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Discovery of the Optimal Trust Inference Path for Online Social Networks

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SUMMARY Analysis of the trust network proves beneficial to the users in Online Social Networks (OSNs) for decision-making. Since the construction of trust propagation paths connecting unfamiliar users is the preceding work of trust inference, it is vital to find appropriate trust propagation paths. Most of existing trust network discovery algorithms apply the classical exhausted searching approaches with low efficiency and/or just take into account the factors relating to trust without regard to the role of distrust relationships. To solve the issues, we first analyze the trust discounting operators with structure balance theory and validate the distribution characteristics of balanced transitive triads. Then, Maximum Indirect Referral Belief Search (MIRBS) and Minimum Indirect Functional Uncertainty Search (MIFUS) strategies are proposed and followed by the Optimal Trust Inference Path Search (OTIPS) algorithms accordingly on the basis of the bidirectional versions of Dijkstra's algorithm. The comparative experiments of path search, trust inference and edge sign prediction are performed on the Epinions data set. The experimental results show that the proposed algorithm can find the trust inference path with better efficiency and the found paths have better applicability to trust inference.
key words: *structural balance, subjective logic, trust inference, pathfinding algorithm, edge sign prediction, online social network*

1. Introduction

The popularity of "Web 2.0" vastly boosts the information sharing and collaboration among users and leads to the development of online communities. Billions of people participate in the web-based Online Social Networks (OSNs), such as Facebook, Google Plus and Linked In, for variety of activities. Trust plays an important role in some OSNs, for example Epinions (epinions.com). The *Web of Trust* in Epinions makes the consumer able to seek out people who share similar interests and deserve his/her trust so as to deliver reliable and useful recommendations. In these applications, trust is the key factor during the users' decision-making process and how to evaluate the trustworthiness of the unfamiliar target user or item emerges as a question.

Trust inference based on trust transitivity is widely used to build trust among unfamiliar users. Following Jøsang's description of trust networks [1], trust inference is based on the trust transitivity for the specific consistent *trust scope* in the trust propagation path. The expectation of the ability to recommend a competent service provider and the ability to be a competent service provider are distinguished

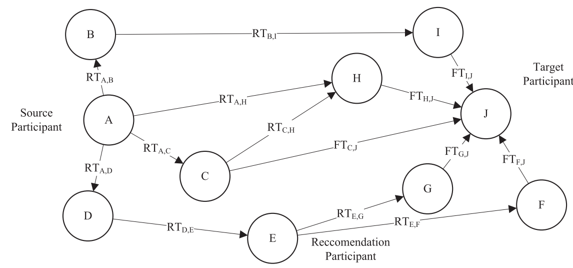

Fig. 1 Trust network connecting the source and target participants.

and denoted as *referral trust RT* and *functional trust FT*. When constructing the trust network connecting for a specific trust scope (Fig. 1), all participants are represented as vertices (e.g. $A, B, \ldots, J$) and the direct trust relationships (e.g. $RT_{A,B}$, $RT_{B,I}$, $FT_{I,J}$) are represented as edges. A trust propagation path is a path connecting the source participant A to the target participant J in the graph, which is composed of edges on referral trust except for the last edge on functional trust. By applying the propagation paths with trust discounting and fusion operations, trust inference can derive the source participant's indirect functional trust about the target participant. In the large-scale online social networks, the amount of such paths would be huge, which obviously burdens the computation cost of the pathfinding, and only part of trust propagation paths are used for trust inference.

As the preceding work of the trust inference, it is very important to find appropriate trust inference paths. Intuitively, the paths with less hops are preferable and people also prefer to get advices from the ones they trust more. According to the principles such as "the enemy of my enemy is my friend", we can even get valuable information from distrusted people. These factors are all important for trust propagation path discovering. Without regard to the fusion of paths in this paper, we focus on the subjective logic based trust inference [1] with the single optimal trust inference path to figure out: 1) how to find the trust inference path efficiently in large-scale network; 2) what is the optimal search strategy for trust inference path search. However, most of the existing trust network discovery algorithms [1]–[4] apply classical exhausted brute-force path search algorithms (such as Breadth First Search and Depth First Search), ignoring the characteristics of trust networks. Many factors relating to trust relationships (such as belief [5], similarities [6] and intimacy [7]) are considered in some latest work, but the role of distrust relationships is still neglected. In

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fact, based on the structure balance theory, we can also utilize distrust relationships in the optimal trust inference path search.

Our contributions include 1) the distribution characteristics of balanced transitive triads are validated by statistical analysis, which can be used to better apply the structure balance theory; 2) optimal trust inference path search strategies MIRBS and MIFUS are proposed and the optimal trust inference path search boils down to a optimization problem; 3) the OTIPS algorithms for the two search strategies are proposed and followed by the proof of optimality; 4) trust inference based edge sign prediction experiment is carried out on real data set, which not only validates the proposed algorithms but also the edge sign prediction method.

The remainder of the paper proceeds as follows. The related work on trust network discovery are illustrated in Sect. 2. Then, the background about trust network are detailed in Sect. 3. We propose the OTIPS algorithms for the MIRBS and MIFUS strategies in Sect. 4. Comparative experiments on the Epinions data set are carried out to demonstrate the superiority of the proposed algorithms in Sect. 5. Section 6 gives the conclusions and future work.

2. Related Work

The research focusing on the trust network discovery and path search strategies attracts much attention. Jøsang et al. [1] determined the possible directed paths between source and target participants by the Depth First path discovering algorithm and provided a method that simplifies complex trust networks to Directed Series-Parallel Graphs. While in [2], the Breadth First Search method is applied to find all the trust paths within the minimum depth for trust evaluation and the maximum search depth is set to 7 according to the small world theory. *TidalTrust* [3] first performs a modified Breadth First Search to find the trust inference paths with the minimum depth and record the trust strength of these paths to the sink's predecessors. Then, the trust threshold is determined and the paths whose trust strength are equal or greater than it will be used for trust inference calculation. Hang et al. [4] proposed the *CertProp* trust propagation model with three path search strategies for the evaluation of trust propagation, including the *shortest* strategy which finds the shortest paths, *fixed* strategy which searches all paths within a specified depth and *selection* strategy which yields the most trusted paths to each witness found by fixed strategy. However, the selection strategy can not guarantee that the selected witness is the most trustworthy globally. TidalTrust and CertProp(sel.) are representative classical exhausted searching approaches and selected as comparison partners in our experiments.

TrustWalker [5] is a random walk model which combines the trust-based and the collaborative filtering approaches for recommendation. Repeated random walks which take into account the item similarities and path lengths are performed to explore the social network and evaluate the recommendations. Liu et al. [6], [7] focused on the optimal social trust path selection with multiple end-to-end QoT constraints (trust, social intimacy and role impact) and modeled it as the classical Multi-Constrained Optimal Path (MCOP) selection problem. They proposed a Monte Carlo method based approximation algorithm (MONTE_K) in [6] and a Heuristic Social Context-Aware trust Network discovery algorithm (*H-SCAN*) in [7] based on the K-Best-First Search (KBFS) method [8]. The experimental results in their latest work [7] show that the proposed algorithm outperforms the Time-to-Live Breadth First Search [9], Random Walk Search [10] and High Degree Search [11]. Thus, H-SCAN is also selected as our comparison partner.

However, most existing trust path search algorithms just apply classical exhausted brute-force search approaches and take into account basic topological characteristics such as out degrees or path length. Although the factors relating to trust, social intimacy and role impact are considered in some trust path search methods, the effect of distrust relationships, especially the valuable information deduced by distrust relationships, is still neglected in these methods, which may lead to smaller solution spaces.

3. Background

3.1 Notions and Notations for Trust Network

Trust networks are networks in which users express their trust opinions about other users. Given a specific trust scope, we denote this network by a directed graph $G = (V, E)$ and follow the notions and notations in subjective logic [12]. V is the set of vertices that represent participants and directed edges E represent the trust relationships among them. One participant's (e.g. $v_A \in V$) subjective opinion about another (e.g. $v_B \in V$) can be noted as $\omega_B^A = (b_B^A, d_B^A, u_B^A, a_B^A)$ which associates with one trust type $\sigma \in \{referral, functional\}$, where b (*belief*) represents the belief mass in support of v_B being competent, d (*disbelief*) represents the belief mass in support of v_B being incompetent and u (*uncertainty*) represents the amount of uncommitted belief mass. They can be obtained by bijections $b_B^A = r_B^A/(2 + r_B^A + s_B^A)$, $d_B^A = s_B^A/(2 + r_B^A + s_B^A)$ and $u_B^A = 2/(2 + r_B^A + s_B^A)$, where r_B^A and s_B^A are the numbers of v_A's positive and negative observations about v_B. The total number of observations can be noted as $t_B^A = r_B^A + s_B^A$ and the certainty can be noted as $c_B^A = b_B^A + d_B^A$. Obviously, $b, d, u \in [0, 1]$ and $b + d + u = 1$. The base rate parameter a is the priori probability in the absence of committed belief mass and used for computing an opinion's expectation when there is no evidence. The probability expectation of the opinion can be represented by $E(\omega_B^A) = b_B^A + a_B^A \cdot u_B^A$.

Furthermore, each directed edge has a sign $sign \in \{1, -1\}$, where edges with 1 mean positive relations and edges with -1 mean negative ones. So, E can be denoted as $\{e_{v_A \to v_B}^{\sigma} = (\omega_B^A, \sigma, sign) | \sigma \in \{referral, functional\}, sign \in \{1, -1\}, v_A, v_B \in V\}$. Moreover, v_A is called a predecessor of v_B and the predecessors of v_B is denoted as $pre(v_B) = \{v_X | e_{v_X \to v_B}^{\sigma} \in E\}$. v_B is called a successor of v_A and the suc-

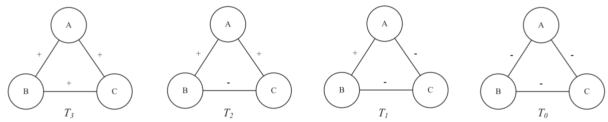

Fig. 2 Undirected signed triads.

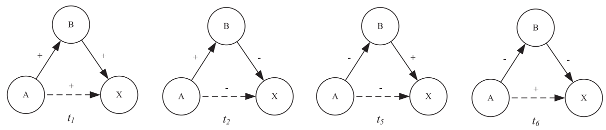

Fig. 3 Edge sign prediction cases in transitive triads.

cessors of v_A is denoted as $suc(v_A) = \{v_X | e^{\sigma}_{v_A \to v_X} \in E\}$.

3.2 Structure Balance in Online Social Network

The structure balance theory [13], [14] inspires novel perspectives on the social network analysis. For the four possibilities on three individuals shown in Fig. 2, the triangles with one or three pluses are denoted as *balanced triads* (T_3 and T_1) and the ones with zero or two pluses are denoted as *unbalanced triads* (T_2 and T_0). This theory considers that balanced triads are more plausible and prevalent than the unbalanced triads. Leskovec et al. [15] investigated the balanced and unbalanced triads in three widely-used social sites: Epinions, Slashdot and Wikipedia. The statistics show that the balanced triads dominate in the social networks and it is feasible to apply the structure balance theory to online social networks. He further extended the triangles to 16 cases ($t_1 - t_{16}$) in directed graphs. For ease of description, the triad in which there is a vertex that has two predecessors is denoted as *transitive triad*. In the transitive triad, the node that has one predecessor is the *transitive vertex*. In this paper, we only focus on the transitive triads t_1, t_2, t_5 and t_6 (see Fig. 3), in which the recommender can be regarded as the transitive vertex. Because they are more consistent with the trust transitivity characteristics than the rest cases.

As shown in Fig. 3, we can derive A's relationship with X by the recommender B. According to the structure balance theory, the dashed relation in t_1 can be derived as a positive relation, corresponding with the principle 1) "the friend of my friend is also my friend". The rest cases correspond with the principles 2) "the enemy of my friend is my enemy", 3) "the friend of my enemy is my enemy" and 4) "the enemy of my enemy is my friend" respectively.

3.3 Trust Inference and Trust Discounting Operators

Trust inference based on trust transitivity is a common method for trust evaluation. Given an unfamiliar user pair (e.g. v_1 and v_n), we can infer v_1's subjective opinion about v_n by applying trust discounting operations to the selected trust propagation path (e.g. $v_1 \to v_2 \to \ldots \to v_n$) by $\omega_n^{1:\ldots:n-1} = \omega_2^1 \otimes \omega_3^2 \otimes \ldots \otimes \omega_n^{n-1}$, where $\otimes$ denotes the *trust discounting operator*. Different definitions of such operators such as the Uncertainty Favoring Discounting operator and Opposite Belief Favoring Discounting operator in [12] and the Concatenation operator in [4] are introduced. We will analyze these operators with structure balance principles.

Definition 1. ***Uncertainty favoring discounting***
Let A, B and X be three participants, the opinions of A and B about the competence of B and X be expressed as $\omega_B^A = (b_B^A, d_B^A, u_B^A, a_B^A)$ *and* $\omega_X^B = (b_X^B, d_X^B, u_X^B, a_X^B)$ *respectively, the uncertainty favoring discounted opinion of A on X can be denoted as* $\omega_X^{A:B} = \omega_B^A \otimes_1 \omega_X^B = (b_X^{A:B}, d_X^{A:B}, u_X^{A:B}, a_X^{A:B})$ *that*

$$\begin{cases} b_X^{A:B} = b_B^A b_X^B \\ d_X^{A:B} = b_B^A d_X^B \\ u_X^{A:B} = d_B^A + u_B^A + b_B^A u_X^B \\ a_X^{A:B} = a_X^B \end{cases}$$

In this trust discounting, A accepts what B believes (b_X^B) and disbelieves (d_X^B) to the extent that A believes B (b_B^A). The rest are all taken as uncertainty. This common situation corresponds to the structure balance principles 1) and 2).

Definition 2. ***Opposite belief favoring discounting***
Given three participants A, B and X as previously described, the opposite belief favoring discounted opinion can be denoted as $\omega_X^{A:B} = \omega_B^A \otimes_2 \omega_X^B = (b_X^{A:B}, d_X^{A:B}, u_X^{A:B}, a_X^{A:B})$ *that*

$$\begin{cases} b_X^{A:B} = b_B^A b_X^B + d_B^A d_X^B \\ d_X^{A:B} = b_B^A d_X^B + d_B^A b_X^B \\ u_X^{A:B} = u_B^A + (b_B^A + d_B^A) u_X^B \\ a_X^{A:B} = a_X^B \end{cases}$$

The operator $\otimes_2$ takes into account that B may recommend the opposite of his real opinion about the truth value of the third participant. Besides discounting B's belief (b_X^B) and disbelief (d_X^B) by the extent A believes B (b_B^A), we can further deduce A's belief and disbelief about X as the opposite of B's belief and disbelief about X to the extent A disbelieves B (d_B^A). This situation comprehensively covers all the four structure balance principles.

Definition 3. ***Concatenation***
Given three participants A, B and X as previously described, the concatenation operator discounted opinion can be denoted as $\omega_X^{A:B} = \omega_B^A \otimes_3 \omega_X^B = (b_X^{A:B}, d_X^{A:B}, u_X^{A:B}, a_X^{A:B})$ *that*

$$\begin{cases} b_X^{A:B} = b_B^A r_X^B / (b_B^A r_X^B + b_B^A s_X^B + 2) \\ d_X^{A:B} = b_B^A s_X^B / (b_B^A r_X^B + b_B^A s_X^B + 2) \\ u_X^{A:B} = 2 / (b_B^A r_X^B + b_B^A s_X^B + 2) \\ a_X^{A:B} = a_X^B \end{cases}$$

The operator $\otimes_3$ is similar to operator $\otimes_1$, which also corresponds to the structure balance principles 1) and 2). The difference is that it discounts B's observations on X (r_X^B and s_X^B) other than B's belief and disbelief on X (b_X^B and d_X^B) by A's belief on B (b_B^A).

Suppose that A has 18 observations about B ($r_B^A = 9$ and $s_B^A = 9$) and B also has 18 observations about X ($r_X^B = 2$

and $s_X^B = 16$), we can get $\omega_B^A = (0.45, 0.45, 0.1, 0.5)$ and $\omega_X^B = (0.1, 0.8, 0.1, 0.5)$. The deduced opinion $\omega_X^{A:B}$ by the previous three operators will be $(0.045, 0.36, 0.595, 0.5)$, $(0.405, 0.405, 0.19, 0.5)$ and $(0.089, 0.713, 0.198, 0.5)$ respectively. In this example, A's observations about B have high conflicts, while B's observations about X are overwhelmingly negative. The deduced opinions by $\otimes_1$ and $\otimes_3$ show that A tends to distrust X as B distrusts X ($b_X^{A:B} < d_X^{A:B}$, $b_X^B < d_X^B$). While, the deduced opinion by $\otimes_2$ shows that A also has conflict belief and disbelief about X as A has conflict belief and disbelief about B ($b_X^{A:B} = d_X^{A:B}$, $b_B^A = d_B^A$). Unlike operators $\otimes_1$ and $\otimes_3$ which just use A's belief about B for discounting, $\otimes_2$ additionally takes into account A's disbelief about B, which has a comprehensive utilization of structure balance principles. Thus, we will further study the characteristics of $\otimes_2$ and appropriately apply it to the trust inference path search.

4. Optimal Trust Inference Path Search

In this section, we firstly analyze the characteristics of balanced trust discounting operator and validate of the relationship between balanced transitive triad distribution and edge uncertainties. The purpose is to help find the trust inference path that goes through balanced triads and appropriately apply the balanced trust discounting operator to the path. Then, the optimal trust inference path search strategies MIRBS and MIFUS are detailed and followed by formal problem descriptions. Finally, we propose two OTIPS algorithm variants for the two strategies and analyze the computation complexity.

4.1 Preliminary Work

In order to predict the signed trust relationship with inferred indirect subjective opinion, we define the signum function that maps subjective opinion to edge sign:

Definition 4. ***Signum function for subjective opinion***

$$sgn(\omega) = \begin{cases} 1 & E(\omega) > s_{thresh} \\ -1 & E(\omega) < s_{thresh} \end{cases}$$

where the sign threshold s_{thresh} is set by experience to adjust the mapping. When $E(\omega) = s_{thresh}$, the sign is randomly picked from {1,-1}.

The expectation $E(\omega_B^A)$ ranges from $[0, 1]$ and it is intuitive to set the s_{thresh} to 0.5. If $E(\omega_B^A)$ approaches to 1, it means that A tends to expect B to behave positively and this edge sign is deduced as positive, and vice versa.

Definition 5. ***Balanced trust discounting operator***
Given $\omega_X^{A:B} = \omega_B^A \otimes \omega_X^B$, $E(\omega_B^A) \neq s_{thresh}$ and $E(\omega_X^B) \neq s_{thresh}$. If $sgn(\omega_X^{A:B}) = sgn(\omega_B^A) \cdot sgn(\omega_X^B)$, then $\otimes$ is called a balanced trust discounting operator for $sign(\omega)$.

This definition defines the trust discounting operators which follow the edge sign prediction cases t_1, t_2, t_5 and t_6 of balanced triads (Sect. 3.2), where the predicted edge sign equals to the product of the other two edge signs. $\otimes_2$ is a balanced trust discounting operator under specific premises.

Theorem 1. *Given $\omega_X^{A:B} = \omega_B^A \otimes_2 \omega_X^B$, if the sign threshold s_{thresh} and the base rates a_B^A,a_X^B are set to 0.5, then $\otimes_2$ is a balanced trust discounting operator for $sgn(\omega)$.*

Proof. See the Appendix □

Given a trust inference path connecting the unfamiliar source participant v_1 and target participant v_n and suppose that all triads along this path are balanced, we have two methods to predict the sign of the trust relationship between them. The first method is to infer it by the signs of the triangles with structure balance theory according to Fig. 3. The second one is to compute $sgn(\omega_n^{1:...:n-1})$ where $\omega_n^{1:...:n-1} = \omega_2^1 \otimes_2 \omega_3^2 \ldots \otimes_2 \omega_n^{n-1}$ under the premise that the sign threshold and the base rates are set to 0.5. According to Definition 5 and Theorem 1, the results obtained by the two methods are the same. We will use the latter method to predict the sign of the trust relations in this paper.

Definition 6. ***Opposite opinion operator***
Given the opinion $\omega_B^A = (b_B^A, d_B^A, u_B^A, a_B^A)$, the opposite opinion $\bar{\omega}_B^A$ is defined as:

$$\bar{\omega}_B^A = (d_B^A, b_B^A, u_B^A, 1 - a_B^A)$$

where $^-$ is the opposite opinion operator.

Theorem 2. *Given $\omega_X^{A:B} = \omega_B^A \otimes_2 \omega_X^B$, sign threshold $s_{thresh} =$ 0.5 and base rates $a_B^A = a_X^B = 0.5$, then $\omega_X^{A:B} = \bar{\omega}_B^A \otimes_2 \bar{\omega}_X^B$.*

The proof is obvious and not detailed. Given that two recommenders B and B' have opposite opinions of the target participant X on direct functional trust and the source participant A's direct opinions of B and B' on referral trust are also opposite, we can choose either path because the inferred opinions on the target participant by the two paths are the same according to Theorem 2, without having to consider A's beliefs about B or B' as common sense.

4.2 Distribution Characters of Balanced Transitive Triads

The directed triads in real social networks are not all balanced as previously assumed. Thus, it is important to study the distribution characters of balanced transitive triads to appropriately apply balanced trust discounting operator.

We argue that the existence of unbalanced triads is due to the lack of acquaintance between the participants. The certainty of subjective opinion (i.e. $1 - u$) corresponds to this acquaintance. So, we investigate the relationship between the distributions of the balanced transitive triads and the uncertainties of the triad edges. 20000 vertices are randomly picked from the Epinions data set in [16]. By taking each picked vertex as a transitive vertex, we record the uncertainties of the edges pointing to and from the transitive vertex, and count the numbers of the corresponding transitive triads and balanced transitive triads. Finally, the distribution of the number of transitive triads and the distribution

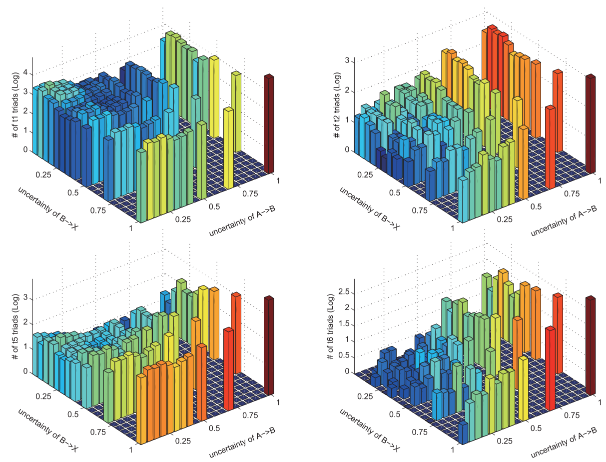

Fig. 4 Distribution of the number of transitive triads versus edge uncertainties for t_1, t_2, t_5 and t_6.

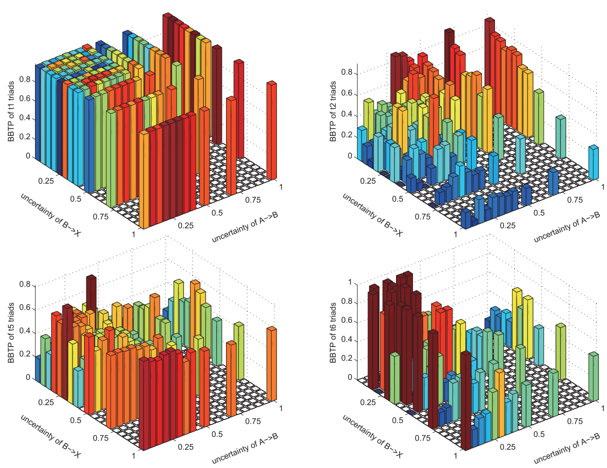

Fig. 5 Distribution of BTTP versus edge uncertainties for t_1, t_2, t_5 and t_6.

of Balanced Transitive Triad Percentage (BTTP, proportion of balanced transitive triads in total transitive triads) versus edge uncertainties are illustrated in Figs. 4, 5 and Table 1.

Figure 4 shows that, for all the four cases, there are less transitive triads when the edges linking the transitive vertex have less uncertainties. While in Fig. 5, the t_1 case shows that the occurrence probabilities of balanced triads are almost high, ignoring the uncertainties of the edges. For the rest three cases, the occurrence probability of balanced triads increases as the uncertainties of the edges decreases in general, especially obvious for t_6. For the t_2 and t_5, the uncertainties of negative edges have a more obvious influence on the occurrence probability of balanced triads than positive edges. In conclusion, uncertainties of edges linking the transitive vertex apparently affect the distribution of BTTP for t_1, t_2, t_5 and t_6.

The triads statistics in Table 1 show that the number of balanced triads are close to the number of unbalanced triads for t_2, t_5 and t_6 case triads. Without any priori knowledge, the trust discounting operators previously mentioned are facing the misusage of structure balance principles when dealing with t_2, t_5 and t_6 case triads, because the triad may be unbalanced at the probability nearly 50%. So, if we can find the trust inference path along which the uncertainties of the edges are as low as possible, the path will go through as many balanced triads as possible and the trust inference by trust discounting operations will be more accurate.

4.3 Trust Inference Path Search Strategies

There may be many trust propagation paths connecting the source participant and the target participant in the trust network. In order to find the optimal one for trust inference, we first need to consider what the optimal trust inference path should be and determine the search strategy.

4.3.1 Maximum Indirect Referral Belief Search

Trustworthy recommenders can provide reliable first-hand information about the target participant or reliable recommendation about other recommenders. In this strategy, from all the recommenders who have direct experience with the target participant, we find out the path which connects to the one whose inferred indirect opinion on referral trust has the maximum belief. Considering about the computation of belief, the operators $\otimes_1$ and $\otimes_3$ corresponding with t_1 and t_2 are applicable for path search and $\otimes_1$ is applied in this paper. It is worth mentioning that the picked recommender with the maximum referral belief may lack observations about the target. TidalTrust and CertProp(Sel.) share the similar strategy but perform different computation criteria.

4.3.2 Minimum Indirect Functional Uncertainty Search

Based on the structure balance theory, we can also get valuable information from the recommenders with high disbelief. Furthermore, the inferred functional trust with least uncertainty can provide as much knowledge about the target as possible. In this strategy, we try to find the path connecting to the target participant with the minimum indirect uncertainty on functional trust. Given the example at the end of Sect. 4.1, we would choose the path connecting to the one with greater belief on referral trust for MIRBS strategy. But the two paths are equally important for this strategy because the two paths are with the same uncertainty on functional trust. This strategy tries to minimize the indirect uncertainty along the trust inference path. In consideration of the relationship between balanced transitive triads and edge uncertainties, it will go through as many balanced transitive triads as possible. So, the balanced trust discounting operator $\otimes_2$ is applicable for path search with this strategy.

4.4 Problem Descriptions

Considering the directed graph $G = (V, E)$ described in Sect. 3, the source participant v_S and target participant v_T, we need to find the optimal trust inference path connecting

Table 1 Transitive triads statistics.

Triad Type	Number of Transitive Triads	Proportion of Transitive Triads	Number of Balanced Transitive Triads	Total BTTP
t_1	371,755	90.73%	367,537	98.87%
t_2	10,441	2.55%	5,085	48.7%
t_5	24,220	5.91%	14,179	58.54%
t_6	3,323	0.81%	1,471	44.27%
Total	409,739	100%	388,272	

them. Given a connecting path $path_i$ which can be noted as $[v_{(i,1)}, v_{(i,2)}, \ldots, v_{(i,n_i)}]$, where n_i is the number of vertices along $path_i$, $v_{(i,1)} = v_S$, $v_{(i,n_i)} = v_T$ and $1 \le i \le |path_i|$, the inferred v_S's opinion about v_T on functional trust would be $\omega_{(i,n_i)}^{(i,1):\ldots:(i,n_i-1)}$. The first element in the round bracket denotes the index of path and the second element denotes the index of vertex along this path.

For the MIRBS strategy, the optimal trust inference path among all the connecting paths is the path $path_k$ with the maximum indirect referral belief $b_{(k,n_k-1)}^{(k,1):\ldots:(k,n_k-2)}$. The problem can be formally described as to find the $path_k$ that

$$b_{(k,n_k-1)}^{(k,1):\ldots:(k,n_k-2)} = \max_{path_i}\{b_{(i,n_i-1)}^{(i,1):\ldots:(i,n_i-2)}\}$$

where $1 \le k \le |path_i|$ and $b_{(i,n_i-1)}^{(i,1):\ldots:(i,n_i-2)}$ can be obtained by $\omega_{(i,n_i-1)}^{(i,1):\ldots:(i,n_i-2)} = \omega_{(i,2)}^{(i,1)} \otimes_1 \omega_{(i,3)}^{(i,2)} \otimes_1 \ldots \otimes_1 \omega_{(i,n_i-1)}^{(i,n_i-2)}$ with all hops on referral trust.

For MIFUS, the optimal path is the path $path_k$ with the minimum indirect functional uncertainty $u_{(k,n_k)}^{(k,1):\ldots:(k,n_k-1)}$. The problem can be formally described as to find the $path_k$ that

$$u_{(k,n_k)}^{(k,1):\ldots:(k,n_k-1)} = \min_{path_i}\{u_{(i,n_i)}^{(i,1):\ldots:(i,n_i-1)}\}$$

where $1 \le k \le |path_i|$ and $u_{(i,n_i)}^{(i,1):\ldots:(i,n_i-1)}$ can be obtained by $\omega_{(i,n_i)}^{(i,1):\ldots:(i,n_i-1)} = \omega_{(i,2)}^{(i,1)} \otimes_2 \omega_{(i,3)}^{(i,2)} \otimes_2 \ldots \otimes_2 \omega_{(i,n_i)}^{(i,n_i-1)}$ with the last hop $\omega_{(i,n_i)}^{(i,n_i-1)}$ on functional trust and former hops on referral trust.

4.5 Optimal Trust Inference Path Search Algorithm

An intuitional approach to solve the above optimization problem is graph search. Unlike to the fundamental point-to-point shortest path problem (the P2P problem) [17], the arcs in trust networks are represented as subjective opinions with belief, disbelief and uncertainty and the trust transitivity is reflected by trust discounting operators. We will transfer the optimal trust inference path search problem to a P2P problem by appropriate modifications. Since bidirectional shortest path algorithms tend to scan fewer vertices than unidirectional ones [17], the idea of bidirectional search is applied to face the challenge of large-scaleness of online social network.

Suppose that the path in the forward search from v_S to arbitrary v_i is $[v_S, \ldots, v_i]$ and, for ease of description, noted as $[v_1, v_2, \ldots, v_m]$, $v_1 = v_S$ and $v_m = v_i$. Similarly, the path in the reverse search is $[v_i, \ldots, v_T]$, noted as $[v_1, v_2, \ldots, v_n]$, $v_1 = v_i$ and $v_n = v_T$. m and n are the numbers of the vertices along the paths. The distance functions for forward search and reverse search according to MIRBS strategy can be defined as:

$$d_f(v_i) = d_f(v_m) = \begin{cases} 1 & m = 1 \\ b_m^{1:\ldots:m-1} & else \end{cases}$$

$$d_r(v_i) = d_r(v_1) = \begin{cases} 1 & n \le 2 \\ b_{n-1}^{1:\ldots:n-2} & else \end{cases}$$

Obviously, $d_f(v_m) = d_f(v_{m-1}) \cdot b_m^{m-1}$ when $m > 1$.

The distance functions of forward search and reverse search for MIFUS strategy can be defined as:

$$d_f(v_i) = d_f(v_m) = \begin{cases} 0 & m = 1 \\ u_m^{1:\ldots:m-1} & else \end{cases}$$

$$d_r(v_i) = d_r(v_1) = \begin{cases} 0 & n = 1 \\ u_n^{1:\ldots:n-1} & else \end{cases}$$

Here, $d_f(v_m) = d_f(v_{m-1}) + u_m^{m-1} - d_f(v_{m-1}) \cdot u_m^{m-1}$ when $m > 1$.

Based on the bidirectional versions of Dijkstra's algorithm in [18], we transfer the path discovery problem to P2P problem by applying the operators $\otimes_1$ and $\otimes_2$ with focuses on the distance defined with referral belief and functional uncertainty and propose the OTIPS algorithms for the two strategies respectively. The proposed algorithms alternate between the forward and reverse scanning which start from v_S and v_T respectively, each maintaining its own set of distance labels. The algorithm for MIRBS maintains the optimal path found so far and the corresponding deduced referral belief μ (initialized as $\mu = 0$) and updates them once a better path is found. Here, a better path is a path with higher deduced referral belief. When an arc $e_{v_k \to v_m}$ being relaxed by the forward search and v_m has been labeled by the reverse search, if $\mu < d_f(v_m) \cdot d_r(v_m)$, the algorithm updates μ and set the middle vertex to v_m. Similar updates are also performed in the reverse search. When the possible deduced referral belief $d_f(v_f) \cdot d_r(v_r)$ is no greater than μ, the algorithm terminates and returns the optimal path accordingly, where v_f and v_r are the vertices with the maximum distance label in the forward and reverse search.

Similarly, the algorithm for MIFUS maintains functional uncertainty of the optimal path found so far as μ, which is initialized as $\mu = \infty$. When the arc $e_{v_k \to v_m}$ being relaxed, if $\mu > d_f(v_m) + d_r(v_m) - d_f(v_m) \cdot d_r(v_m)$, the algorithm updates μ and sets the middle vertex to v_m. When the possible deduced functional uncertainty $d_f(v_f) + d_r(v_r) - d_f(v_f) \cdot d_r(v_r)$ is no less than μ, the algorithm terminates and returns the optimal path accordingly, where v_f and v_r are with the minimum distance labels in the forward and reverse search.

Given space limitations, only the OTIPS algorithm for the MIFUS strategy is detailed in Fig. 6. Furthermore, the

Require: $G=(V,E), v_S, v_T$.
Ensure: $path=[v_S,\ldots,v_T]$ {if the path does not exist, it returns null}.

Set $scanned_f$ and $scanned_r$ to $\emptyset$, $labeled_f \leftarrow \{v_S\}$, $labeled_r \leftarrow \{v_T\}$
Set $d_f(v)$ and $d_r(v)$ to *Inf* for each $v \in V$ and $d_f(v_S) \leftarrow 0$, $d_r(v_T) \leftarrow 0$
Set $mid\_vetex \leftarrow Null$, $path \leftarrow Null$, $\mu \leftarrow Inf$
while $labeled_f \neq \emptyset$ **and** $labeled_r \neq \emptyset$ **do**
  find v_f and v_r that $d_f(v_f)=\min\{d_f(v_i)\}$ and $d_r(v_r)=\min\{d_r(v_j)\}$ where $v_i \in labeled_f$ and $v_j \in labeled_r$
  if $d_f(v_f)+d_r(v_r)-d_f(v_f)\cdot d_r(v_r) \geq \mu$ **then**
    break
  end if
  find$v_k$that $argmin\{\{d_f(v_i)\} \cup \{d_r(v_j)\}\}$ where $v_i \in labeled_f$, $v_j \in labeled_r$
  if $v_k \in labeled_f$ **then**
    remove v_k from $labeled_f$, add v_k to $scanned_f$
    for $v_m \in suc(v_k)-scanned_f$ **do**
      if $d_f(v_m)>d_f(v_k)+u_m^k-d_f(v_k)\cdot u_m^k$ **then**
        $d_f(v_m) \leftarrow d_f(v_k)+u_m^k-d_f(v_k)\cdot u_m^k$
        add v_m to $labeled_f$ and $parent_f(v_m) \leftarrow v_k$
      end if
      if $d_f(v_m)+d_r(v_m)-d_f(v_m)\cdot d_r(v_m)<\mu$ **then**
        $\mu \leftarrow d_f(v_m)+d_r(v_m)-d_f(v_m)\cdot d_r(v_m)$
        $mid\_vertex \leftarrow v_m$
      end if
    end for
  else
    remove v_k from $labeled_r$, add v_k to $scanned_r$
    for $v_m \in pre(v_k)-scanned_r$ **do**
      if $d_r(v_m)>d_r(v_k)+u_k^m-d_r(v_k)\cdot u_k^m$ **then**
        $d_r(v_m) \leftarrow d_r(v_k)+u_k^m-d_r(v_k)\cdot u_k^m$
        add v_m to $labeled_r$ and $parent_r(v_m) \leftarrow v_k$
      end if
      if $d_f(v_m)+d_r(v_m)-d_f(v_m)\cdot d_r(v_m)<\mu$ **then**
        $\mu \leftarrow d_f(v_m)+d_r(v_m)-d_f(v_m)\cdot d_r(v_m)$
        $mid\_vertex \leftarrow v_m$
      end if
    end for
  end if
end while
if $mid\_vetex \neq Null$ **then**
  assemble the path *path* by mid_vertex, $parent_f$ and $parent_r$
end if
return

Fig. 6 The OTIPS algorithm for MIFUS strategy.

restrictions on trust types are omitted to simplify the algorithm description. There may be multiple optimal paths with the same deduced referral belief or functional uncertainty, only the optimal path found first is returned.

In the worst case, the bidirectional Dijkstra's search algorithm degrades into the conventional unidirectional Dijkstra's search algorithm. If we note the branching factor as θ, the deduced referral belief as b and the minimum belief in edges as m, the hops of the found path would be $\log_m b$. Thus, the worst-case time complexity of the OTIPS algorithm for the MIRBS strategy with operator $\otimes_1$ is $O(\theta^{\log_m b})$. If we note the deduced functional uncertainty as u and the minimum uncertainty in edges as m, the hops of the found path would be $\log_{(1-m)}(1-u)$. Thus, the worst-case time complexity of the OTIPS algorithm for MIFUS with the operator $\otimes_2$ is $O(\theta^{\log_{(1-m)}(1-u)})$.

5. Theoretical and Empirical Validation

In this section, we theoretically prove the optimality of the path found by the proposed algorithms. Comparative experiments on trust inference path search, trust inference and trust inference based edge sign prediction are carried out with a large-scale OSN data set to demonstrate the superiority of OTIPS(MIRBS) and OTIPS(MIFUS) over TidalTrust, CertProp(Sel.) and H-SCAN on the efficiency of trust inference path search and applicability for trust inference.

5.1 Correctness Proof

The correctness of the proposed OTIPS algorithms for the two strategies is proofed in this subsection.

Lemma 1. *For each $v_X \in scanned_f$(or $v_X \in scanned_r$), the distance $d_f(v_X)$ (or $d_r(v_X)$) is the maximum indirect referral belief or minimum indirect functional uncertainty for all S-X (or X-T) paths.*

Proof. See the Appendix □

By Lemma 1, we can further get the following theorem.

Theorem 3. *The nonempty path returned by the proposed algorithm is with the maximum indirect referral belief for MIRBS strategy or minimum indirect functional uncertainty for MIFUS strategy among all S-T paths.*

Proof. See the Appendix □

5.2 Data Set Description

The Epinions data set (obtained from trustlet.org) describes the trust and distrust relationships (1 and −1 respectively) among users and their ratings ($rating \in \{1,2,3,4,5\}$) on other peoples' review articles about products. The data set contains about 132000 users who issued 841372 trust and distrust statements. The small-worldness of the Epinions trust network has been verified in [19], which has large clustering coefficient and short average path length. In order to testify the performance of each path search algorithm in the large-scale network, the data set is not further scaled down.

As the statistics shown in [14], the edge signs in Epinions are overwhelmingly positive. For edge sign prediction, a naive method that always predicts "trust" will incur a success rate of about 85% on randomly-picked samples. To avoid this situation, 1200 samples are randomly-picked with equal numbers of positive sign edges and negative ones.

5.3 Methodology, Metrics and Settings

First of all, we need to obtain peoples' subjective opinions on other people. Given the trustor v_A and trustee v_B, we investigate the v_A's ratings on all the articles written by the v_B, record the number of rating times as the sum of positive and negative observations (i.e. $t = r + s \geq 0$) and the mean rating (noted as $r_{mean} \in [1,5]$). Intuitively, r and s can be obtained by dividing the rating times according to the ratio $(r_{mean} - 1) : (5 - r_{mean})$. For example, given $t = 2$

and $r_{mean} = 3$, we get $r = s = 1$ and $b = d = 0.25$. In fact, $r_{mean} = 3$ may be not the appropriate watershed. So, we introduce the rating threshold r_{thresh} and map the intervals $[1, r_{thresh}]$ and $[r_{thresh}, 5]$ to $[1, 3]$ and $[3, 5]$ respectively. For arbitrary pair of people, trustor v_A and trustee v_B, the normalized subjective opinion $\omega_B^A = (b_B^A, d_B^A, u_B^A, a_B^A)$ can be obtained by r_{thresh},r_{mean} and t:

Case 1: $r_{mean} = r_{thresh}$

$$\begin{cases} b_B^A = (1 - u_B^A)/2 \\ d_B^A = (1 - u_B^A)/2 \\ u_B^A = 2/(2 + t) \\ a_B^A = 0.5 \end{cases}$$

Case 2: $r_{mean} > r_{thresh}$

$$\begin{cases} b_B^A = (1 - u_B^A) \cdot \left(1 + \frac{r_{mean} - r_{thresh}}{5 - r_{thresh}}\right)/2 \\ d_B^A = (1 - u_B^A) \cdot \left(\frac{5 - r_{mean}}{5 - r_{thresh}}\right)/2 \\ u_B^A = 2/(2 + t) \\ a_B^A = 0.5 \end{cases}$$

Case 3: $r_{mean} < r_{thresh}$

$$\begin{cases} b_B^A = (1 - u_B^A) \cdot \left(\frac{r_{mean} - 1}{r_{thresh} - 1}\right)/2 \\ d_B^A = (1 - u_B^A) \cdot \left(1 + \frac{r_{thresh} - r_{mean}}{r_{thresh} - 1}\right)/2 \\ u_B^A = 2/(2 + t) \\ a_B^A = 0.5 \end{cases}$$

where a_B^A is set to 0.5 to accord with intuition and exploit Theorem 1 and 2. When $r_{thresh} = 3$, this method is the same as the intuitive one. Considering the previous example with $r_{thresh} = 4$, the normalized opinion will be $\omega = (0.1875, 0.3125, 0.5, 0.5)$ and $b < d$.

In order to determine an appropriate r_{thresh} for normalized opinion mapping, we obtain users' opinions with different r_{thresh} and get the edge signs by the signum function. Then, the edge signs are compared with the original ones and the coincidence rates in terms of accuracy (ACC), true positive rate (TPR) and true negative rate (TNR) with the 1200 samples are illustrated in Fig. 7 by Eq. (1). TP, TN, P and N are the numbers of true positives, true negatives, actual positives and actual negatives respectively.

$$ACC = \frac{TP + TN}{P + N},\ TPR = \frac{TP}{P},\ TNR = \frac{TN}{N} \qquad (1)$$

Obviously, Fig. 7 is not symmetric and the watershed of positive and negative observations by r_{thresh} is much greater than 3. When $r_{thresh} = 4.04$, ACC reaches the maximum value 0.8675 and $TPR = 0.9417$, $TNR = 0.7933$. Thus, we set $r_{thresh} = 4.04$ in the comparative experiments to get the edge sign prediction as accurate as possible.

The standard evaluation technique Leave-one-out is used in trust inference path search. For an original trust statement that v_A puts on v_B, we first remove the edge linking v_A to v_B from the trust network and then try to find the

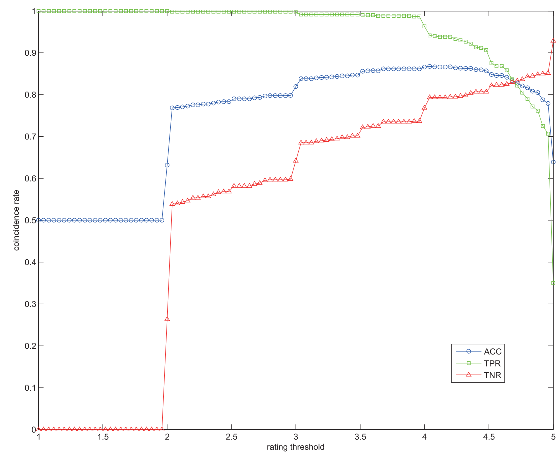

Fig. 7 Coincidence rates for different r_{thresh}.

optimal trust inference path connecting them with different trust path searching approaches. For all these approaches, only one eligible path is returned for comparison in this experiment. Referral trust and functional trust are not distinguishable in this data set. We set the maximum scanned vertices to 20000 and maximum hops to 7 (according to the small-world properties) for each path search to give a computation bound. In TidalTrust path search, r_{mean} is regarded as user-to-user rating for routing. In CertProp(Sel.), the Depth-First Search is applied to find all the possible paths for further path selection. In H-Scan, the number of expansion nodes K is set to $\lceil 20000/7 \rceil$ to reach the highest performance. For all the path searches with above approaches, the number of paths found, hops and scanned vertices for each path search are recorded for comparison. Since fetching data from database takes over most time consumption in the experiments, the number of scanned vertices instead of consumed time is used to show the computation complexity.

For trust inference, the deduced opinion ω' is obtained by trust discounting operations along the trust inference path and compared with the original opinion ω obtained by the normalized opinion mapping with ground truth. P-errors and B-errors introduced in [4] are used to validate the accuracy of trust inference with different trust discounting operators. Let $\omega = (b, d, u, a)$, $\omega' = (b', d', u', a')$ and the corresponding observations be (r, s) and (r', s'), the P-error and the B-error between ω and ω' are defined as:

$$P_{error}(\omega, \omega') = \left|\frac{r}{r + s} - \frac{r'}{r' + s'}\right|,\ B_{error}(\omega, \omega') = |b - b'|$$

Finally, the predicted edge sign of the trust relationship can be obtained by the deduced opinion ω' and signum function. Thus, the predicted edge sign can be compared with the original trust statement in terms of Receiver Operating Characteristics (ROCs) such as the proportions of TP, FN, TN and FP to validate the applicabilities of the paths for trust inference based edge sign prediction. FN and FP are the numbers of false negatives and false positives.

5.4 Results and Analysis

The experimental results on trust inference path search, trust inference and trust inference based sign prediction with the five trust inference path search approaches are detailed and analyzed as follows:

- Figure 8 shows that the proposed OTIPS(MIRBS) and OTIPS(MIFUS) can find the paths with the highest coverage rate (1185 out of 1200, 7.6% higher than TidalTrust's 1101) and much less mean scanned vertices per path (1016.51 and 1302.34, 67.5% and 58.4% less than TidalTrust's 3127.65) compared with other path search algorithms. CertProp(Sel.) exhaustively tries to find all the connecting paths within 7 depth and yields the lowest coverage rate and most mean scanned vertices per path, which demonstrates its deficiency for path search in the large-scale network.
- P-errors (P-errors1,2,3) and B-errors (B-errors1,2,3) of the trust inference corresponding with trust discounting operators $\otimes_1, \otimes_2$ and $\otimes_3$ are shown in Fig. 9 and Fig. 10. TidalTrust, OTIPS(MIRBS) and OTIPS(MIFUS) show much less P-errors than CertProp(Sel.) and H-Scan. The lowest P-errors of TidalTrust, OTIPS(MIRBS) and OTIPS(MIFUS) (0.1885, 0.1912 and 0.1883) are very close (difference less than 1.4%). $\otimes_3$ generally yields more B-error than the rest operators for its own defect. TidalTrust, OTIPS(MIRBS) and OTIPS(MIFUS) show much less B-errors than the rest approaches. The lowest B-errors of OTIPS(MIRBS) and OTIPS(MIFUS) reach 0.1915 and 0.1883, which are both less than TidalTrust's 0.2067 by 7.4% and 8.9% respectively.
- The experimental results of sign predictions with different trust discounting operators are given in Fig. 11. The TPR is higher than TNR for each algorithm. The main reason is that positive edges and t_1 type balanced triads dominate in the trust network and the path search tends to find trusting recommenders. Edge sign prediction with CerProp(Sel.) shows much lower TPR than other algorithms. H-Scan tends to find much more positive samples ($TP+FN$) than negative ones ($TN+FP$) and the TPR is also lower than that of other algorithms. These two approaches have obvious shortages. TidalTrust, OTIPS(MIRBS) and OTIPS(MIFUS) show similar ROCs, better than CertProp(Sel.) and H-Scan. The highest ACCs of the three approaches reach 71%, 70% and 70%. Moreover, referring to the numbers of found paths (i.e. sum of TP, FP, TN and FN) in Fig. 8, OTIPS(MIRBS) and OTIPS(MIFUS) can obtain more determinate predictions than TidalTrust by 7.6%.

In summary, CertProp(Sel.) and H-Scan show obvious deficiencies for the three aspects, compared with the rest approaches. Comparing OTIPS(MIRBS) and OTIPS(MIFUS) with TidalTrust, we find that the proposed approaches have better performance in trust inference search (higher coverage rate by 7.6% and much lower computation complexity by 67.5% and 58.4%). Because bidirectional search approaches tend to perform better than unidirectional search approaches. For trust inference, the lowest P-errors of the proposed approaches are close to that of TidalTrust, while the B-errors are lower by 7.4% and 8.9%. In the trust inference based sign predictions, the proposed approaches have close ROCs to TidalTrust but higher cardinal number by 7.6%. The superiority of the proposed approaches derives from the global optimality among all possible connecting paths, while TidalTrust just finds the local optimal path among the connecting paths with the least hops.

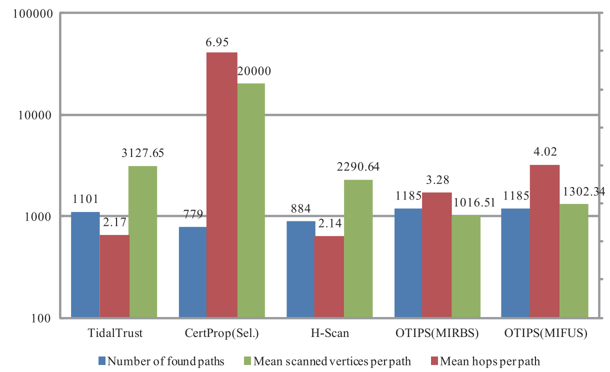

Fig. 8 Experimental results of path searches.

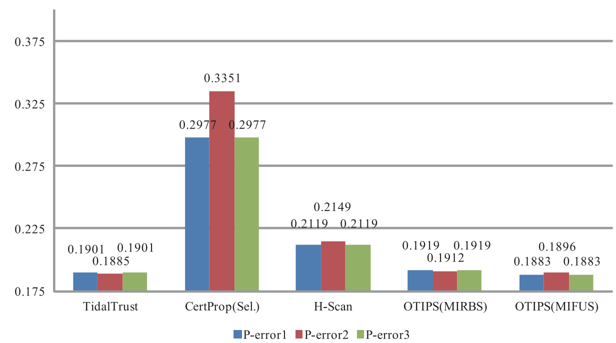

Fig. 9 Mean P-errors of trust inferences.

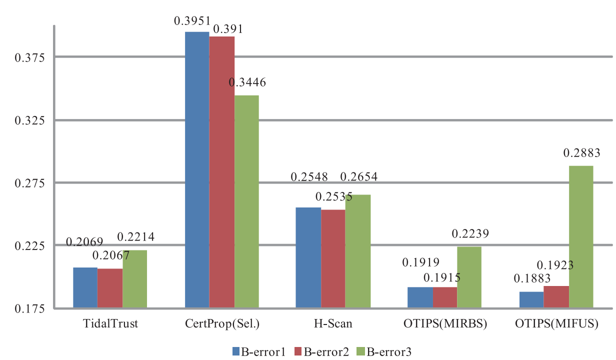

Fig. 10 Mean B-errors of trust inferences.

6. Conclusions and Future Work

In this paper, we first investigate the characteristics of trust

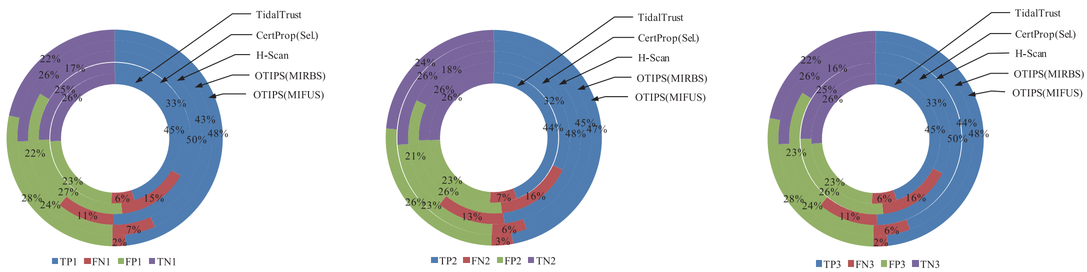

Fig. 11 Experimental results of edge sign predictions.

discounting operators and balanced transitive triads distribution, then formally boil down the trust inference path searches with the proposed MIRBS and MIFUS strategies to optimization problem. The corresponding OTIPS algorithms are finally proposed and followed by theoretical and empirical validations. The proposed OTIPS algorithms for MIRBS and MIFUS strategies can efficiently find the optimal trust inference path with better applicability to trust inference.

Although the paths found by OTIPS(MIRBS) and OTIPS(MIFUS) are with greater mean hops per path in Fig. 6 than those found by TidalTrust (which are with the minimum hops), they can also even reach more accurate results, which breaks the common sense that the shorter the trust inference path is and the more accurate the trust inference will be. The experiments with OTIPS(MIFUS) validate the effectiveness of minimizing the uncertainty of the inferred opinion. Since t_2, t_5 and t_6 case triads only hold less than 10% of the total triads in this data set, the superiority of MIFUS on this data set is limited. Moreover, only single path is selected in this paper to figure out what is the optimal search strategy for trust inference path search. For future work, we are going to focus on the k-optimal trust inference path search problem and fuse the indirect opinions along theses paths to further reduce the uncertainty in trust inference and improve the accuracy.

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Appendix: Proofs for lemmas and theorems

Proof for Theorem1

Proof. $\because s_{thresh} = a_B^A = a_X^B = 0.5 \therefore$ we can derive:

$$E(\omega_B^A) - s_{thresh} = 0.5(b_B^A - d_B^A) \quad (A\cdot 1)$$
$$E(\omega_X^B) - s_{thresh} = 0.5(b_X^B - d_X^B) \quad (A\cdot 2)$$

$\because \omega_X^{A:B} = \omega_B^A \otimes_2 \omega_X^B \therefore$ by the definition of $\otimes_2$ we can get:

$$E(\omega_X^{A:B}) - s_{thresh} = 0.5(b_B^A - d_B^A)(b_X^B - d_X^B) \quad (A\cdot 3)$$

When $sgn(\omega_B^A) = 1$ and $sgn(\omega_X^B) = 1$, we can get $b_B^A - d_B^A > 0$ and $b_B^A - d_B^A > 0$ by Eqs. (A· 1) and (A· 2), $sgn(\omega_X^{A:B}) = 1$ by Eq. (A· 3). It is similar for the rest cases. In conclusion, $sgn(\omega_X^{A:B}) = sgn(\omega_B^A) \cdot sgn(\omega_X^B)$ □

Proof for Lemma1

Proof. The forward search is similar to the reverse search. The difference is that only the last hop in the reverse search is on functional trust and $b_{(i,n_i-1)}^{(i,n_i-2)}$ is set to be 1 for MIRBS strategy to mask the effect of functional trust. Thus, we only discuss the forward search to save space.
Base case: $|scanned_f| = 1$ is trivial.
Inductive hypothesis: Assume true for $|scanned_f| \geq 1$. Given the source vertex v_S and the vertex to be scanned v_Y, where v_Y is the labeled vertex with the maximum distance for MIRBS or minimum distance for MIFUS in $labeled_f$ (the path composed of predecessors can be noted as $[v_S, \ldots, v_X, v_Y]$). There is another path connecting to v_Y and $v_{B'_1}$ is the first vertex that leaves $scanned_f$, which can be noted as $[v_S, v_{A'_1}, \ldots, v_{A'_m}, v_{B'_1}, \ldots, v_{B'_n}, v_Y]$.

Since $b \in [0, 1)$ and $u \in (0, 1]$, for MIRBS with $\otimes_1$:
$b_Y^{S:A'_1:\ldots:A'_m:B'_1:\ldots:B'_n} = b_{B'_1}^{S:A'_1:\ldots:A'_m} b_Y^{B'_1:\ldots:B'_n} \leq b_{B'_1}^{S:A'_1:\ldots:A'_m}$

$\because b_{A'_m}^{S:A'_1:\ldots:A'_{m-1}} \leq d_f(v_{A'_m})$ according to the assumption;
$\therefore b_{B'_1}^{S:A'_1:\ldots:A'_m} = b_{A'_m}^{S:A'_1:\ldots:A'_{m-1}} b_{B'_1}^{A'_m} \leq d_f(v_{A'_m}) \cdot b_{B'_1}^{A'_m} = d_f(v_{B'_1})$.

$\because v_Y$ is the labeled vertex with the maximum distance, $d_f(v_{B'_1}) \leq d_f(v_Y)$; $\therefore b_Y^{S:A'_1:\ldots:A'_m:B'_1:\ldots:B'_n} \leq d_f(v_Y)$.

For the strategy MIFUS with $\otimes_2$, we get:

$$u_Y^{S:A'_1:\ldots:A'_m:B'_1:\ldots:B'_n} = u_{B'_1}^{S:A'_1:\ldots:A'_m} + (b_{B'_1}^{S:A'_1:\ldots:A'_m} + d_{B'_1}^{S:A'_1:\ldots:A'_m}) u_Y^{B'_1:\ldots:B'_n} \geq u_{B'_1}^{S:A'_1:\ldots:A'_m}$$

$\because u_{A'_m}^{S:A'_1:\ldots:A'_{m-1}} \geq d_f(A'_m)$ according to the assumption.

$$\therefore u_{B'_1}^{S:A'_1:\ldots:A'_m} = u_{A'_m}^{S:A'_1:\ldots:A'_{m-1}} + u_{B'_1}^{A'_m} - u_{A'_m}^{S:A'_1:\ldots:A'_{m-1}} u_{B'_1}^{A'_m}$$
$$\geq d_f(v_{A'_m}) + u_{B'_1}^{A'_m} - d_f(v_{A'_m}) u_{B'_1}^{A'_m} = d_f(v_{B'_1})$$

$\because v_Y$ is the labeled vertex with the minimum distance, $d_f(v_{B'_1}) \geq d_f(v_Y)$; $\therefore u_Y^{S:A'_1:\ldots:A'_m:B'_1:\ldots:B'_n} \geq d_f(v_Y)$. □

Proof for Theorem3

Proof. When the algorithm is to terminate, the best vertices with the maximum distance for MIRBS or minimum distance for MIFUS in the forward and reverse searches are noted as v_F and v_R, and the optimal path found is with length μ. If there is a path $[v_S, v_{A'_1}, \ldots, v_{A'_m}, v_{B'_1}, \ldots, v_{B'_n}, v_{C'_1}, \ldots, v_{C'_l}, v_T]$ which has better length μ' than μ, where $v_{B'_1}$ is the first vertex leaves the forward scanned vertices and $v_{B'_n}$ is the first vertex leaves the reverse scanned vertices. m, n, l are the numbers of the vertices along the corresponding subpaths.

For MIRBS with $\otimes_1$: According to Lemma 1, we get $b_{B'_1}^{S:A'_1:\ldots:A'_m} \leq d_f(v_{B'_1})$ and $b_T^{B'_n:C'_1:\ldots:C'_l} \leq d_r(v_{B'_n})$. Since v_F and v_R are the labeled vertices with maximum distances, $d_f(v_{B'_1}) \leq d_f(v_F)$ and $d_r(v_{B'_n}) \leq d_r(v_R)$. Thus, $b_{B'_1}^{S:A'_1:\ldots:A'_m} \leq d_f(v_F)$ and $b_T^{B'_n:C'_1:\ldots:C'_l} \leq d_r(v_R)$.

$$\because \mu' = b_T^{S:A'_1:\ldots:A'_m:B'_1:\ldots:B'_n:C'_1:\ldots:C'_l}$$
$$= b_{B'_1}^{S:A'_1:\ldots:A'_m} b_{B'_n}^{B'_1:\ldots:B'_{n-1}} b_T^{B'_n:C'_1\ldots:C'_l} \leq b_{B'_1}^{S:A'_1:\ldots:A'_m} b_T^{B'_n:C'_1\ldots:C'_l}$$

and $d_f(v_F) \cdot d_r(v_R) \leq \mu$.
$\therefore b_{B'_1}^{S:A'_1:\ldots:A'_m} b_T^{B'_n:C'_1\ldots:C'_l} \leq d_f(v_F) \cdot d_r(v_R) \leq \mu$
Thus, $\mu' \leq \mu$ and it leads to contradiction.

For MIFUS with $\otimes_2$: According to Lemma 1, we get $u_{B'_1}^{S:A'_1:\ldots:A'_m} \geq d_f(v_{B'_1})$ and $u_T^{B'_n:C'_1:\ldots:C'_l} \geq d_r(v_{B'_n})$. Since v_F and v_R are the labeled vertices with minimum distances, $d_f(v_{B'_1}) \geq d_f(v_F)$ and $d_r(v_{B'_n}) \geq d_r(v_R)$. Thus, $u_{B'_1}^{S:A'_1:\ldots:A'_m} \geq d_f(v_F)$ and $u_T^{B'_n:C'_1:\ldots:C'_l} \geq d_r(v_R)$.

$$\because \mu' = u_T^{S:A'_1:\ldots:A'_m:B'_1:\ldots:B'_n:C'_1:\ldots:C'_l}$$
$$= u_{B'_n}^{S:A'_1:\ldots:A'_m:B'_1:\ldots:B'_{n-1}} + u_T^{B'_n:C'_1\ldots:C'_l} - u_{B'_n}^{S:A'_1:\ldots:A'_m:B'_1:\ldots:B'_{n-1}} u_T^{B'_n:C'_1\ldots:C'_l} \quad (A\cdot 4)$$

$$u_{B'_n}^{S:A'_1:\ldots:A'_m:B'_1:\ldots:B'_{n-1}} = u_{A'_m}^{S:A'_1:\ldots:A'_{m-1}} + (b_{A'_m}^{S:A'_1:\ldots:A'_{m-1}} + d_{A'_m}^{S:A'_1:\ldots:A'_{m-1}}) \cdot u_{B'_n}^{A'_m:B'_1:\ldots:B'_{n-1}} \geq u_{A'_m}^{S:A'_1:\ldots:A'_{m-1}} \quad (A\cdot 5)$$

$\therefore$ By Eqs. (A· 4) and (A· 5) and $u \in (0, 1]$, we get:

$$\mu' \geq u_{B'_1}^{S:A'_1:\ldots:A'_m} + u_T^{B'_n:C'_1\ldots:C'_l} - u_{B'_1}^{S:A'_1:\ldots:A'_m} u_T^{B'_n:C'_1\ldots:C'_l}$$

$\because u_{B'_1}^{S:A'_1:\ldots:A'_m} \geq d_f(v_F)$, $u_T^{B'_n:C'_1:\ldots:C'_l} \geq d_r(v_R)$ and

$$d_f(v_F) + d_r(v_R) - d_f(v_F) \cdot d_r(v_R) \geq \mu$$

$$\therefore u_{B'_1}^{S:A'_1:\ldots:A'_m} + u_T^{B'_n:C'_1\ldots:C'_l} - u_{B'_1}^{S:A'_1:\ldots:A'_m} u_T^{B'_n:C'_1\ldots:C'_l} \geq d_f(v_F) + d_r(v_R) - d_f(v_F) \cdot d_r(v_R) \geq \mu$$

Thus, $\mu' \geq \mu$ and it leads to contradiction.

In conclusion, the path returned by the proposed algorithm is with the maximum indirect referral belief for MIRBS strategy or minimum indirect functional uncertainty for MIFUS strategy among all S-T paths. □

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