k \neq \emptyset\}$. We define the description length of a Muller objective $\text{Muller}(\mathcal{F})$ for $\mathcal{F} \subseteq 2^V$ is $|V| \cdot |\mathcal{F}|$, because each element of $\mathcal{F}$, which is a subset of V, can be represented by a bit vector of length $|V|$.† By $\Omega \subseteq 2^{Play}$, we refer to a certain class of objectives. For example, $\Omega = \{\text{Büchi}(U) \mid U \subseteq V\} \subseteq 2^{Play}$ is the class of Büchi objectives.

An *objective profile* is a tuple $\boldsymbol{\alpha} = (O_p)_{p\in P}$ of objectives of all players, namely $O_p \subseteq Play$ for all $p \in P$. A pair $(\mathcal{G}, \boldsymbol{\alpha})$ of a game arena and an objective profile is called a *game*. For a strategy profile $\boldsymbol{\sigma} \in \Sigma_{\mathcal{G}}$ and an objective profile $\boldsymbol{\alpha} = (O_p)_{p\in P}$, we define the set $\mathrm{Win}_{\mathcal{G}}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) \subseteq P$ of winners as $\mathrm{Win}_{\mathcal{G}}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) = \{p \in P \mid \mathrm{out}_{\mathcal{G}}(\boldsymbol{\sigma}) \in O_p\}$. That is, a player p is a winner if and only if $\mathrm{out}_{\mathcal{G}}(\boldsymbol{\sigma})$ belongs to the objective O_p of p. If $p \in \mathrm{Win}_{\mathcal{G}}(\boldsymbol{\sigma}, \boldsymbol{\alpha})$, we also say that p wins the game $(\mathcal{G}, \boldsymbol{\alpha})$ (by the strategy profile $\boldsymbol{\sigma}$). Note that it is possible that there is no player who wins the game or all the players win the game. In this sense, a game is *non-zero-sum*. If an objective profile $\boldsymbol{\alpha} = (O_p)_{p\in P}$ is a partition of *Play*, i.e., $O_i \cap O_j = \emptyset$ for all $i \neq j$ $(i, j \in P)$ and $\bigcup_{p\in P} O_p = Play$, then the game is called *zero-sum*. When a game is zero-sum, there is one and only one winner and the other players are all losers. We abbreviate $\Sigma^p_{\mathcal{G}}, \Sigma_{\mathcal{G}}, \mathrm{out}^p_{\mathcal{G}}, \mathrm{out}_{\mathcal{G}}$ and $\mathrm{Win}_{\mathcal{G}}$ as $\Sigma^p, \Sigma, \mathrm{out}^p$, out and Win, respectively, if $\mathcal{G}$ is clear from the context.

Winning strategy

For a game arena $\mathcal{G}$, a player $p \in P$ and an objective $O_p \subseteq Play$, a strategy $\sigma_p \in \Sigma^p$ of p such that $\mathrm{out}^p(\sigma_p) \subseteq O_p$ is called a *winning strategy* of p for $\mathcal{G}$ and O_p because if p takes σ_p as her strategy then she wins against any combination of strategies of the other players. (Recall that $\mathrm{out}^p(\sigma_p) \subseteq Play$ is the set of all plays consistent with σ_p.) Hence, a strategy for p such that her winning or losing depends on which strategies other players choose is not a winning strategy for p. For a game arena $\mathcal{G}$ and a player $p \in P$, we define the set $\mathrm{Winnable}^p_{\mathcal{G}}$ of objectives permitting p a winning strategy as $\mathrm{Winnable}^p_{\mathcal{G}} = \{O \mid \exists\sigma_p \in \Sigma^p_{\mathcal{G}}.\ \mathrm{out}^p_{\mathcal{G}}(\sigma_p) \subseteq O\}$. For a player p, $O \in \mathrm{Winnable}^p_{\mathcal{G}}$ means that p has a winning strategy for $\mathcal{G}$ and O. We have the following problem and theorem for a winning strategy.

Problem 2.1: Let $\mathcal{G} = (P, V, (V_p)_{p\in P}, v_0, E)$ be a game arena, $p \in P$ be a player and $O_p \subseteq Play$ be an objective of p. Decide whether there exists a winning strategy of p for O_p.

Theorem 2.1 ([3, Theorems 21 and 25]): If a given game is a 2-player zero-sum game, i.e., $|P| = 2$ and a given objective profile is a partition of *Play*, then Problem 2.1 is

(1) P-complete when O_p is a Büchi, co-Büchi or Muller objective or an intersection of Büchi objectives,
(2) NP-complete when O_p is a Rabin objective,
(3) co-NP complete when O_p is a Streett objective and
(4) PSPACE-complete when O_p is an intersection of Rabin objectives or is a Boolean combination of Büchi objectives.

Theorem 2.1 (1) implies that the complexity of Problem 2.1 is smaller when O_p is a Muller objective than O_p is a Rabin or Streett objective, despite the fact that both of the

†Translating a representation of a Büchi objective into that of a Muller objective may cause an exponential blowup in the description length.

class of Rabin objectives and that of Streett objectives are subclasses of Muller objectives. This is because a Muller objective is given in an explicit way, i.e., a Muller objective is a subset of subsets of vertices that should be visited infinitely often.

We can apply Theorem 2.1 to multiplayer non-zero-sum games by regarding them as 2-player zero-sum games as follows: for a player $p \in P$ in a multiplayer non-zero-sum game, we let the other player $-p$ be the coalition of the players $q \in P \setminus \{p\}$ whose objective is the complement of the objective of p. We have the following theorem.

Theorem 2.2: Problem 2.1 is

(1) P-complete when O_p is a Büchi, co-Büchi or Muller objective or an intersection of Büchi objectives,
(2) NP-complete when O_p is a Rabin objective,
(3) co-NP complete when O_p is a Streett objective and
(4) PSPACE-complete when O_p is an intersection of Rabin objectives or is a Boolean combination of Büchi objectives.

Nash equilibrium

For non-zero-sum multiplayer games, besides a winning strategy of each player, we often use Nash equilibrium, defined below, as a criterion for a strategy profile (a tuple of strategies of all players) to be locally optimal. Let $\boldsymbol{\sigma} \in \Sigma$ be a strategy profile and $\boldsymbol{\alpha} = (O_p)_{p \in P}$ be an objective profile. A strategy profile $\boldsymbol{\sigma}$ is called a *Nash equilibrium* (NE) for $\boldsymbol{\alpha}$ if it holds that $\forall p \in P.\ \forall \sigma_p \in \Sigma^p.\ p \in \mathrm{Win}(\boldsymbol{\sigma}[p \mapsto \sigma_p], \boldsymbol{\alpha}) \Rightarrow p \in \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha})$. Intuitively, $\boldsymbol{\sigma}$ is an NE if any player p cannot improve the result (from losing to winning) by changing her strategy alone. For a strategy profile $\boldsymbol{\sigma} \in \Sigma$, we call a strategy $\sigma_p \in \Sigma^p$ such that $p \notin \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) \wedge p \in \mathrm{Win}(\boldsymbol{\sigma}[p \mapsto \sigma_p], \boldsymbol{\alpha})$ a *profitable deviation* of p from $\boldsymbol{\sigma}$. Hence, $\boldsymbol{\sigma}$ is an NE if and only if no player has a profitable deviation from $\boldsymbol{\sigma}$. Because $p \in \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha})$ is equivalent to $\mathrm{out}(\boldsymbol{\sigma}) \in O_p$, a strategy profile $\boldsymbol{\sigma} \in \Sigma$ is an NE for $\boldsymbol{\alpha}$ if and only if $\forall p \in P.\ \forall \sigma_p \in \Sigma^p.\ \mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \in O_p \Rightarrow \mathrm{out}(\boldsymbol{\sigma}) \in O_p$. We write this condition as $\mathrm{Nash}(\boldsymbol{\sigma}, \boldsymbol{\alpha})$.

Below we define an extension of NE as a single strategy profile simultaneously satisfying the condition of NE for more than one objective profiles. We can prove that the existence of this extended NE is decidable (Theorem 2.3), and later we will reduce some problems to the existence checking of this type of NE.

Definition 2.3: For a game arena $\mathcal{G} = (P, V, (V_p)_{p \in P}, v_0, E)$ and objective profiles $\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_n$, a strategy profile $\boldsymbol{\sigma} \in \Sigma$ is called an $(\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_n)$*-Nash equilibrium* if $\mathrm{Nash}(\boldsymbol{\sigma}, \boldsymbol{\alpha}_j)$ holds for all $1 \le j \le n$.

Theorem 2.3: Let $\mathcal{G} = (P, V, (V_p)_{p \in P}, v_0, E)$ be a game arena and $\boldsymbol{\alpha}_j = (O_p^j)_{p \in P}$ $(1 \le j \le n)$ be objective profiles over Muller objectives. Whether there exists an $(\boldsymbol{\alpha}_1, \dots, \boldsymbol{\alpha}_n)$-NE is decidable.

A proof of this theorem is given in the appendix.

3. Indistinguishable Strategy and Related Equilibrium

In this section, we propose two new notions concerning on the privacy of a player: indistinguishable strategy and objective-indistinguishability equilibrium. We first define the set of possible objectives of a player in the viewpoint of an adversary that can observe restricted information on a game, a play and its result (i.e., which players win).

We assume that an adversary guesses objectives of players from the three types of information: a play (p), a game arena (g) and a set of winners (w) of the play. We use a word $knw \in \{\mathsf{pw}, \mathsf{gw}, \mathsf{pg}, \mathsf{pgw}\}$ to represent a type of information that an adversary can use. For example, an adversary guesses objectives from a play and winners when $knw = \mathsf{pw}$. In either case, we implicitly assume that an adversary knows the set V of vertices of the game arena and the class Ω of objectives of players. We do not consider the cases where knw is a singleton because in such cases the information an adversary can have is too limited to make useful inference about the objectives: An adversary cannot guess anything about objectives when $knw = \mathsf{g}$ or $knw = \mathsf{p}$. When $knw = \mathsf{w}$, he only knows that the objective of a winner is not empty and that of a loser is not the universal set.

Let $p \in P$ be a player and $O_p \subseteq Play$ be an objective of p. We define the function $\mathrm{Obj}^{p,O_p}_{\Omega,knw} : \Sigma \to 2^{\Omega}$ as follows, which maps a strategy profile $\boldsymbol{\sigma} \in \Sigma$ to the set of objectives of p that an adversary guesses. Note that $p \in \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha})$ is equivalent to $\mathrm{out}(\boldsymbol{\sigma}) \in O_p$ and hence we let $\mathrm{Obj}^{p,O_p}_{\Omega,knw}$ have a parameter O_p instead of $\boldsymbol{\alpha}$. (Two types of underlines in the following equations show the parts that correspond to player p's rationality, explained later.)

$$\begin{aligned}\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{pw}}(\boldsymbol{\sigma}) = \{O \subseteq V^{\omega} \mid\ & (\mathrm{out}(\boldsymbol{\sigma}) \in O \wedge p \in \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha})) \vee \\ & (\mathrm{out}(\boldsymbol{\sigma}) \notin O \wedge p \notin \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha}))\},\end{aligned}$$

$$\begin{aligned}\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{gw}}(\boldsymbol{\sigma}) = \{O \in \Omega \mid\ & (p \in \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) \wedge O \neq \emptyset \wedge \\ & \underline{(O \in \mathrm{Winnable}^p \Rightarrow \mathrm{out}^p(\sigma_p) \subseteq O)}) \vee \\ & (p \notin \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) \wedge \underline{O \notin \mathrm{Winnable}^p})\},\end{aligned}$$

$$\begin{aligned}\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{pg}}(\boldsymbol{\sigma}) = \{O \in \Omega \mid\ & (\mathrm{out}(\boldsymbol{\sigma}) \in O \wedge \\ & \underline{(O \in \mathrm{Winnable}^p \Rightarrow \mathrm{out}^p(\sigma_p) \subseteq O)}) \vee \\ & (\mathrm{out}(\boldsymbol{\sigma}) \notin O \wedge \underline{O \notin \mathrm{Winnable}^p})\},\end{aligned}$$

$$\begin{aligned}\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{pgw}}(\boldsymbol{\sigma}) = \{O \in \Omega \mid\ & (\mathrm{out}(\boldsymbol{\sigma}) \in O \wedge p \in \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) \wedge \\ & \underline{(O \in \mathrm{Winnable}^p \Rightarrow \mathrm{out}^p(\sigma_p) \subseteq O)}) \vee \\ & (\mathrm{out}(\boldsymbol{\sigma}) \notin O \wedge p \notin \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) \wedge \\ & \underline{O \notin \mathrm{Winnable}^p})\}\end{aligned}$$

where $\boldsymbol{\alpha}$ is any objective profile in which the objective of p is O_p and σ_p is the strategy of p in $\boldsymbol{\sigma} = (\sigma_p)_{p \in P}$. (Note that for a given $\boldsymbol{\sigma}$ whether $p \in \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha})$ or not does not depend on objectives of the players other than p and hence we can use an arbitrary $\boldsymbol{\alpha}$ containing O_p.)

The definitions of $\mathrm{Obj}^{p,O_p}_{\Omega,knw}$ are based on the following two ideas. First, if a player p wins in a play ρ, then her objective O_p contains ρ. This is obvious from the definition of the winning condition of the game. Similarly, if p loses in ρ, then O_p does not contain ρ. Second, we assume that if a player has a winning strategy for her objective and the game arena, then she takes a winning strategy. In other words, we assume that each player behaves rationally. The conditions for O due to the assumption that a player behaves rationally when she is a winner (resp. loser) are underlined with solid lines (resp. wavy lines) in the definition of $\mathrm{Obj}^{p,O_p}_{\Omega,knw}$.

When $knw = \mathsf{pw}$, we assume that an adversary can observe the play and the set of winners but he does not know the game arena. The adversary can infer that the play $\mathrm{out}(\boldsymbol{\sigma})$ he observed belongs to the objective of a player p if the adversary knows that p is a winner, and $\mathrm{out}(\boldsymbol{\sigma})$ does not belong to the objective of p if p is not a winner. Note that the adversary does not know the real objective O_p of player p. For the adversary, any $O \subseteq V^\omega$ satisfying $\mathrm{out}(\boldsymbol{\sigma}) \in O$ is a candidate of the objective of player p when p is a winner. Similarly, any $O \subseteq V^\omega$ satisfying $\mathrm{out}(\boldsymbol{\sigma}) \notin O$ is a candidate objective of p when p is not a winner. An adversary does not know the game arena because $knw = \mathsf{pw}$, that is, he does not know the set of edges in the arena. Therefore, the candidate objective O cannot be narrowed down to a subset of plays (i.e., infinite sequences of vertices along the edges in the game arena), but O can be an arbitrary set of infinite sequences of the vertices consistent with the information obtained by the adversary.

When $knw = \mathsf{gw}$, an adversary cannot observe the play, but he knows the game arena and can observe the set of winners. If p is a winner, the adversary can infer that p has a strategy σ'_p such that $\mathrm{out}^p(\sigma'_p) \cap O_p \neq \emptyset$. Because there exists such a strategy σ'_p for all O_p other than $\emptyset$, he can remove only $\emptyset$ from the set of candidates for p's objective. In addition, if a candidate objective O of p for the game arena has a winning strategy and p's strategy σ_p of $\boldsymbol{\sigma} = (\sigma_p)_{p \in P}$ is not a winning strategy for O, then O must not be a real objective of p and the adversary can exclude O from the set of candidate objectives, because we assume that every player takes a winning strategy for her objective when one exists. In other words, if O has a winning strategy, then O is in $\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{gw}}(\boldsymbol{\sigma})$ only if σ_p is a winning strategy for O. On the other hand, if p is a loser, the adversary can infer that p has no winning strategy for O_p for the same reason. Therefore, when p loses, the adversary can narrow down the set of candidates for p's objective to the set of objectives without a winning strategy.

The definition where $knw = \mathsf{pg}$ can be interpreted in a similar way. Note that we have

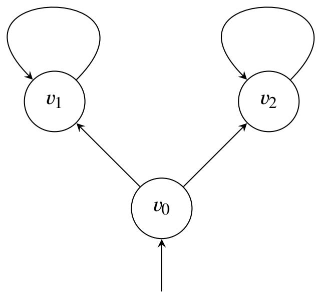

Fig. 3 1-player game arena with Büchi objectives

$$\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{pgw}}(\boldsymbol{\sigma}) = \mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{pw}}(\boldsymbol{\sigma}) \cap \mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{gw}}(\boldsymbol{\sigma}) \cap \mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{pg}}(\boldsymbol{\sigma}).$$

Since $p \in \mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha})$ is equivalent to $\mathrm{out}(\boldsymbol{\sigma}) \in O_p$ as mentioned before, the above definitions can be rephrased as follows:

$$\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{pw}}(\boldsymbol{\sigma}) = \{O \subseteq V^\omega \mid \mathrm{out}(\boldsymbol{\sigma}) \in (O \cap O_p) \cup (\overline{O} \cap \overline{O_p})\},$$

$$\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{gw}}(\boldsymbol{\sigma}) = \{O \in \Omega \mid (O \in \mathrm{Winnable}^p \Rightarrow (\mathrm{out}(\boldsymbol{\sigma}) \in O_p \wedge \mathrm{out}^p(\sigma_p) \subseteq O)) \wedge (O \notin \mathrm{Winnable}^p \Rightarrow (O \neq \emptyset \vee \mathrm{out}(\boldsymbol{\sigma}) \notin O_p))\}$$

$$\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{pg}}(\boldsymbol{\sigma}) = \{O \in \Omega \mid O \in \mathrm{Winnable}^p \Rightarrow (\mathrm{out}(\boldsymbol{\sigma}) \in O \wedge \mathrm{out}^p(\sigma_p) \subseteq O)\},$$

$$\mathrm{Obj}^{p,O_p}_{\Omega,\mathsf{pgw}}(\boldsymbol{\sigma}) = \{O \in \Omega \mid \mathrm{out}(\boldsymbol{\sigma}) \in (O \cap O_p) \cup (\overline{O} \cap \overline{O_p}) \wedge (O \in \mathrm{Winnable}^p \Rightarrow (\mathrm{out}(\boldsymbol{\sigma}) \in O \cap O_p \wedge \mathrm{out}^p(\sigma_p) \subseteq O))\}.$$

The reader may wonder why O_p appears in this (alternative) definition in spite of the assumption that the adversary does not know O_p. The condition $\mathrm{out}(\boldsymbol{\sigma}) \in O_p$ (or $\notin O_p$) only means that the adversary knows whether p is a winner (or a loser) without knowing O_p itself.

Example 3.1: Figure 3 shows a 1-player game arena $\mathcal{G} = (\{1\}, V, (V), v_0, E)$ where $V = \{v_0, v_1, v_2\}$ and $E = \{(v_0, v_1), (v_0, v_2), (v_1, v_1), (v_2, v_2)\}$. We specify a Büchi objective by a set of accepting states, e.g., let $\langle v_1 \rangle$ denote $\text{Büchi}(\{v_1\}) = \{\rho \in V^\omega \mid \mathit{Inf}(\rho) \cap \{v_1\} \neq \emptyset\}$. In this example, we assume the objective of player 1 is $\langle\rangle = \emptyset \subseteq \mathit{Play}$. Therefore, player 1 always loses regardless of her strategy. There are only two strategies σ_1 and σ_2 of player 1. The strategy σ_1 takes the vertex v_1 as the next vertex at the initial vertex v_0 and then keeps looping in v_1. On the other hand, the strategy σ_2 takes v_2 at v_0 and then keeps looping in v_2. Let σ_1 be the strategy player 1 chooses. We have the play $\rho = \mathrm{out}(\sigma_1) = v_0 v_1 v_1 v_1 \cdots$.

We assume that an adversary knows that the objective of player 1 is a Büchi objective. Then, for each type of information $knw \in \{\mathsf{pw}, \mathsf{gw}, \mathsf{pg}, \mathsf{pgw}\}$, $\mathrm{Obj}^{1,\emptyset}_{\text{Büchi},knw}(\sigma_1)$ becomes as follows (we regard the strategy σ_1 of player 1 as

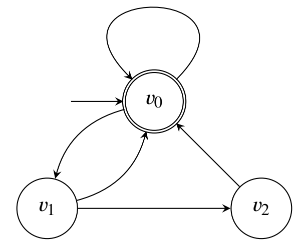

Fig. 4 1-player game arena a with Büchi objective

the strategy profile $\boldsymbol{\sigma} = (\sigma_p)$):

- If knw = pw, then an adversary can deduce that v_1 is not an accepting state because he knows that $Inf(v_0v_1v_1\cdots) = \{v_1\}$ and player 1 loses. Therefore, we have $\mathrm{Obj}^{1,\emptyset}_{\text{Büchi},\mathsf{pw}}(\sigma_1) = \{\langle\rangle, \langle v_0\rangle, \langle v_2\rangle, \langle v_0, v_2\rangle\}$. Note that in this game arena, there is no play passing v_0 infinitely often, and thus $\langle\rangle$ and $\langle v_0\rangle$ (resp. $\langle v_2\rangle$ and $\langle v_0, v_2\rangle$) are equivalent actually. However, because an adversary does not know the game arena when knw = pw, he should consider every infinite sequence over V would be a play and thus $\langle\rangle$ and $\langle v_0\rangle$ are different for him when knw = pw. In the other cases where an adversary knows the game arena, he also knows e.g. $\langle\rangle$ and $\langle v_0\rangle$ are equivalent and thus he would consider $\Omega = \{\langle\rangle, \langle v_1\rangle, \langle v_2\rangle, \langle v_1, v_2\rangle\}$.
- If knw = gw, then an adversary can deduce that neither v_1 nor v_2 is an accepting state because player 1 loses in spite of the fact that there are strategies that pass through v_1 or v_2 infinitely often. Therefore, $\mathrm{Obj}^{1,\emptyset}_{\text{Büchi},\mathsf{gw}}(\sigma_1) = \{\langle\rangle\}$. That is, an adversary can infer the complete information.
- If knw = pg, then an adversary can deduce that $\langle v_2\rangle$ does not belong to $\mathrm{Obj}^{1,\emptyset}_{\text{Büchi},\mathsf{pg}}(\sigma_1)$ because player 1 did not take σ_2 to pass through v_2 infinitely often. That is, if $\langle v_2\rangle$ were the objective of player 1, then it meant she chose losing strategy σ_1 instead of winning strategy σ_2, which is unlikely to happen. Therefore, we have $\mathrm{Obj}^{1,\emptyset}_{\text{Büchi},\mathsf{pg}}(\sigma_1) = \{\langle\rangle, \langle v_1\rangle, \langle v_1, v_2\rangle\}$.
- If knw = pgw, we have

$$\mathrm{Obj}^{1,\emptyset}_{\text{Büchi},\mathsf{pgw}}(\sigma_1) = \bigcap_{knw\in\{\mathsf{pw},\mathsf{gw},\mathsf{pg}\}} \mathrm{Obj}^{1,\emptyset}_{\text{Büchi},knw}(\sigma_1) = \{\langle\rangle\}.$$

$\mathcal{O}$-indistinguishable strategy

Definition 3.1: Let $\mathcal{G} = (P, V, (V_p)_{p\in P}, v_0, E)$ be a game arena, $\sigma_p \in \Sigma^p$ be a strategy of $p \in P$, $\Omega \subseteq 2^{Play}$ be one of the classes of objectives defined in Definition 2.2, $O_p \in \Omega$ be an objective of p and $knw \in \{\mathsf{pw}, \mathsf{gw}, \mathsf{pg}, \mathsf{pgw}\}$ be a type of information that an adversary can use. For any set $\mathcal{O} \subseteq 2^{Play}$ of objectives such that $\mathcal{O} \subseteq \bigcap_{\boldsymbol{\sigma}\in\Sigma} \mathrm{Obj}^{p,O_p}_{\Omega,knw}(\boldsymbol{\sigma}[p \mapsto \sigma_p])$, we call σ_p an *$\mathcal{O}$-indistinguishable strategy* ($\mathcal{O}$-IS) of p (for O_p and knw).

Intuitively, when a player takes an $\mathcal{O}$-IS as her strategy, an adversary cannot narrow down the set of candidates of p's objective from $\mathcal{O}$ by the following reason. By definition, any objective O belonging to $\mathcal{O}$ also belongs to $\mathrm{Obj}^{p,O_p}_{\Omega,knw}(\boldsymbol{\sigma}[p \mapsto \sigma_p])$ for the combination of σ_p and any strategies of the players other than p. This means that such an objective O is possible as the objective of p from the viewpoint of the adversary who can use a type of information specified by knw. If an $\mathcal{O}$-IS $\sigma_p \in \Sigma^p$ is a winning strategy of p, then we call σ_p a *winning $\mathcal{O}$-IS* of p.

Example 3.2: Figure 4 shows a 1-player game arena $\mathcal{G} = (\{1\}, V, (V), v_0, E)$ where $V = \{v_0, v_1, v_2\}$ and $E = \{(v_0, v_0), (v_0, v_1), (v_1, v_0), (v_1, v_2), (v_2, v_0)\}$. We use the same notation of Büchi objectives as Example 3.1, and in this example the objective of player 1 is $\langle v_0\rangle \subseteq Play$. We assume that an adversary knows that the objective of player 1 is a Büchi objective. In this example, we focus on knw = pw. We examine the following three strategies of player 1, all of which result in player 1's winning.

- Let $\sigma_1 \in \Sigma^1$ be a strategy of player 1 such that $\mathrm{out}(\sigma_1) = v_0v_0v_0\cdots$. Since player 1 wins, an adversary can deduce that v_0 must be an accepting state. Therefore, $\mathrm{Obj}^{1,\langle v_0\rangle}_{\text{Büchi},\mathsf{pw}}(\sigma_1) = \{\langle v_0\rangle, \langle v_0, v_1\rangle, \langle v_0, v_2\rangle, \langle v_0, v_1, v_2\rangle\}$. For all $\mathcal{O} \subseteq \mathrm{Obj}^{1,\langle v_0\rangle}_{\text{Büchi},\mathsf{pw}}(\sigma_1)$, σ_1 is an $\mathcal{O}$-IS (for $\langle v_0\rangle$ and knw = pw).
- Let $\sigma_2 \in \Sigma^1$ be a strategy of player 1 such that $\mathrm{out}(\sigma_1) = v_0v_1v_0v_1\cdots$. In a similar way to the above case, an adversary can deduce that v_0 or v_1 (or both) must be an accepting state. Therefore, $\mathrm{Obj}^{1,\langle v_0\rangle}_{\text{Büchi},\mathsf{pw}}(\sigma_2) = \{\langle v_0\rangle$, $\langle v_1\rangle$, $\langle v_0, v_1\rangle$, $\langle v_1, v_2\rangle$, $\langle v_2, v_0\rangle$, $\langle v_0, v_1, v_2\rangle\}$. For all $\mathcal{O} \subseteq \mathrm{Obj}^{1,\langle v_0\rangle}_{\text{Büchi},\mathsf{pw}}(\sigma_2)$, σ_2 is an $\mathcal{O}$-IS.
- Let $\sigma_3 \in \Sigma^1$ be a strategy of player 1 such that $\mathrm{out}(\sigma_3) = v_0v_1v_2v_0v_1v_2\cdots$. In a similar way to the above cases, an adversary can deduce that at least one of v_0, v_1, and v_2 must be an accepting state. Therefore, $\mathrm{Obj}^{1,\langle v_0\rangle}_{\text{Büchi},\mathsf{pw}}(\sigma_3) = \{\langle v_0\rangle$, $\langle v_1\rangle$, $\langle v_2\rangle$, $\langle v_0, v_1\rangle$, $\langle v_1, v_2\rangle$, $\langle v_2, v_0\rangle$, $\langle v_0, v_1, v_2\rangle\}$. For all $\mathcal{O} \subseteq \mathrm{Obj}^{1,\langle v_0\rangle}_{\text{Büchi},\mathsf{pw}}(\sigma_3)$, σ_3 is an $\mathcal{O}$-IS.

In the above example, $\mathrm{Obj}^{1,\langle v_0\rangle}_{\text{Büchi},\mathsf{pw}}(\sigma_1) \subset \mathrm{Obj}^{1,\langle v_0\rangle}_{\text{Büchi},\mathsf{pw}}(\sigma_2) \subset \mathrm{Obj}^{1,\langle v_0\rangle}_{\text{Büchi},\mathsf{pw}}(\sigma_3)$. Hence, the strategy σ_3 is the most favorable one for player 1 with regard to her privacy protection. This observation motivates us to introduce a new concept of equilibrium defined below.

Objective-indistinguishability equilibrium

Definition 3.2: Let $(O_p)_{p\in P}$ be an objective profile and $knw \in \{\mathsf{pw}, \mathsf{gw}, \mathsf{pg}, \mathsf{pgw}\}$ be a type of information that an adversary can use. We call a strategy profile $\boldsymbol{\sigma} \in \Sigma$ such that

$$\forall p \in P.\ \forall \sigma_p \in \Sigma^p.\ \mathrm{Obj}^{p,O_p}_{knw}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \subseteq \mathrm{Obj}^{p,O_p}_{knw}(\boldsymbol{\sigma}) \quad (1)$$

an *objective-indistinguishability equilibrium* (OIE) for knw.

If a strategy profile $\boldsymbol{\sigma}$ is an OIE for knw, no player can expand her $\mathrm{Obj}^{p,O_p}_{knw}(\boldsymbol{\sigma})$ by changing her strategy alone. For

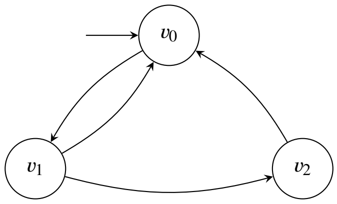

Fig. 5 3-player game arena with Büchi objectives

a strategy profile $\boldsymbol{\sigma} \in \Sigma$, we call a strategy $\sigma_p \in \Sigma^p$ such that $\mathrm{Obj}^{p,O_p}_{\Omega,knw}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \not\subseteq \mathrm{Obj}^{p,O_p}_{\Omega,knw}(\boldsymbol{\sigma})$ a profitable deviation for OIE. In this paper, we think that a set $\mathcal{O}_1 \subseteq 2^{Play}$ of objectives is less indistinguishable than a set $\mathcal{O}_2 \subseteq 2^{Play}$ of objectives when $\mathcal{O}_1 \subset \mathcal{O}_2$, not when $|\mathcal{O}_1| < |\mathcal{O}_2|$ because the latter does not always imply that $\mathcal{O}_1$ is more informative than $\mathcal{O}_2$. If an OIE $\boldsymbol{\sigma}$ is an NE as well, we call $\boldsymbol{\sigma}$ an *objective-indistinguishability Nash equilibrium* (OINE). While an OIE is locally optimal with respect only to indistinguishability, an OINE is locally optimal with respect to both indistinguishability and the result (winning or losing) of the game.

Example 3.3: Figure 5 shows a 3-player game arena $\mathcal{G} = (P, V, (V_p)_{p\in P}, v_0, E)$ where $P = \{0, 1, 2\}$, $V = \{v_0, v_1, v_2\}$, $V_p = \{v_p\}$ $(p \in P)$ and $E = \{\ (v_0, v_1),\ (v_1, v_0),\ (v_1, v_2),\ (v_2, v_0)\ \}$. The objective of player $p \in P$ is $\langle v_p \rangle$, and hence the objective profile is $\boldsymbol{\alpha} = (\langle v_0 \rangle, \langle v_1 \rangle, \langle v_2 \rangle)$. Players 0 and 2 have only one strategy $\sigma_0 \in \Sigma^0$ and $\sigma_2 \in \Sigma^2$, respectively, where $\sigma_0(h_0) = v_1$ and $\sigma_2(h_2) = v_0$ for every $h_0 \in Hist_0 \cup \{\varepsilon\}$ and $h_2 \in Hist_2 \cup \{\varepsilon\}$. Let $\sigma_1, \sigma_1' \in \Sigma^1$ be the strategies of player 1 defined as $\sigma_1(h_1) = v_0$ and $\sigma_1'(h_1) = v_2$ for every $h_1 \in Hist_1 \cup \{\varepsilon\}$. Let $\boldsymbol{\sigma} = (\sigma_0, \sigma_1, \sigma_2)$ and $\boldsymbol{\sigma}' = (\sigma_0, \sigma_1', \sigma_2)$. It holds that $\mathrm{out}(\boldsymbol{\sigma}) = v_0 v_1 v_0 v_1 \cdots$, $\mathrm{Win}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) = \{0, 1\}$, $\mathrm{out}(\boldsymbol{\sigma}') = v_0 v_1 v_2 v_0 v_1 v_2 \cdots$ and $\mathrm{Win}(\boldsymbol{\sigma}', \boldsymbol{\alpha}) = \{0, 1, 2\}$. We have

$$\mathrm{Obj}^{1,\langle v_1\rangle}_{\text{Büchi},knw}(\boldsymbol{\sigma}) = \begin{cases} \Omega \setminus \{\emptyset, \langle v_2 \rangle\} & knw = \mathsf{pw}, \mathsf{gw} \text{ and } \mathsf{pgw}, \\ \Omega \setminus \{\langle v_2 \rangle\} & knw = \mathsf{pg}, \end{cases}$$

$$\mathrm{Obj}^{1,\langle v_1\rangle}_{\text{Büchi},knw}(\boldsymbol{\sigma}') = \begin{cases} \Omega \setminus \{\emptyset\} & knw = \mathsf{pw}, \mathsf{gw} \text{ and } \mathsf{pgw}, \text{ and} \\ \Omega & knw = \mathsf{pg}. \end{cases}$$

For $knw = \mathsf{pw}, \mathsf{gw}, \mathsf{pg}$ and pgw, $\boldsymbol{\sigma}$ is not an OIE because there exists a profitable deviation $\sigma_1' \in \Sigma^1$ for OIE. For $knw = \mathsf{pw}, \mathsf{gw}, \mathsf{pg}$ and pgw, $\boldsymbol{\sigma}'$ is an OIE by the following reason: Players 0 and 2 have no profitable deviation for OIE because $|\Sigma^0| = |\Sigma^2| = 1$, i.e., each of them has only one strategy. Player 1 also has no profitable deviation for OIE because there is no strategy improving $\mathrm{Obj}^{1,\langle v_1\rangle}_{\text{Büchi},knw}(\boldsymbol{\sigma}')$ for each of $knw = \mathsf{pw}, \mathsf{gw}, \mathsf{pg}$ and pgw.

4. Decidability and complexity results on the existence of $\mathcal{O}$-IS

Problem 4.1: Let $\mathcal{G} = (P, V, (V_p)_{p\in P}, v_0, E)$ be a game arena, $p \in P$ be a player, O_p be an objective of p and $\mathcal{O} \subseteq 2^{Play}$ be a subset of objectives. Decide whether there exists an $\mathcal{O}$-IS of p for O_p.

To solve Problem 4.1, we give a necessary and sufficient condition of $\mathcal{O}$-IS for each knowledge of an adversary.

Lemma 4.1: Let $\mathcal{G} = (P, V, (V_p)_{p\in P}, v_0, E)$ be a game arena, $p \in P$ be a player, O_p be an objective of p and $\mathcal{O}$ be a set of objectives. A strategy $\sigma_p \in \Sigma^p$ is an $\mathcal{O}$-IS of p for O_p, i.e., $\mathcal{O} \subseteq \bigcap_{\boldsymbol{\sigma}\in\Sigma} \mathrm{Obj}^{p,O_p}_{knw}(\boldsymbol{\sigma}[p \mapsto \sigma_p])$, if and only if

$$\mathrm{out}^p(\sigma_p) \subseteq \bigcap_{O\in\mathcal{O}} \Big((O \cap O_p) \cup (\overline{O} \cap \overline{O_p}) \Big) \tag{2}$$

when $knw = \mathsf{pw}$,

$$\mathrm{out}^p(\sigma_p) \subseteq \bigcap_{O\in\mathcal{O}\cap\mathrm{Winnable}^p} (O_p \cap O) \cap \bigcap_{O\in\mathcal{O}\cap\{\emptyset\}} \overline{O_p} \tag{3}$$

when $knw = \mathsf{gw}$,

$$\mathrm{out}^p(\sigma_p) \subseteq \bigcap_{O\in\mathcal{O}\cap\mathrm{Winnable}^p} O \tag{4}$$

when $knw = \mathsf{pg}$,

$$\mathrm{out}^p(\sigma_p) \subseteq \bigcap_{O\in\mathcal{O}} \Big((O \cap O_p) \cup (\overline{O} \cap \overline{O_p}) \Big) \cap \bigcap_{O\in\mathcal{O}\cap\mathrm{Winnable}^p} (O \cap O_p) \tag{5}$$

when $knw = \mathsf{pgw}$.

Proof. Let $knw = \mathsf{pgw}$. Assume that $\mathcal{O} \subseteq \bigcap_{\boldsymbol{\sigma}\in\Sigma} \mathrm{Obj}^{p,O_p}_{\mathsf{pgw}}(\boldsymbol{\sigma}[p \mapsto \sigma_p])$. Then, every $O \in \mathcal{O}$ should belong to $\mathrm{Obj}^{p,O_p}_{\mathsf{pgw}}(\boldsymbol{\sigma}[p \mapsto \sigma_p])$ for every $\boldsymbol{\sigma} \in \Sigma$. Then by the definition of $\mathrm{Obj}^{p,O_p}_{\mathsf{pgw}}$, every $O \in \mathcal{O}$ and every $\boldsymbol{\sigma} \in \Sigma$ should satisfy $\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \in (O \cap O_p) \cup (\overline{O} \cap \overline{O_p})$ and whenever $O \in \mathrm{Winnable}^p$, $\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \in O \cap O_p$ and $\mathrm{out}^p(\sigma_p) \subseteq O$. Because $\mathrm{out}^p(\sigma_p) = \{\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \mid \boldsymbol{\sigma} \in \Sigma\}$, we have the containment in the above (5). The other containments can be proved in a similar way. □

We first show some lower bounds of complexities for solving Problem 4.1. Although the class of Muller objectives is the largest one considered in this paper, the complexity of Problem 4.1 for Muller objectives is not the hardest. The reason is described in the footnote of Section 2.

Theorem 4.1: Problem 4.1 is

(a1) P-hard when O_p and $\mathcal{O}$ are over Büchi, co-Büchi or Muller objectives,
(a2) coNP-hard when O_p and $\mathcal{O}$ are over Streett objectives, and
(a3) PSPACE-hard when O_p and $\mathcal{O}$ are over Rabin objectives.

Proof. We can reduce Problem 2.1 for Büchi (resp. co-Büchi, Streett and Muller) objectives to Problem 4.1 for Büchi (resp. co-Büchi, Streett and Muller) objectives as follows: Let $(\mathcal{G}, p, O_p)$ be an instance of Problem 2.1

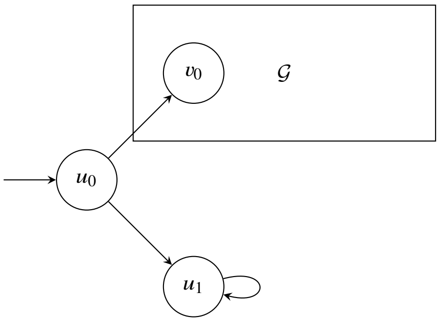

Fig. 6 The game arena $\mathcal{G}'$ made from a game arena $\mathcal{G}$

where O_p is a Büchi, co-Büchi, Streett or Muller objective. When knw = pw, gw or pgw, let $(\mathcal{G}', p', O'_{p'}, \mathcal{O}')$ be an instance of Problem 4.1 where $\mathcal{G}' = \mathcal{G}$, $p' = p$, $O'_{p'} = O_p$ and $\mathcal{O}' = \{Play\}$. When knw = pw, the right-hand side of the containment (2) of Lemma 4.1 becomes $(Play \cap O'_{p'}) \cup (\overline{Play} \cap \overline{O'_{p'}}) = O'_{p'} = O_p$. This property also holds when knw = gw or pgw as follows. When knw = gw, the right-hand side of the containment (3) of Lemma 4.1 becomes $(O'_{p'} \cap Play) \cap Play = O'_{p'} = O_p$. Note that $Play$ is always in Winnable$_p$ and the second term in the right-hand side of the containment (3) is $Play$ if $\mathcal{O}' \cap \{\emptyset\}$ is empty, and $\overline{O_p}$ otherwise. Since we let $\mathcal{O}' = \{Play\}$, the second term in the right-hand side of the containment (3) equals $Play$. When knw = pgw, the right-hand side of the containment (5) of Lemma 4.1 becomes $((Play \cap O'_{p'}) \cup (\overline{Play} \cap \overline{O'_{p'}})) \cap (Play \cap O'_{p'}) = (O'_{p'} \cup (\emptyset \cap \overline{O'_{p'}})) \cap O'_{p'} = O'_{p'} = O_p$. By Lemma 4.1, a strategy $\sigma'_{p'} \in \Sigma^{p'}_{\mathcal{G}'}$ is an $\mathcal{O}'$-IS of p' for $O'_{p'}$ if and only if $\mathrm{out}^{p'}(\sigma'_{p'}) \subseteq O_p$, i.e., $\sigma'_{p'}$ is a winning strategy of p for O_p in $\mathcal{G}$. Therefore, Problem 2.1 for Büchi (resp. co-Büchi, Streett and Muller) objectives is reduced to Problem 4.1 for Büchi (resp. co-Büchi, Streett and Muller) objectives. We conclude (a1) (resp. (a2)) of the lemma when knw = pw, gw or pgw by Theorem 2.2 (1) (resp. (3)).

When knw = pg, let $\mathcal{G}' = (P, V', (V'_i)_{i\in P}, u_0, E')$ where $V' = V \cup \{u_0, u_1\}$, $V'_p = V_p \cup \{u_0, u_1\}$, $V'_i = V_i$ $(i \in P \setminus \{p\})$ and $E' = E \cup \{(u_0, v_0), (u_0, u_1), (u_1, u_1)\}$. Figure 6 shows $\mathcal{G}'$. Let $p' = p$. In this paragraph, we use regular expressions to represent objectives. For example, $u_0 O_p$ means the set $\{u_0\rho \mid \rho \in O_p\}$. Let $O'_{p'} = u_0 O_p$ and $\mathcal{O}' = \{O_1, O_2\}$ where $O_1 = u_0 Play$ and $O_2 = u_0(O_p \cup u_1^{\omega})$. Note that $Play$ in the definition of O_1 means the set of all plays in $\mathcal{G}$. Moreover, u_1^{ω} in the definition of O_2 is the regular expression that represents the singleton set consisting of the infinite sequence of u_1. Note that $O_1, O_2 \in \mathrm{Winnable}^{p'}_{\mathcal{G}'}$ because any strategy of p' moving to v_0 from u_0 is a winning strategy for O_1 and any strategy of p' moving to u_1 from u_0 is a winning strategy for O_2. Hence, we have

$$\bigcap_{O\in\mathcal{O}'\cap\mathrm{Winnable}^{p'}_{\mathcal{G}'}} O = O_1 \cap O_2 = u_0 O_p.$$

By Lemma 4.1, this means that if an $\mathcal{O}'$-IS $\sigma_{p'}$ exists, then $\mathrm{out}^p(\sigma_{p'}) \subseteq u_0 O_p$, and thus we have a winning strategy of p for O_p in $\mathcal{G}$ by removing the initial transition from u_0 to v_0 in $\sigma_{p'}$. Conversely, if we have a winning strategy σ_p of p for O_p in $\mathcal{G}$, then we also have an $\mathcal{O}'$-IS of p' for $O'_{p'}$ in $\mathcal{G}'$ by adding the initial transition from u_0 to v_0 in σ_p. Therefore, when knw = pg, Problem 2.1 is reduced to Problem 4.1 and we conclude (a1) and (a2) by Theorem 2.2 (1) and (3), respectively.

We can reduce Problem 2.1 for an intersection of Rabin objectives to Problem 4.1 for Rabin objectives as follows: Let $(\mathcal{G}, p, O_p)$ be an instance of Problem 2.1 where $O_p = O_1 \cap \cdots \cap O_k$ is an intersection of Rabin objectives $O_1, \ldots, O_k$. Then, let $(\mathcal{G}', p', O'_{p'}, \mathcal{O}')$ be an instance of Problem 4.1 where $\mathcal{G}' = \mathcal{G}$, $p' = p$, $O'_{p'} = Play$ and $\mathcal{O}' = \{O_1, \ldots, O_k\}$. When knw = pw, it is easy to see that the right-hand side of the containment (2) of Lemma 4.1 becomes $O_p = O_1 \cap \cdots \cap O_k$. Assume knw = gw. If $O_i \notin \mathrm{Winnable}^p$ for some $1 \le i \le k$, then there is no winning strategy of p for O_p in $\mathcal{G}$. Hence, we consider only the case where $O_i \in \mathrm{Winnable}^p$ for all $1 \le i \le k$. Because the empty objective $\emptyset \subseteq Play$ is never in Winnable^p, we have

$$\bigcap_{O\in\mathcal{O}'\cap\mathrm{Winnable}^{p'}} (O'_{p'} \cap O) \cap \bigcap_{O\in\mathcal{O}'\cap\{\emptyset\}} \overline{O'_{p'}} = O_p \cap Play = O_p.$$

Also when knw = pg or pgw, we can show that the right-hand sides of the containments (4) and (5) of Lemma 4.1 become O_p. Hence, for every $knw \in$ {pw, gw, pg, pgw}, there exists a winning strategy of p for O_p in $\mathcal{G}$ if and only if there exists an $\mathcal{O}'$-IS of p' for $O'_{p'}$ in $\mathcal{G}'$. Therefore, Problem 2.1 for an intersection of Rabin objectives is reduced to Problem 4.1 for Rabin objectives. We conclude (a3) by Theorem 2.2 (4). □

Next, we provide upper bounds for solving Problem 4.1.

Theorem 4.2: Problem 4.1 is in

(b1) P when knw = pg and $\mathcal{O}$ is over Büchi, co-Büchi or Muller objectives,
(b2) coNP when knw = pw, gw or pgw and O_p and $\mathcal{O}$ are over Büchi objectives or over co-Büchi objectives and when knw = pg and $\mathcal{O}$ is over Streett objectives,
(b3) PSPACE when O_p and $\mathcal{O}$ are over Rabin or Streett objectives,
(b4) EXPTIME when knw = pw, gw or pgw and O_p and $\mathcal{O}$ are over Muller objectives.

Proof. (b1): Assume knw = pg. When $\mathcal{O}$ is over Büchi (resp. co-Büchi and Muller) objectives, the right-hand side of the containment (4) of Lemma 4.1 is an intersection of Büchi objectives (resp. a co-Büchi objective and a Muller objective). This means that there is an $\mathcal{O}$-IS for $\mathcal{O}$ over Büchi (resp. co-Büchi and Muller) objectives if and only if there is a winning strategy for an intersection of Büchi objectives (resp. a co-Büchi objective and a Muller objective). Hence, (b1) holds by Theorem 2.2 (1). Note that $\mathcal{O} \cap \mathrm{Winnable}^p$ can be computed in polynomial time because each member of $\mathcal{O}$ is a Büchi, co-Büchi or Muller objective.

(b2): The right-hand side of the containment (2) of

Lemma 4.1 can be considered a Streett objective as follows: Assume knw = pw and O_p and $\mathcal{O}$ are over Büchi objectives. Let O be an arbitrary member of $\mathcal{O}$. Because both O and O_p are Büchi objectives, both $\overline{O}$ and $\overline{O_p}$ are co-Büchi objectives. An intersection of co-Büchi objectives is also a co-Büchi objective. Hence, $\overline{O} \cap \overline{O_p}$ is a co-Büchi objective. It is easy to see that $(O \cap O_p) \cup (\overline{O} \cap \overline{O_p}) = (O \cup (\overline{O} \cap \overline{O_p})) \cap (O_p \cup (\overline{O} \cap \overline{O_p}))$. Both $O \cup (\overline{O} \cap \overline{O_p})$ and $O_p \cup (\overline{O} \cap \overline{O_p})$ have forms of (F_k, G_k) of a Streett objective in Definition 2.2. Therefore, the right-hand side of the containment (2) of Lemma 4.1 is a Streett objective. We can also verify that the same property holds when any combination of knw = pw, gw or pgw and O_p and $\mathcal{O}$ over Büchi objectives or over co-Büchi objectives. Assume knw = pg and $\mathcal{O}$ is over Streett objectives. Because an intersection of Streett objectives is also a Streett objective, the right-hand side of the containment (4) of Lemma 4.1 is a Streett objective. By Theorem 2.2 (3), (b2) holds.

(b3): It is easy to see that both a Rabin objective and a Streett objective are Boolean combinations of Büchi objectives. A Boolean combination of Rabin or Streett objectives is also a Boolean combination of Büchi objectives. Hence, the right-hand sides of the containments (2), (3), (4) and (5) of Lemma 4.1 are Boolean combinations of Büchi objectives because any $O \in \mathcal{O}$ and O_p are Rabin or Streett objectives. Therefore, (b3) holds by Theorem 2.2 (4).

(b4): Assume knw = pw, gw or pgw and O_p and $\mathcal{O}$ are over Muller objectives. Then the right-hand sides of the containments (2), (3) and (5) of Lemma 4.1 are also Muller objectives. Recall that we represent a Muller objective by a set of bit vectors of length $|V|$ where each bit represents whether the corresponding vertex should be visited infinitely often. For a given specification $\mathcal{F}$ of a Muller objective, the description length of the specification $\mathcal{F}'$ of the complement objective of Muller($\mathcal{F}$), i.e., Muller($\mathcal{F}'$) = $\overline{\text{Muller}(\mathcal{F})}$, may become an exponential order of that of $|V|$ even if that of $\mathcal{F}$ is a polynomial of $|V|$. Therefore, (b4) holds by Theorem 2.2 (1). □

Table 1 in Section 1 summarizes Theorems 4.1 and 4.2.

Next, we consider the same problem as Problem 4.1 for a winning $\mathcal{O}$-IS.

Problem 4.2: Let $\mathcal{G} = (P, V, (V_p)_{p\in P}, v_0, E)$ be a game arena, $p \in P$ be a player, O_p be an objective of p and $\mathcal{O} \subseteq 2^{Play}$ be a subset of objectives. Decide whether there exists a winning $\mathcal{O}$-IS of p for O_p.

Recall that a winning $\mathcal{O}$-IS is a strategy that is a winning strategy and an $\mathcal{O}$-IS. Hence, for a p's objective O_p, a winning $\mathcal{O}$-IS σ_p of p should satisfy the condition $\text{out}^p(\sigma_p) \subseteq O_p$ of a winning strategy and the condition of an $\mathcal{O}$-IS stated in Lemma 4.1: σ_p is a winning $\mathcal{O}$-IS if and only if

$$\text{out}^p(\sigma_p) \subseteq O_p \cap \bigcap_{O\in\mathcal{O}} O \qquad \text{when } knw = \text{pw or pgw},$$

$$\text{out}^p(\sigma_p) \subseteq O_p \cap \bigcap_{O\in\mathcal{O}\cap\text{Winnable}^p} (O_p \cap O) \cap \bigcap_{O\in\mathcal{O}\cap\{\emptyset\}} \overline{O_p}$$

when knw = gw,

$$\text{out}^p(\sigma_p) \subseteq O_p \cap \bigcap_{O\in\mathcal{O}\cap\text{Winnable}^p} O \qquad \text{when } knw = \text{pg}.$$

Table 2 The complexities of Problem 4.2

	gw	pw, pg or pgw
Büchi or co-Büchi	coNP P-hard	P-complete
Streett	PSPACE coNP-hard	coNP-complete
Rabin	PSPACE-complete	PSPACE-complete
Muller	EXPTIME P-hard	P-complete

The complexities of Problem 4.2 is as shown in Table 2. We can prove these complexities in the same way as the proofs of Theorems 4.1 and 4.2. Note that the upper bounds of the complexities of Problem 4.2 when knw = pw and knw = pgw (except for Rabin objectives) are lower than those of Problem 4.1, because the above conditions when knw = pw and knw = pgw are simpler than (2) and (5) in Lemma 4.1.

5. Decidability result of the existence of OIE

In this section, we prove that the existence of an OIE when we assume the rationality of a player only when she is a loser, i.e., we remove the conditions underlined with solid lines from the definitions of $\text{Obj}^{p,O_p}_{\Omega,knw}$. The definitions of $\text{Obj}^{p,O_p}_{\Omega,knw}$ become as follows:

$$\begin{aligned}
&\text{Obj}^{p,O_p}_{\Omega,\text{pw}}(\boldsymbol{\sigma}) = \{O \subseteq V^\omega \mid \\
&\quad \text{out}(\boldsymbol{\sigma}) \in (O \cap O_p) \cup (\overline{O} \cap \overline{O_p})\},\\
&\text{Obj}^{p,O_p}_{\Omega,\text{gw}}(\boldsymbol{\sigma}) = \{O \in \Omega \mid \\
&\quad (O \in \text{Winnable}^p \Rightarrow \text{out}(\boldsymbol{\sigma}) \in O_p) \land \\
&\quad (O = \emptyset \Rightarrow \text{out}(\boldsymbol{\sigma}) \notin O_p)\},\\
&\text{Obj}^{p,O_p}_{\Omega,\text{pg}}(\boldsymbol{\sigma}) = \{O \in \Omega \mid \\
&\quad O \in \text{Winnable}^p \Rightarrow \text{out}(\boldsymbol{\sigma}) \in O\},\\
&\text{Obj}^{p,O_p}_{\Omega,\text{pgw}}(\boldsymbol{\sigma}) = \{O \in \Omega \mid \\
&\quad \text{out}(\boldsymbol{\sigma}) \in (O \cap O_p) \cup (\overline{O} \cap \overline{O_p}) \land \\
&\quad (O \in \text{Winnable}^p \Rightarrow \text{out}(\boldsymbol{\sigma}) \in O \cap O_p)\}.
\end{aligned}$$

This simplification causes $\text{Obj}^{p,O_p}_{\Omega,knw}(\boldsymbol{\sigma})$ to depend only on the outcome of $\boldsymbol{\sigma}$, not on $\boldsymbol{\sigma}$ itself. Whether the problem is decidable or not is open in the general case.

Theorem 5.1: For a game arena $\mathcal{G}$ and an objective profile $\boldsymbol{\alpha} = (O_p)_{p\in P}$ over Muller objectives, whether there exists an OIE for $\mathcal{G}$ and $\boldsymbol{\alpha}$ is decidable.

Proof. Condition (1) in Definition 3.2 is equivalent to the following condition:

$$\begin{aligned}
&\forall p \in P.\ \forall \sigma_p \in \Sigma^p.\ \forall O \in \Omega.\\
&O \in \text{Obj}^{p,O_p}_{\Omega,knw}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \Rightarrow O \in \text{Obj}^{p,O_p}_{\Omega,knw}(\boldsymbol{\sigma}).
\end{aligned} \tag{6}$$

First we consider the case where $knw = \mathsf{pgw}$. By the definition of $\mathrm{Obj}_{\Omega,\mathsf{pgw}}^{p,O_p}$, Condition (6) for $knw = \mathsf{pgw}$ is equivalent to the following condition:

$$\begin{aligned}&\forall p \in P.\ \forall \sigma_p \in \Sigma^p.\ \forall O \in \Omega.\\ &\text{if } O \in \mathrm{Winnable}^p, \text{ then}\\ &(\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \in O \cap O_p \Rightarrow \mathrm{out}(\boldsymbol{\sigma}) \in O \cap O_p);\\ &\text{otherwise,}\\ &(\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \in (O \cap O_p) \cup (\overline{O} \cap \overline{O_p})\\ &\quad \Longrightarrow \mathrm{out}(\boldsymbol{\sigma}) \in (O \cap O_p) \cup (\overline{O} \cap \overline{O_p})).\end{aligned} \quad (7)$$

For $O \in \mathcal{O}$ and $p \in P$, let R_p^O be the objective defined as follows: If $O \in \mathrm{Winnable}^p$, $R_p^O = O \cap O_p$. Otherwise, $R_p^O = (O \cap O_p) \cup (\overline{O} \cap \overline{O_p})$. Let $\boldsymbol{\alpha}_O = (R_p^O)_{p \in P}$ be the objective profile consisting of these objectives. Then, Condition (7) can be written as $\forall O \in \mathcal{O}.\ \mathrm{Nash}(\boldsymbol{\sigma}, \boldsymbol{\alpha}_O)$. Therefore, this theorem holds for $knw = \mathsf{pgw}$ by Theorem 2.3.

For the other cases, the implication inside the scope of the three universal quantifiers in Condition (6) is equivalent to the following implications:

When $knw = \mathsf{pw}$,

$$\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \in (O \cap O_p) \cup (\overline{O} \cap \overline{O_p}) \Longrightarrow \mathrm{out}(\boldsymbol{\sigma}) \in (O \cap O_p) \cup (\overline{O} \cap \overline{O_p}).$$

When $knw = \mathsf{gw}$,

if $O \in \mathrm{Winnable}^p$, then

$$\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \in O_p \Longrightarrow \mathrm{out}(\boldsymbol{\sigma}) \in O_p;$$

if $O = \emptyset$, then

$$\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \in \overline{O_p} \Rightarrow \mathrm{out}(\boldsymbol{\sigma}) \in \overline{O_p}.$$

When $knw = \mathsf{pg}$,

if $O \in \mathrm{Winnable}^p$, then

$$\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma_p]) \in O \Rightarrow \mathrm{out}(\boldsymbol{\sigma}) \in O.$$

These conditions can be written as the combination of NE in the same way as the case where $knw = \mathsf{pgw}$. Therefore, this theorem also holds for $knw \in \{\mathsf{pw}, \mathsf{gw}, \mathsf{pg}\}$ by Theorem 2.3. □

Corollary 5.1: For a game arena $\mathcal{G}$ and an objective profile $\boldsymbol{\alpha} = (O_p)_{p \in P}$ over Muller objectives, whether there exists an OINE for $\mathcal{G}$ and $\boldsymbol{\alpha}$ is decidable.

Proof. As shown in the proof of Theorem 5.1, $\boldsymbol{\sigma} \in \Sigma$ is an OIE if and only if it is an $((\boldsymbol{\alpha}_O)_{O \in \mathcal{O}})$-NE. By definition, $\boldsymbol{\sigma}$ is an OINE if and only if it is an OIE and also satisfies $\mathrm{Nash}(\boldsymbol{\sigma}, \boldsymbol{\alpha})$. Therefore, $\boldsymbol{\sigma}$ is an OINE if and only if it is an $((\boldsymbol{\alpha}_O)_{O \in \mathcal{O}}, \boldsymbol{\alpha})$-NE, and the theorem holds by Theorem 2.3. □

6. Conclusion

We proposed two new notions $\mathcal{O}$-indistinguishable strategy ($\mathcal{O}$-IS) and objective-indistinguishability equilibrium (OIE). Then, we analyzed the complexities of deciding whether there exists an $\mathcal{O}$-IS for some classes of objectives. In addition, we proved that whether there exists an OIE over Muller objectives is decidable under a weaker assumption on rationality. To prove this, we defined an $(\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_n)$-Nash equilibrium as a strategy profile which is simultaneously a Nash equilibrium for all objective profiles $\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_n$. In Appendix, we proved that whether there exists an $(\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_n)$-Nash equilibrium is decidable.

In this paper, we assume that an adversary is not a player but an individual who observes partial information on the game. He cannot directly affect the outcome of the game by choosing next vertices. We can consider another setting where an adversary is also a player. His objective is minimizing the set $\mathrm{Obj}_{\Omega,knw}^{p,O_p}$ of candidate objectives of other players and he takes a strategy for achieving the objective. Extending the results shown in this paper to the above setting is future work.

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Appendix A: Decidability of the existence of multiple Nash equilibrium

In this section, we give a proof of Theorem 2.3.

For a play $\rho = v_0 v_1 v_2 \cdots \in \mathit{Play}$, let $\rho_{\leq i} = v_0 \cdots v_i$ and $\rho_{\geq i} = v_i v_{i+1} v_{i+2} \cdots$. For an objective O and a history $h = v_0 \cdots v_i$, we define $h \backslash O$ as $h \backslash O = \{\rho_{\geq i} \mid \rho \in O \wedge \rho_{\leq i} = h\}$. Note that each sequence in $h \backslash O$ is starting from v_i, not v_{i+1}. For an objective O and a vertex v, we define ${}^*v \backslash O$ as ${}^*v \backslash O = \bigcup_{hv \in \mathit{Hist}} hv \backslash O$. For a game arena $\mathcal{G} = (P, V, (V_p)_{p \in P}, v_0, E)$ and $v \in V$, let $(\mathcal{G}, v) = (P, V, (V_p)_{p \in P}, v, E)$ be the game arena obtained from $\mathcal{G}$ by replacing the initial vertex v_0 of $\mathcal{G}$ with v. Note that every objective in the objective classes in this paper is "prefix-independent", i.e., $h_1 \rho \in O \iff h_2 \rho \in O$ for any objective O, histories h_1, h_2, and ρ satisfying $h_1 \rho, h_2 \rho \in \mathit{Play}$, because the objective is defined only on the set of vertices that appear in a play infinitely often. Therefore, $h_1 v \backslash O = h_2 v \backslash O$ for any histories $h_1 v$ and $h_2 v$, which implies $hv \backslash O = {}^*v \backslash O$ for any history hv.

For a game arena $\mathcal{G} = (P, V, (V_p)_{p \in P}, v_0, E)$ and an objective profile $\boldsymbol{\alpha} = (O_p)_{p \in P}$, we define the game arena $\mathcal{G}_p = (\{p, -p\}, V, (V_p, \overline{V_p}), v_0, E)$ and the objective profile $(O_p, \overline{O_p})$ for each $p \in P$. The game arena $\mathcal{G}_p$ with the objective profile $(O_p, \overline{O_p})$ is a 2-player zero-sum game such that vertices and edges are the same as $\mathcal{G}$ and the player $-p$ is formed by the *coalition* of all the players in $P \setminus \{p\}$. The following proposition is a variant of [3, Proposition 28] adjusted to the settings of this paper.

Proposition A.1: Let $\mathcal{G} = (P, V, (V_p)_{p \in P}, v_0, E)$ be a game arena and $\boldsymbol{\alpha} = (O_p)_{p \in P}$ be an objective profile. Then, a play $\rho = v_0 v_1 v_2 \cdots \in \mathit{Play}$ is the outcome of some NE $\boldsymbol{\sigma} \in \Sigma_{\mathcal{G}}$ for $\boldsymbol{\alpha}$, i.e., $\rho = \mathrm{out}(\boldsymbol{\sigma})$, if and only if $\forall p \in P.\ ((\exists i \geq 0.\ v_i \in V_p \wedge \rho_{\leq i} \backslash O_p \in \mathrm{Winnable}^p_{(\mathcal{G}, v_i)}) \implies \rho \in O_p)$.

Proof. ($\Rightarrow$) We prove this direction by contradiction. Assume that a play $\rho = v_0 v_1 v_2 \cdots \in \mathit{Play}$ is the outcome of an NE $\boldsymbol{\sigma} = (\sigma_p)_{p \in P} \in \Sigma$ for $\boldsymbol{\alpha}$ and there exist $p \in P$ and $i \geq 0$ with $v_i \in V_p \wedge \rho_{\leq i} \backslash O_p \in \mathrm{Winnable}^p_{(\mathcal{G}, v_i)}$ such that $\rho \notin O_p$. Since $\rho_{\leq i} \backslash O_p \in \mathrm{Winnable}^p_{(\mathcal{G}, v_i)}$, there exists a winning strategy $\tau_p \in \Sigma^p_{(\mathcal{G}, v_i)}$ of p for $\rho_{\leq i} \backslash O_p$. Let $\sigma'_p \in \Sigma^p_{\mathcal{G}}$ be the strategy obtained from σ_p and τ_p as follows: Until producing $v_0 v_1 \cdots v_i$, σ'_p is the same as σ_p. From v_i, σ'_p behaves in the same way as τ_p. Therefore, $\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma'_p])$ equals $v_0 v_1 \cdots v_{i-1} \pi$ for some play π of $(\mathcal{G}, v_i)$ and $\pi \in \rho_{\leq i} \backslash O_p$ because τ_p is a winning strategy of p for $\rho_{\leq i} \backslash O_p$ in $(\mathcal{G}, v_i)$. We have $\mathrm{out}(\boldsymbol{\sigma}[p \mapsto \sigma'_p]) = v_0 \cdots v_{i-1} \pi \in O_p$. This contradicts the assumption that $\boldsymbol{\sigma}$ is an NE.

($\Leftarrow$) Let $\rho = v_0 v_1 v_2 \cdots \in \mathit{Play}$ be a play on $\mathcal{G}$ and assume that $\rho \in O_p$ for all $p \in P$ such that if there exists $i \geq 0$ satisfying $v_i \in V_p \wedge \rho_{\leq i} \backslash O_p \in \mathrm{Winnable}^p_{(\mathcal{G}, v_i)}$ then $\rho \in O_p$. We define a strategy profile $\boldsymbol{\sigma} = (\sigma_p)_{p \in P}$ as the one that satisfies the following two conditions: First, $\boldsymbol{\sigma}$ produces ρ as its outcome, i.e., $\mathrm{out}(\boldsymbol{\sigma}) = \rho$. Second, if some player p deviates from ρ at $v_j \in V_p$ ($j \geq 0$) and $\rho_{\leq j} \backslash O_p \notin \mathrm{Winnable}^p_{(\mathcal{G}, v_j)}$, then all the other players (as a coalition) play from v_j according to a winning strategy of $-p$ for $(\mathcal{G}_p, v_j)$ and $\overline{\rho_{\leq j} \backslash O_p}$. (Note that in a 2-player zero-sum game, there is always a winning strategy for one of the players, and thus there is a winning strategy of $-p$ for $(\mathcal{G}_p, v_j)$ and $\overline{\rho_{\leq j} \backslash O_p}$ when $\rho_{\leq j} \backslash O_p \notin \mathrm{Winnable}^p_{(\mathcal{G}, v_j)}$.) We can show that the strategy profile $\boldsymbol{\sigma}$ is an NE as follows: Assume that some player p deviates from σ_p to a strategy $\sigma'_p \in \Sigma^p$, and $\mathrm{out}_{\mathcal{G}}(\boldsymbol{\sigma}[p \mapsto \sigma'_p])$ deviates from ρ at $v_j \in V_p$ for some $j \geq 0$. If $\rho_{\leq j} \backslash O_p \in \mathrm{Winnable}^p_{(\mathcal{G}, v_j)}$, then by assumption, $\rho \in O_p$. Hence, σ'_p is not a profitable deviation. Otherwise, as described above, all the other players (as a coalition) punish the player p by taking a winning strategy of $-p$ for $(\mathcal{G}_p, v_j)$ and $\overline{O_p}$, and hence $p \notin \mathrm{Win}(\boldsymbol{\sigma}[p \mapsto \sigma'_p])$. Thus, σ'_p is not a profitable deviation also in this case. □

Corollary A.1: Let $\mathcal{G} = (P, V, (V_p)_{p \in P}, v_0, E)$ be a game arena and $\boldsymbol{\alpha}_j = (O^j_p)_{p \in P}$ ($1 \leq j \leq n$) be objective profiles. Then, a play $\rho = v_0 v_1 v_2 \cdots \in \mathit{Play}$ is the outcome of some $(\boldsymbol{\alpha}_1, \ldots, \boldsymbol{\alpha}_n)$-NE $\boldsymbol{\sigma} \in \Sigma$, i.e., $\rho = \mathrm{out}(\boldsymbol{\sigma})$, if and only if

Algorithm 1

Input: a game arena $\mathcal{G} = (P, V, (V_p)_{p\in P}, v_0, E)$ and objective profiles $\alpha_j = (O_p^j)_{p\in P}$ $(1 \le j \le n)$.

```
for all v ∈ V do
    Let p ∈ P be the player such that v ∈ V_p.
    O_v := ⋂_{*v\O_p^j ∈ Winnable^p_(G,v), 1≤j≤n} O_p^j.
end for
Nondeterministically select a set of vertices V′ ⊆ V and construct a 1-player subgame arena G_V′ = ({1}, V′, (V′), v_0, E′) of G.
O_G_V′ := ⋂_{v∈V′} O_v.
if Player 1 has a winning strategy σ_1 ∈ Σ^1_G_V′ for G_V′ and O_G_V′ then
    return Yes with σ_1
else
    return No
end if
```

$$\begin{aligned}&\forall p \in P.\ 1 \le \forall j \le n.\\ &(\exists i \ge 0.\ v_i \in V_p \wedge \rho_{\le i}\backslash O_p^j \in \text{Winnable}^p_{(\mathcal{G},v_i)}) \qquad \text{(A· 1)}\\ &\implies \rho \in O_p^j.\end{aligned}$$

Corollary A.1 can be easily proved by Proposition A.1 and Definition 2.3.

Using this corollary, we can prove Theorem 2.3 as follows.

Theorem 2.3. *Let $\mathcal{G} = (P, V, (V_p)_{p\in P}, v_0, E)$ be a game arena and $\alpha_j = (O_p^j)_{p\in P}$ $(1 \le j \le n)$ be objective profiles over Muller objectives. Deciding whether there exists an $(\alpha_1, \ldots, \alpha_n)$-NE is decidable.*

Proof. By Corollary A.1, there exists an $(\alpha_1, \ldots, \alpha_n)$-NE if and only if there exists a play $\rho = v_0v_1v_2\cdots \in \mathit{Play}$ satisfying Condition (A· 1). Algorithm 1 decides the existence of a play satisfying Condition (A· 1). In Algorithm 1, we call a game arena $\mathcal{G}_{V'} = (\{1\}, V', (V'), v_0, E')$ satisfying $V' \subseteq V, v_0 \in V'$ and $E' = \{(v, v') \in E \mid v, v' \in V'\}$ a 1-player subgame arena of $\mathcal{G}$ (induced by V').

Let us show the correctness of Algorithm 1. First, we show that when Algorithm 1 answers Yes, the outcome of the strategy answered by Algorithm 1 satisfies Condition (A· 1). Let $\rho = \text{out}_{\mathcal{G}_{V'}}(\sigma_1) = v_0v_1v_2\cdots \in \mathit{Play}$ for the strategy σ_1 returned by Algorithm 1. Because ρ is the outcome of a winning strategy for $O_{\mathcal{G}_{V'}}$, we have $\rho \in O_{\mathcal{G}_{V'}}$. By the definitions of $O_{\mathcal{G}_{V'}}$ and O_v,

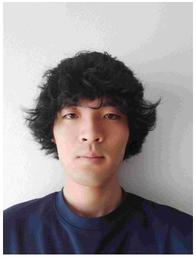
$$\begin{aligned}\rho \in O_{\mathcal{G}_{V'}} &\iff \forall v \in V'.\ \rho \in O_v\\ &\iff \forall v \in V'.\ \forall p \in P.\ 1 \le \forall j \le n.\\ &\qquad (v \in V_p \wedge {}^*v\backslash O_p^j \in \text{Winnable}^p_{(\mathcal{G},v)}) \Rightarrow \rho \in O_p^j.\end{aligned}$$

Because ρ is a play in $\mathcal{G}_{V'}$, we have $v_i \in V'$ for all $i \ge 0$. Thus,

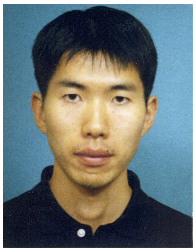
$$\begin{aligned}\rho \in O_{\mathcal{G}_{V'}} \Rightarrow &\forall p \in P.\ \forall i \ge 0.\ 1 \le \forall j \le n.\\ &(v_i \in V_p \wedge {}^*v_i\backslash O_p^j \in \text{Winnable}^p_{(\mathcal{G},v_i)}) \Rightarrow \rho \in O_p^j.\end{aligned}$$

Therefore ρ satisfies Condition (A· 1). (Note that $\rho_{\le i}\backslash O_p^j = {}^*v_i\backslash O_p^j$ holds by the prefix-independence of the objectives.)

Conversely, we show that if there exists a play ρ satisfying Condition (A· 1), then at least one nondeterministic branch of Algorithm 1 should answer Yes with a strategy σ_1 such that $\rho = \text{out}_{\mathcal{G}_{V'}}(\sigma_1)$. Assume that there exists a play $\rho = v_0v_1v_2\cdots \in \mathit{Play}$ satisfying Condition (A· 1). Let $V' = \{v \in V \mid \exists i \ge 0.\ v = v_i\}$, and then construct the 1-player subgame arena $\mathcal{G}_{V'} = (\{1\}, V', (V'), v_0, E')$ with $E' = \{(v, v') \in E \mid v, v' \in V'\}$ and the objective $O_{\mathcal{G}_{V'}} = \bigcap_{v\in V'} O_v$ where for all $v \in V'$, $O_v = \bigcap_{{}^*v\backslash O_p^j \in \text{Winnable}^p_{(\mathcal{G},v)}, 1\le j\le n} O_p^j$ for $p \in P$ such that $v \in V_p$. It is easy to see that ρ is a play of $\mathcal{G}_{V'}$ and $\rho \in O_{\mathcal{G}_{V'}}$ by Condition (A· 1). Therefore, any strategy σ_1 that produces ρ is a winning strategy of the player 1 for $\mathcal{G}_{V'}$ and $O_{\mathcal{G}_{V'}}$, and Algorithm 1 should answer Yes with a strategy σ_1 such that $\rho = \text{out}_{\mathcal{G}_{V'}}(\sigma_1)$. □



Rindo Nakanishi received his Master's degree in informatics from Nagoya University in 2023. He has pursued Doctor's degree in the University. His research interests include formal language theory and its application to software engineering.



Yoshiaki Takata received the Ph.D. degree in information and computer science from Osaka University in 1997. He was with Nara Institute of Science and Technology as an Assistant Professor in 1997–2007. In 2007, he joined the faculty of Kochi University of Technology, where he is currently working as a professor. His research interests include formal specification and verification of software systems.

Hiroyuki Seki received his Ph.D. degree from Osaka University in 1987. He was an Assistant Professor, and later, an Associate Professor in Osaka University from 1987 to 1994. In 1994, he joined Nara Institute of Science and Technology, where he was a Professor during 1996 to 2013. Currently, he is a Professor in Nagoya University. His current research interests include formal language theory and formal approach to software development.